Vacuum Interpolation in Supergravity via Super p-branes

G. W. Gibbons and P. K. Townsend

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge, United Kingdom

(Received 7 July 1993)

We show that many of the recently proposed supersymmetric p-brane solutions of $d=10$ and $d=11$ supergravity have the property that they interpolate between Minkowski spacetime and a compactified spacetime, both being supersymmetric supergravity vacua. Our results imply that the effective world-volume action for small fluctuations of the super p-brane is a supersingleton field theory for $(p+2)$-dimensional anti–de Sitter spacetime, as has been often conjectured in the past.

PACS numbers: 11.17.+y, 04.65.+e, 11.10.Kk

It is possible that particle physics in our four-dimensional ($d=4$) Universe may ultimately be well described by some compactification of a ten-dimensional ($d=10$) supergravity theory that serves as the effective field theory of a $d=10$ superstring theory. Even if superstring theory meets with complete success in this respect there will remain the question of why the Universe "chooses" to compactify six dimensions in a particular way and, indeed, why it chooses to compactify any of them since $d=10$ Minkowski spacetime ($\mathcal{M}_{10}$) is as good a vacuum solution as any other from a purely mathematical point of view. In contrast to solutions of simple flat space field theories there is no way to compare the energies of different compactifications and thus determine "the" vacuum by finding the one of lowest energy. In these circumstances it might be supposed that the choice of compactification must be left to some theory of initial conditions. An alternative is that *all possible* compactifications are already to be found in different spatial regions of a single (presumably ten-dimensional) universe. The particular region in which we find ourselves must then be decided by chance and/or anthropic considerations. Ideas along these lines, but within the context of a four-dimensional universe, have been suggested previously by Linde [1], and the possibility of an interpolation between different compactifications of $d=11$ supergravity was suggested by van Baal, Bais, and van Nieuwenhuizen in their work on "local compactification" [2].

A clue to progress is this direction is provided by consideration of the extreme Reissner-Nordstrom (RN) black hole as a solution of $N=2$, $d=4$ supergravity. This solution interpolates between four-dimensional Minkowski spacetime ($\mathcal{M}_4$), at spatial infinity, and $(\mathrm{adS})_2\times S^2$, down an infinite wormhole throat [3]. Both asymptotic spacetimes are maximally supersymmetric "vacua" of $N=2$ supergravity. We shall show here that many of the recently discussed extreme black p-brane solutions of $d=10$ [4–7] and $d=11$ [8,9] supergravity also interpolate between supersymmetric (although not always maximally supersymmetric) vacua. The cases that most closely resemble the RN prototype are (i) $d=11$ mem-brane ($p=2$), (ii) $d=11$ five-brane ($p=5$), and (iii) $d=10$ IIB self-dual three-brane ($p=3$).

For these cases the p-brane interpolates between $\mathcal{M}_d$ and $(\mathrm{adS})_{p+2}\times S^{d-p-2}$. The latter spacetimes are known to be maximally supersymmetric solutions (for the appropriate value of p) of the respective supergravity theories [10–12]. Like the extreme RN black hole, these p-brane solutions are nonsingular and break only half the supersymmetry; they may therefore be regarded as examples of "supersymmetric extended solitons."

An example that does not quite fit the above pattern is the (iv) $d=10$ five-brane. We shall show that this solution interpolates between $\mathcal{M}_{10}$ and $\mathcal{M}_7\times S^3$. This is something of a surprise since no compactification of $d=10$ supergravity to $\mathcal{M}_7$ on S^3 has been previously described. As we shall see, the explanation lies in the asymptotic behavior of the dilaton field down the wormhole throat; rather than approach a constant, as it does for cases (i)–(iii) above, it approaches a linear function of the inertial coordinates of $\mathcal{M}_7$ (this behavior is similar to certain $d=4$ "dilaton black holes" [13]). We shall show that there is indeed such a compactification of $d=10$ supergravity but it is not maximally supersymmetric because, like the five-brane solution itself [14], it breaks half the supersymmetry. That is, unlike cases (i)–(iii), the full supersymmetry is not restored in *both* asymptotic regions for case (iv). Nevertheless, the five-brane solution is nonsingular and supersymmetric, so it too can be regarded as a supersymmetric extended soliton.

