# Rough Surface Retrieval from the Specular Intensity of Multiply Scattered Waves

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(Received 24 February 1993)

We present a solution to the direct determination of the statistical parameters (correlation length and root mean square deviation), characterizing the profile of a random rough surface with large slopes, thus producing multiple scattering. This is done from the specular, coherent, component of the mean intensity of waves scattered from the surface, and, today, is the only reliable, realistic procedure which uses an analytical expression of this component. The method is applicable whenever this specular component may be measured, and is valid for a wide variety of physical problems involving surface scattering.

PACS numbers: 42.25.Fx, 03.40.Kf, 68.35.Bs

The difficulties involved in the determination of a potential from scattering data are well known. Detectors only provide intensities, namely, the square modulus of the scattered amplitudes. Hence, the information on the phase is lost. For single scattering, holographic methods provide phase information and thus the potential or diffusing object can be recovered, e.g., from a set of Fourier transformations. However, when multiple scattering is prominent, no direct method exists in general, although some tentatives have been proposed [1,2].

In this work we present a direct method for determining the correlation length $T$ and the rms $\sigma$ of random rough surface profiles from measurements of only the specular component (SC) of the mean scattered intensity (MSI) of waves multiply scattered from the surface. As far as we know, this is the first time that a general direct inversion procedure is established for surface topographies that produce multiple scattering. The retrieval of these statistical parameters constitutes a general physics problem, of relevance in such a diversity of areas as, e.g., surface physics, light scattering, integrated optics, acoustics, radioastronomy, oceanography, remote sensing, and particle physics [3–9]. The basis of the procedure is an analytical expression of the SC which we have found to coincide with that given by Monte Carlo (MC) numerical calculations in all tested cases; as a matter of fact, it presents an excellent matching with the MC whenever the SC is larger than the background diffuse halo, namely, when the MSI has a *measurable coherent specular component.*

Let $\mathbf{k}_0 = (\mathbf{K}, -q_0)$ and $\mathbf{k_Q} = (\mathbf{K}+\mathbf{Q}, q)$ represent the incident and scattered wave vectors, respectively, both expressed in terms of the components parallel and normal to the surface mean plane $\mathbf{R} = (x, y)$, $(K^2 + q_0^2 = \mathbf{K} + \mathbf{Q}^2 + q^2)$. The surface profile being $z = D(\mathbf{R})$.

For a hard wall, or perfectly conducting surface, by using a series expansion in $k\sigma$ of the scattering equations used for periodic surfaces [10] an expression of the SC from a random surface was obtained [11,12] up to second order. By performing a phase expansion of the scattered field it was proven [13] that the second order term of Refs. [11] and [12] is just the exponent of the SC in a fastly convergent expression which is like a Debye-Waller factor. The expression for this intensity is

$$\langle I(Q=0)\rangle = \exp[-4q_0^2\sigma^2 g(q_0,T)], \tag{1}$$

where

$$g(q_0,T) = \frac{1}{(2\pi)^2}\frac{1}{q_0}\int_0^{2\pi} d\beta \int_0^{\infty} dQ' Q' \Re q' W(\mathbf{Q}'). \tag{2}$$

In Eq. (2) $\Re q' = (q_0^2 - Q'^2 - 2KQ'\cos\beta)^{1/2}$ for $(q_0^2 - Q'^2 - 2KQ'\cos\beta) > 0$ and it is zero for $(q_0^2 - Q'^2 - 2KQ'\cos\beta) < 0$; also, $W(\mathbf{Q})$ denotes the power spectrum of the surface, i.e., the Fourier transform of the profile correlation function.

For a surface with an isotropic Gaussian correlation function $c(\tau) = \exp(-\tau^2/T^2)$, at normal incidence the $g$ function of Eq. (2) reads

$$g(q_0,T) = \frac{1}{6}q_0^2T^2{}_1F_1\left(1, \frac{5}{2}, -\frac{q_0^2T^2}{4}\right), \tag{3}$$

where ${}_1F_1$ is Kummer's function.