A further implication of our work concerns the nature of the effective world-volume action for the p-brane solutions of cases (i)–(iii) in which there is an interpolation between Minkowski spacetime and a lower-dimensional anti–de Sitter (adS) spacetime. Far down the wormhole throat we have a supergravity theory compactified on a $(d-p-2)$-sphere to adS_{p+2}. It is known from studies of these compactifications that the fields of a singleton supermultiplet of the adS supergroup appear as coefficients in the harmonic expansion of the d-dimensional fields on the $(d-p-2)$-sphere, but that these can be gauged away everywhere except at the boundary of adS [15]. This result is in accord with the currently accepted field theoretic interpretation of singleton irreducible representation of adS groups, i.e., that they are what we would now call topological field theories in that all physical degrees of freedom reside on the boundary [16–18]. In the current context the boundary is just the opening of the wormhole throat, which is perceived from the exterior as the p-brane core. We therefore conclude that the world-volume fields of the effective p-brane action should be those of the appropriate adS singleton supermultiplet. These are as follows [19]: (i) three-dimensional $N=8$ scalar supermultiplet, (ii) six-dimensional $N=2$ antisymmetric tensor multiplet, and (iii) four-dimensional $N=4$ Maxwell supermultiplet.

In cases (i) and (iii) it is known that these are indeed the world-volume fields by an analysis of the fluctuations about the p-brane solution [5,20]. In case (ii) one can deduce the world-volume fields from the fact that they must include five Goldstone scalars associated with the breakdown of translation invariance at the five-brane's world volume in the five "transverse" directions and the fact that these fields must appear in an $N=4$ world-volume supermultiplet, because this corresponds to the correct number of supersymmetries left unbroken by the solution. The unique six-dimensional $N=4$ supermultiplet with five scalars is the antisymmetric tensor multiplet [21]. Note that the fermions of this multiplet transform as a 4-plet of $USp(4)\sim Spin(5)$, as expected since an $SO(5)$ group of rotations in the five-dimensional transverse space is left unbroken by the five-brane solution.

It has been suggested at various times in the past that effective actions for super p-branes might be considered as supersingleton field theories associated with the appropriate supergroup extension of the $(adS)_{p+2}$ group [19,22,23]. Since this group acts as a conformal supergroup on the boundary and since the superstring action has worldsheet conformal invariance (in the "conformal" gauge), this proposal is most natural for super-strings. However, it is known that the effective action for *small* (i.e., to quadratic order) fluctuations of a membrane at the boundary of adS space is also conformally invariant [19,20]. Our results provide significant further evidence for the connection between supersingleton field theories and super p-branes.

All of the metrics to be considered here have the form

$$ds_d^2=-\left[1-\left(\frac{r_+}{r}\right)^{(d-p-3)}\right]\left[1-\left(\frac{r_-}{r}\right)^{(d-p-3)}\right]^{\gamma_x-1}dt^2+\left[1-\left(\frac{r_-}{r}\right)^{(d-p-3)}\right]^{\gamma_x}d\mathbf{x}\cdot d\mathbf{x}$$
$$+\left[1-\left(\frac{r_+}{r}\right)^{(d-p-3)}\right]^{-1}\left[1-\left(\frac{r_-}{r}\right)^{(d-p-3)}\right]^{\gamma_r}dr^2+r^2\left[1-\left(\frac{r_-}{r}\right)^{(d-p-3)}\right]^{\gamma_r+1}d\Omega^2_{(d-p-2)} \quad (1)$$

for constants γ_x and γ_r, and where $d\Omega^2_{(d-p-2)}$ is the "round" metric on the $(d-p-2)$-sphere (although it is straightforward to extend the results we will obtain below to the case in which it is any Einstein metric on $S^{(d-p-2)}$). These metrics are asymptotic to the flat metric on d-dimensional Minkowski space, $\mathcal{M}_d$, as $r\to\infty$. They have an outer horizon at $r=r_+$ and an inner horizon at $r=r_-$, in close analogy with the RN black hole solution of $d=4$ Einstein-Maxwell theory, which is in fact a special case of the above metric with $d=4$, $p=0$, $\gamma_r=-1$, and $\gamma_x=2$. When there is a dilaton field ϕ, it will take the form (in the solutions of interest here)

$$e^{-2\phi}=\left[1-\left(\frac{r_-}{r}\right)^{(d-p-3)}\right]^{\gamma_\phi} \quad (2)$$

for some constant γ_ϕ. We are principally interested in the extreme case for which $r_+=r_-=a$. In this case the metric is

$$ds_d^2=\left[1-\left(\frac{a}{r}\right)^{(d-p-3)}\right]^{\gamma_x}[-dt^2+d\mathbf{x}\cdot d\mathbf{x}]+\left[1-\left(\frac{a}{r}\right)^{(d-p-3)}\right]^{\gamma_r-1}dr^2+r^2\left[1-\left(\frac{a}{r}\right)^{(d-p-3)}\right]^{\gamma_r+1}d\Omega^2_{d-p-2} \quad (3)$$

which can be interpreted as the metric of a flat, static, and infinite p-brane. However, only if $\gamma_r=-1$ is this metric nonsingular at $r=a$, so only in this case can it be considered to represent an "extended soliton."

Fortunately, γ_r does equal -1 for the known p-brane solutions of $d=11$ supergravity, cases (i) and (ii) above. This is not true of $d=10$ supergravity but here it should be remembered that in the presence of a dilaton there is an intrinsic ambiguity in the metric because a new, albeit conformally equivalent, metric can be obtained by rescaling the old one by any positive function of the dilaton, e.g., a power of e^ϕ. The values of the indices γ_x and γ_r used here correspond to the metric for which the Lagrangian takes the form $\mathcal{L}=\sqrt{-g}e^{-2\phi}R+\cdots$. This form is appropriate to the interpretation of solutions of $d=10$ supergravity theory as approximate solutions of string theory. For cases (iii) and (iv) above, $\gamma_r=-1$ for this choice of metric so these solutions can be considered as nonsingular within the context of string theory; we shall comment briefly on the other p-brane solutions of $d=10$ supergravity below. The remaining exponents for the cases (i)–(iv) studied here are (i) $\gamma_x=\frac{2}{3}$, (ii) $\gamma_x=\frac{1}{3}$, (iii) $\gamma_x=\frac{1}{2}$ and $\gamma_\phi=0$, and (iv) $\gamma_x=0$ and $\gamma_\phi=1$.

To examine the asymptotic behavior as $r\to a$ we define the new variable λ by $a\lambda=(d-p-3)(r-a)$, in terms of which the metric (3) becomes

$$ds^2=\left[\lambda^{\gamma_x}[-dt^2+d\mathbf{x}\cdot d\mathbf{x}]+\left(\frac{a}{(d-p-3)\lambda}\right)^2 d\lambda^2+a^2 d\Omega^2_{d-p-2}\right][1+O(\lambda)]\,. \tag{4}$$

In the $r\rightarrow a$ limit we can neglect the $O(\lambda)$ terms. Then, defining $\rho=a\ln\lambda/(d-p-3)$, we obtain the asymptotic metric

$$ds^2\sim e^{a^{-1}(d-p-3)\gamma_x\rho}[-dt^2+d\mathbf{x}\cdot d\mathbf{x}] +d\rho^2+a^2 d\Omega^2_{d-p-2} \tag{5}$$

which is the metric of $(\mathrm{adS})_{p+2}\times S^{d-p-2}$ if $\gamma_x\neq 0$, as happens for cases (i)–(iii). In these cases the p-brane solution interpolates between $\mathcal{M}_d$ and, respectively, (i) $(\mathrm{adS})_4\times S^7$, (ii) $(\mathrm{adS})_7\times S^4$, and (iii) $(\mathrm{adS})_5\times S^5$. These are all known compactifications of $d=11$ and $d=10$ supergravity, preserving all the supersymmetry.