For the case of a negative exponential correlation function, $c(\tau) = \exp(-\tau/T)$, at normal incidence, the $g$ function reads

$$g(q_0,T) = \frac{1}{4\pi}q_0^2T^2{}_2F_1\left(1, \frac{3}{2}, \frac{5}{2}, -q_0^2T^2\right), \tag{4}$$

where ${}_2F_1$ is the hypergeometric function.

If the incident field is light or other electromagnetic wave, one should use instead of Eq. (2) the corresponding equation for vector waves. For instance, in the case of $ss$ scattering, the $g$ function reads [14]

$$g^{(ss)}(q_0,T) = g(q_0,T) - \frac{1}{(2\pi)^2}\frac{1}{q_0}\int_0^{2\pi} d\beta \cos^2\beta \int_0^{\infty} dQ' Q'^3 \Re(1/q') W(\mathbf{Q}'). \tag{5}$$

0031-9007/93/71(22)/3645(4)$06.00

The first term in Eq. (5) coincides with the $g$ function of Eq. (2). $\Re(1/q') = 1/(q_0^2 - Q'^2 - 2KQ'\cos\beta)^{1/2}$ for $(q_0^2 - Q'^2 - 2KQ'\cos\beta) > 0$ and it is zero for $(q_0^2 - Q'^2 - 2KQ'\cos\beta) < 0$.

The crucial fact in Eqs. (1)–(5) is that $g$ does not depend on $\sigma$, but only on both $T$ and the angle of incidence $\theta_0 = \cos^{-1}(q_0/k_0)$, $(k_0 = 2\pi/\lambda)$. Thus, $\sigma^2$ constitutes only a scaling factor for $\langle I(q=0)\rangle$.

Although Eqs. (1)–(5) should be a very good approximation to the SC of the MSI, they cannot be checked against *exact* numerical MC calculations since these do not exist at present due to large computer memory and time needed. However, this difficulty is overtaken if the surface is one dimensional (i.e., constant in, e.g., the $y$ direction), $z = D(x)$, for which numerical MC calculations are possible at present [15–18]. Then Eq. (2), for either scalar waves or $s$-polarized electromagnetic waves, becomes

$$g(q_0, T) = \frac{1}{2\pi}\frac{1}{q_0}\int_{-\infty}^{\infty} dQ' \Re q' W(\mathbf{Q}'). \tag{6}$$

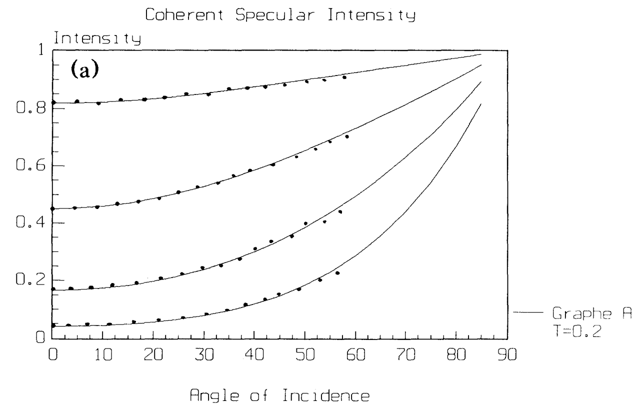
In Eq. (6) $\Re q' = (q_0^2 - Q'^2 - 2KQ')^{1/2}$ for $(q_0^2 - Q'^2 - 2KQ') > 0$ and it is zero for $(q_0^2 - Q'^2 - 2KQ') < 0$.

For a Gaussian correlation function, at normal incidence, Eq. (6) reads

$$g(q_0, T) = \frac{\sqrt{\pi}}{4} q_0 T {}_1F_1\left(\frac{1}{2}, 2, -\frac{q_0^2 T^2}{4}\right). \tag{7}$$