The above three cases are all closely analogous to the $d=4$ extreme RN black hole, but this analogy is closest for the self-dual three-brane as we now explain. The extreme RN black hole has a conformal isometry that exchanges the two asymptotic regions [24]. The generalization to p-brane solutions of the form (1) with $\gamma_r=-1$ involves the consideration of a new radial coordinate $\tilde{r}$ given by

$$r^{(d-p-3)}-a^{(d-p-3)}=\frac{a^{2(d-p-3)}}{\tilde{r}^{(d-p-3)}-a^{(d-p-3)}}\,. \tag{6}$$

The new metric is then conformal to the original one if $\gamma_x=2/(d-p-3)$. This condition is satisfied by the $d=4$ extreme RN solution. Of the above three p-brane solutions it is satisfied only by the self-dual three-brane.

For the $d=10$ five-brane $\gamma_x=0$ so we have instead an interpolation between $\mathcal{M}_{10}$ and (iv) $\mathcal{M}_7\times S^3$; and, since $\gamma_\phi\neq 0$, a dilaton that does not approach a constant down the S^3 throat but, instead, has the asymptotic behavior

$$\phi\sim-\frac{(d-p-3)\gamma_\phi}{a}\rho\,, \tag{7}$$

i.e., a dilaton that is linear in the inertial coordinates of $\mathcal{M}_7$. Such a compactification of $d=10$ supergravity has not been previously described in the literature but our results imply its existence. To verify this we shall need the bosonic action of $d=10$ supergravity

$$S=\int d^{10}x\sqrt{-g}\,e^{-2\phi}[R+4(\partial\phi)^2-\tfrac{1}{12}H^2]\,, \tag{8}$$

where H is a three-form field strength. The field equations are

$$\begin{aligned}
0&=R_{MN}-\tfrac{1}{4}H_{MPQ}H_N{}^{PQ}+2\nabla_M\partial_N\phi\,,\\
0&=\nabla_M(\sqrt{-g}\,e^{-2\phi}H^{MNP})\,,\\
0&=4(\partial\phi)^2-4\nabla^2\phi-R+\tfrac{1}{12}H_{MNP}H^{MNP}\,.
\end{aligned} \tag{9}$$

We now split the coordinates x^M into two sets, x^μ ($\mu=0,1,\ldots,6$) for $\mathcal{M}_7$ and y^m ($m=1,2,3$) for a compact 3-space, and we make the ansatz

$$H_{mnp}=ke_m{}^a e_n{}^b e_p{}^c\varepsilon_{abc},\quad \phi=\tfrac{1}{2}k'n\cdot x\,, \tag{10}$$

where $e_m{}^a(y)$ is the dreibein for the 3-space, k and k' are constants, and n is a unit spacelike 7-vector. The Einstein equation now yields

$$R_{ab}=\tfrac{1}{2}k^2\delta_{ab} \tag{11}$$

which implies that the 3-space is S^3 with inverse radius k. The antisymmetric tensor equation is trivially satisfied while the ϕ equation is satisfied if $k'=k$. Hence $\mathcal{M}_7\times S^3$ *with a linear dilaton* is a solution.

We remark that a similar compactification to $\mathcal{M}_4$ on $S^3\times S^3$ also exists provided the two three-spheres have the same radius. Such a compactification was considered previously [25], with a different ansatz for the dilaton field ϕ, but the solution found there was unacceptable [26] (because singular in ϕ). In fact, in [26] a "no-go" theorem was proved, under certain premises, that rules out such compactifications. The solution found here evades this theorem because the linear dilaton was excluded by the premises of the theorem.