In order to assess the validity of our equations, we compare the SC obtained from them with that given by the MC computations available in 1D random surfaces. These calculations can be performed for a large range of statistical parameters $T$ and $\sigma$. We shall show comparisons for the case of a Gaussian correlation function with $T$ between $0.1\lambda$ and $T = \lambda$, and $\sigma/T$ up to one. Figures 1(a) and 1(b) show the SC calculated by inserting Eq. (6) into Eq. (1). The MC values fall on top of the curves corresponding to the analytical results, Eqs. (1) and (6), except for small departures at angles of incidence larger than 60°. This is due to the lack of convergence of the MC calculations at large angles of incidence, shown by errors larger than 2% in their unitarity condition [16]. Since Eq. (1) works better the larger the angle of incidence [19] (see also [11,12]), these analytical results should correct the MC computations at such large values of $\theta_0$. For $T \leq 0.1\lambda$ the MC is very sensitive to the surface sampling at $\sigma/T$ near one, although the agreement between both calculations is better than 5% (this is not shown here for brevity). We should say that in all cases that we tested in which the SC is larger than the background, the analytical and the MC computations have been found to agree with each other. These cover all cases where measurements of the SC of the MSI are possible.

We have also compared with the Kirchhoff approximation (KA), obtained when $T \to \infty$ ($g$ then tends to one), and the errors for $T = \lambda$ and $\sigma \leq 0.1$ are of the order of, or larger than, 10%. Of course, for smaller values of $T$ comparisons with the KA are meaningless.



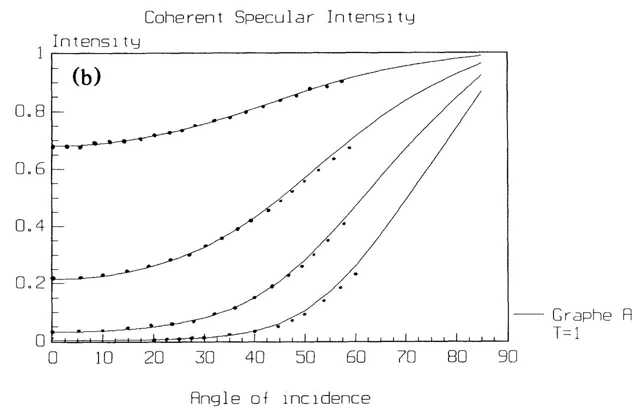




FIG. 1. (a) Plots of $\langle I(Q=0)\rangle$ obtained from Eqs. (6) and (7) (full lines) and from the Monte Carlo calculations (dots). From top to bottom: $T = 0.2\lambda$, $\sigma = 0.05\lambda$, $0.1\lambda$, $0.15\lambda$, and $0.2\lambda$. (b) Same as (a) for $T = \lambda$.

Thus, the inversion method works for all surfaces with slope $\sigma/T \leq 1$ providing they yield a measurable specular component of the MSI, namely, when both $\sigma$ and $T$ are considerably smaller than the wavelength. Notice that then large roughness (i.e., large slopes) are included. Although we believe that this range of validity holds both for 2D and 1D surfaces, the availability of present day MC calculations for 1D surfaces only, restrict our test of these equations to these 1D surfaces. Conversely, the method should fail when the surface produces an incoherent background of the MSI which is large enough to swamp the SC and thus this SC cannot be measured, this occurs when both $\sigma$ and $T$ are comparable, or larger than, the wavelength; note, however, that in this case the calculation of the SC is meaningless.

*Inversion method for the statistical parameters $\sigma$ and $T$.*—Having proven that the analytical expression presented above gives the same result as the full MC computations in all cases where the MC is reliable, we can solve the inverse statistical problem for the random sur-