To determine how many supersymmetries are preserved by the S^3 compactification we need the fermion supersymmetry transformation laws in a bosonic background. These are

$$\begin{aligned}
\delta\psi_M&=\nabla_M(\omega^-)\epsilon\equiv\partial_m\epsilon+\omega^-_{mAB}\Gamma^{AB}\epsilon\,,\\
\delta\lambda&=-\frac{\sqrt{2}}{4}(\Gamma^M\partial_M\phi=\tfrac{1}{12}\Gamma^{MNP}H_{MNP})\epsilon\,,
\end{aligned} \tag{12}$$

where ω^- is the connection with torsion $\omega^-=\omega-\frac{1}{2}H$ (we use here the results of [27] with $\omega\rightarrow-\omega$ and a rescaled H). We have $\delta\psi_M=0$ for our solution because $-\frac{1}{2}H$ is the parallelizing torsion for the 3-sphere, while $\delta\lambda=0$ implies, using $k'=k$, that

$$\tfrac{1}{2}n\cdot\Gamma\epsilon-\tfrac{1}{12}\Gamma^{abc}H_{abc}\epsilon=0\,. \tag{13}$$

Since ϵ is chiral, $\Gamma^{abc}\epsilon=\varepsilon^{abc}\gamma_7\epsilon$ where $\gamma_7=\Gamma^0\Gamma^1\cdots\Gamma^6$, which satisfies $(\gamma_7)^2=1$. Hence (13) implies that

$$\gamma_7 n\cdot\Gamma\epsilon=\epsilon\,. \tag{14}$$

The matrix $\gamma_7 n\cdot\Gamma$ squares to unity and has zero trace, which means that of the sixteen possible linearly independent chiral $d=10$ spinors only eight linearly independent combinations satisfy (13). The solution therefore breaks half the supersymmetry. Thus, the $d=10$ five-brane, which itself breaks half the supersymmetry, does not interpolate between *maximally* supersymmetric vacua.

The remaining known p-brane solutions of $d=10$ supergravity are singular in the "string theory metric" used here since, for them, $\gamma_r\neq-1$. It appears, however, that in each case a new nonsingular metric can be obtained by rescaling by an appropriate power of e^{ϕ}. A case of particular interest is the extreme version of the string solution of Dabholkar *et al.* [28], since this has previously been shown [29] to be nonsingular, albeit in a different sense to that discussed here, in the rescaled metric appropriate to a hypothetical five-brane theory, dual to string theory. As will be shown elsewhere [30], this rescaled metric is also nonsingular in the sense used here. That is, when considered in five-brane variables the string solution of $d=10$ supergravity also interpolates between supersymmetric vacua of $d=10$ supergravity.

Up to now we have regarded the $d=10$ five-brane as a solution of $N=1$, $d=10$ supergravity, but it is also a solution of the $d=10$, $N=2A$ and $N=2B$ supergravity theories. We conclude with some remarks on these cases. In the case of the $N=2A$ theory, one would expect, in view of the fact that this theory is the dimensional reduction of $d=11$ supergravity, that the world-volume field content should be the same as in that theory, i.e., the six-dimensional $N=4$ antisymmetric tensor multiplet, which contains five world-volume scalars. This is known to be true from an analysis of small fluctuations about the $d=10$, $N=2A$ five-brane solution [5]. The $d=10$, $N=2B$ theory is more problematic; a similar small fluctuation analysis [5] led to the conclusion that the world-volume fields are those of the $N=4$ Maxwell supermultiplet. This does not fit easily with the proposal that the linearized world-volume field theory (at least) is to be identified as a supersingleton field theory because the $N=4$ Maxwell theory is not conformally invariant in six dimensions. Without entering into a further discussion of this case, which we have not properly understood, we wish to point out here that the $N=2B$ supergravity has a rigid U(1) symmetry under which, *inter alia*, the two two-form potentials rotate into each other, and that this symmetry is broken by the five-brane solution. One would therefore expect a world-volume Goldstone field for this broken U(1), in addition to the four Goldstone modes of broken translation invariance, but this would contradict the conclusions of Ref. [5].

P.K.T. would like to thank Nathan Berkovits for helpful discussions.