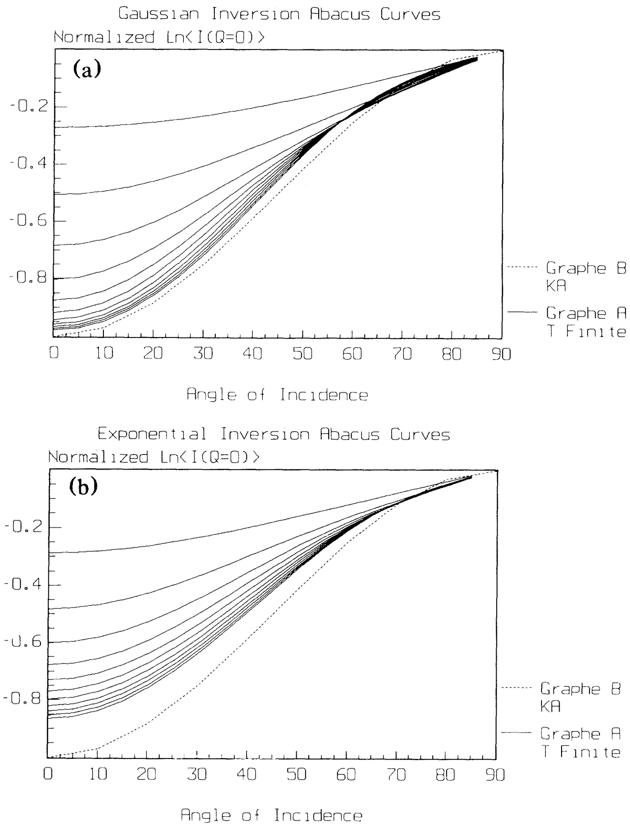



FIG. 2. (a) Plots of $\ln\langle I(Q=0)\rangle/4k_0^2\sigma^2$ versus $\theta_0$, obtained from Eqs. (6) and (7), for several values of $T$. Gaussian correlation function. The upper curve corresponds to $T=0.1\lambda$. The successive curves downwards are for $T$ increasing at steps of $0.1\lambda$. The Kirchhoff limit is represented by the broken line. (b) Same as (a) for a negative exponential correlation function.

face; namely, determine the values of $T$ and $\sigma$ if the SC is known at several angles of incidence. Note that we do not need to use information on the background diffuse MSI. This inversion procedure is based on the remarkable fact of Eq. (2) that $g$ does not depend on $\sigma$. This permits the direct determination of the surface statistical parameters in the following manner: (1) Draw the surface: $-g(\theta_0,T)\cos^2\theta_0$ versus $\theta_0$ and $T$, with $g$ given either by Eq. (2) or Eq. (3) according to whether the problem is scalar or vector. (2) Measure the specular component of the MSI for several angles of incidence $\theta_0$ and plot: $\ln\langle I(Q=0)\rangle$. Scale this curve and move it along the $T$ axis of the surface drawn in step (1) until, at a certain value of $T$, it coincides with one of the sections of this surface. This matching determines $T$. Then, $\sigma$ is obtained by equating this scale factor to $1/(4k_0^2\sigma^2)$.

Figures 2(a) and 2(b) show the sections $\ln\langle I(Q=0)\rangle/4k_0^2\sigma^2$ versus $\theta_0$ at several values of $T$ (inversion abacus curves), for a surface with correlation function being either a Gaussian and a negative exponential, respectively. As seen, as $T$ increases, these sections tend to gather as they approach the limit of the KA ($g=1$) (see also Refs. [11] and [12]). Also, there are significant differences between the results according to the shape of correlation function. These differences should be well distinguished with experimental measurements. Observe that the deviations of the KA with respect to these curves are larger for the case of the negative exponential correlation. Notice that this method determines both $\sigma$ and $T$ from the SC, while the KA [20], when it works, only permits the determination of $\sigma$ from the SC.

Finally,we should mention that for dielectric rough interfaces we have no exact equation for the SC in this case as Eqs. (1)–(7). However, we propose an approximate solution by multiplying the SC of Eq. (1) by the corresponding reflectivity of a flat interface. We have made comparisons with MC numerical simulations and we have obtained that for $T \geq \lambda$ and $\sigma/T \leq 0.2$ there is agreement; but for $T=0.2\lambda$ and $\sigma/T > 0.5$ there are departures of 20% between our proposed formula and the MC results.

In conclusion, we have presented an inversion method that directly retrieves the statistical parameters of random rough surfaces even with large slopes, thus giving rise to multiple scattering. We have compared with MC simulations for Gaussian correlation functions, available for 1D surfaces, obtaining excellent agreement in all practical cases submitted to test. The procedure is equally applicable to any other correlation function and does not require us to measure the diffuse background and should be useful to experimentalists.

It is a pleasure to thank M. Franco and J.A. Sanchez-Gil for helping us in the computations. The MC simulations were done by J.A. Sanchez-Gil. This research has been supported by grants from the CICYT and the EC.

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