[1] A Linde, Phys. Lett. **129B**, 177 (1983).
[2] P. van Baal, F. A. Bais, and P. van Nieuwenhuizen, Nucl. Phys. **B233**, 477 (1984).
[3] G. W. Gibbons, in *Supersymmetry, Supergravity and Related Topics*, edited by F. Del Aguila, J. A. de Azcárraga, and L. E. Ibañez (World Scientific, Singapore, 1985).
[4] G. W. Gibbons and K. Maeda, Nucl. Phys. **B298**, 741 (1988).
[5] C. Callan, J. Harvey, and A. Strominger, Nucl. Phys. **B359**, 611 (1991).
[6] G. Horowitz and A. Strominger, Nucl. Phys. **B360**, 197 (1991).
[7] M. J. Duff and X. Lu, Nucl. Phys. **B354**, 141 (1991); Phys. Lett. B **273**, 409 (1991).
[8] M. J. Duff and K. S. Stelle, Phys. Lett. B **253**, 113 (1991).
[9] R. Güven, Phys. Lett. B **276**, 49 (1992).
[10] P. G. O. Freund and M. A. Rubin, Phys. Lett. **97B**, 233 (1980); M. J. Duff and C. N. Pope, in *Supersymmetry and Supergravity '82*, edited by S. Ferrara, J. G. Taylor, and P. van Nieuwenhuizen (World Scientific, Singapore, 1983); B. Biran, F. Englert, B. de Wit, and H. Nicolai, Phys. Lett. **124B**, 45 (1983).
[11] K. Pilch, P. K. Townsend, and P. van Nieuwenhuizen, Nucl. Phys. **B242**, 377 (1984).
[12] J. H. Schwarz, Nucl. Phys. **B226**, 269 (1983).
[13] R. Kallosh and A. Peet, Phys. Rev. D **46**, R5223 (1992).
[14] M. J. Duff and X. Lu, Nucl. Phys. **B390**, 276 (1993).
[15] M. Günaydin and N. Marcus, Classical Quantum Gravity **2**, L1 (1985); M. Günaydin, P. van Nieuwenhuizen, and N. P. Warner, Nucl. Phys. **B255**, 63 (1985); H. S. Kim, L. J. Romans, and P. van Nieuwenhuizen, Phys. Rev. D **32**, 389 (1985).
[16] C. Fronsdal, Phys. Rev. D **26**, 1988 (1982).
[17] H. Nicolai and E. Sezgin, Phys. Lett. **143B**, 389 (1984).
[18] E. Bergshoeff, E. Sezgin, and Y. Tanii, Int. J. Mod. Phys. A **5**, 3599 (1990); E. Bergshoeff, A. Salam, E. Sezgin, and Y. Tanii, Phys. Lett. B **205**, 237 (1988).
[19] H. Nicolai, E. Sezgin, and Y. Tanii, Nucl. Phys. **B305**, **[FS23]**, 483 (1988).
[20] E. Bergshoeff, M. J. Duff, C. N. Pope, and E. Sezgin, Phys. Lett. B **199**, 69 (1987).
[21] P. S. Howe, G. Sierra, and P. K. Townsend, Nucl. Phys. **B221**, 331 (1983).
[22] M. Günaydin, B. E. W. Nilsson, G. Sierra, and P. K. Townsend, Phys. Lett. B **176**, 45 (1986).
[23] M. P. Blencowe and M. J. Duff, Phys. Lett. B **203**, 229 (1988).
[24] W. E. Couch and R. J. Torrence, Gen. Relativ. Gravitation **16**, 789 (1984).
[25] M. J. Duff, P. K. Townsend, and P. van Nieuwenhuizen, Phys. Lett. **122B**, 232 (1983).
[26] D. Z. Freedman, G. W. Gibbons, and P. C. West, Phys. Lett. **124B**, 491 (1983).
[27] E. Bergshoeff and M. de Roo, Phys. Lett. B **218**, 210 (1989).
[28] A. Dabholkar, G. W. Gibbons, J. A. Harvey, and F. Ruiz-Ruiz, Nucl. Phys. **B340**, 33 (1990).
[29] M. J. Duff, R. R. Khuri, and J. X. Lu, Nucl. Phys. **B377**, 281 (1992).
[30] M. J. Duff, G. W. Gibbons, and P. K. Townsend (to be published).