Measurement of the Aharonov-Casher Phase in an Atomic System

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We describe a new experimental configuration suitable for observing the topological phase of Aharonov and Casher in atomic systems. Using this we have been able to show experimentally that the Aharonov-Casher phase is both independent of velocity and proportional to electric field and we have verified the predicted size of the effect with an accuracy of 4%.

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In 1984, Aharonov and Casher [1] considered a particle with magnetic dipole moment $\boldsymbol{\mu}$ being taken on a closed path around a charged wire. They predicted that the wave function of the particle should acquire a topological phase shift

$$\Delta\Phi_{\mathrm{AC}}=\frac{1}{\hbar c^2}\oint \boldsymbol{\mu}\times\mathbf{E}\cdot d\mathbf{r}\,, \tag{1}$$

where $\mathbf{E}$ is the electric field at the site of the dipole due to the wire. This phenomenon is analogous to the Aharonov-Bohm effect [2] in which a charged particle acquires a phase when taken around a line of magnetic flux. It is a characteristic feature of both these effects that the phase shift is independent of the velocity of the particle [3].

The Aharonov-Casher (AC) effect has been tested in a neutron interferometer [4], where a beam of neutrons was coherently split, as illustrated in Fig. 1(a), allowed to encircle a line charge, and then recombined to give an interference pattern. The measured phase shift was 2.11 $\pm$0.34 mrad, compared with the predicted value of 1.52 mrad. Although the observed phase is nearly 2 standard deviations above the theoretical value, the experiment does seem to confirm the existence of the effect. However, there was no experimental verification of the two most notable features, velocity independence and proportionality to electric field. Recently there have been suggestions for observing the AC effect in similar interferometers using atoms instead of neutrons [5,6], but so far the neutron observation has been the only one.

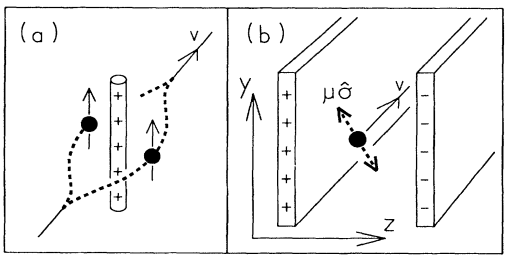

FIG. 1. Experimental configurations for observing the Aharonov-Casher effect. (a) Geometry of the original measurement using a neutron interferometer, in which the two interfering states encircle a charge and have the same magnetic moments. (b) New geometry described here. Particles travel in a uniform electric field in a coherent superposition of opposite magnetic moments $\pm\mu\hat{\boldsymbol{\sigma}}$. The two states are oppositely shifted by the Aharonov-Casher phase as they travel through the field.

The scheme shown in Fig. 1(a) involves two coherent beams with the *same* magnetic moment traveling on *opposite sides* of a charged wire. Casella has noted [7] that it is not necessary for the path to enclose a charged wire in order to observe the AC effect. Placing charged plates above and below the plane of the interferometer with opposite polarity for each arm can lead to the same phase shift as in the original configuration used by Cimmino *et al.* [4]. In this Letter, we point out a third possible configuration, shown in Fig. 1(b), where the two coherent beams have *opposite* magnetic moments and are not spatially separated; they pass through the *same* electric field. This apparatus is also sensitive to the AC phase, since the important quantity $\boldsymbol{\mu}\times\mathbf{E}$ still has opposite sign in the two arms of the interferometer. For simplicity we assume that the beam travels along the x axis and $\mathbf{E}$ lies on the z axis; then the relevant component of the magnetic moment is μ_y. If $\pm\mu\hat{\boldsymbol{\sigma}}$ are the magnetic moments in the two coherent beams ($\hat{\boldsymbol{\sigma}}$ is a unit vector), the AC phase difference between them after traveling a length l is

$$\Delta\Phi_{\mathrm{AC}}=\frac{2\mu}{\hbar c^2}\sigma_y E l\,, \tag{2}$$

where σ_y is the projection of $\hat{\boldsymbol{\sigma}}$ onto the y axis. It is by no means necessary to use neutrons; any neutral particle with a magnetic moment should exhibit the AC effect. All that is required is a convenient way of preparing the magnetic moment in a coherent superposition of "up" and "down" states and detecting an accumulated phase difference. A natural choice is to utilize Ramsey's method of separated oscillatory fields [8].

The AC phase shift has an interesting interpretation when viewed from the rest frame of the atom. Here there is no displacement ($d\mathbf{r}=0$), but there is a motional magnetic field arising from the laboratory frame electric field.

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From this point of view then, the AC phase shift is the integral over time of the motional Zeeman energy:

$$\frac{1}{\hbar}\int(\boldsymbol{\mu}\cdot\mathbf{B}dt)_{\text{atom}}=\frac{1}{\hbar c^2}\int(\boldsymbol{\mu}\times\mathbf{E}\cdot d\mathbf{r})_{\text{lab}}=\Phi_{\text{AC}}, \quad (3)$$

where the subscripts lab and atom refer to the laboratory and atom frames of reference, respectively. The magnitude of the shift is, of course, the same in any frame of reference. The motional Zeeman effect has been recognized as a source of systematic error [9] in atomic beam experiments [10,11] which test time-reversal symmetry by looking for a permanent atomic electric dipole moment (EDM). However, this connection with the AC phase has been overlooked until now and the effect has therefore escaped thorough experimental investigation, although some early data ascribed to the motional Zeeman effect do suggest linearity in the electric field [9]. In EDM beam experiments, the effect is usually suppressed as much as possible by providing a magnetic bias field which forces the magnetic moment to be quantized along the direction of the electric field so that $\boldsymbol{\mu}\times\mathbf{E}=0$. The bias field can either be a carefully aligned external field [11] or, in the case of a polar diatomic molecule [10], it can be the internal magnetic field of the molecule.

We observe the AC phase using the fluorine nuclei in a thallium fluoride (TlF) molecular beam in a strong (10–30 kV/cm) external electric field $\mathbf{E}$. The molecules are in the electronic and vibrational ground states $^1\Sigma$, $v=0$, and in the first excited rotational state $J=1$. The rotational states are strongly mixed by the applied electric field, so J is not a good quantum number, but it serves adequately to identify which rotational state we use. Within the $J=1$ manifold there are twelve hyperfine sublevels corresponding to the magnetic quantum numbers of the rotation ($m_J=0,\pm1$), Tl nuclear spin ($m_{\text{Tl}}=\pm\frac{1}{2}$), and F nuclear spin ($m_{\text{F}}=\pm\frac{1}{2}$). Using techniques that are fully described elsewhere [10], we prepare the beam in one of these twelve states, well described by $(m_J,m_{\text{Tl}},m_{\text{F}})=(+1,+\frac{1}{2},-\frac{1}{2})$, which for simplicity we call $(\gamma,-)$ since we are primarily interested in the state of the F spin. The beam then passes through the first of two Ramsey loops [8] in which an rf magnetic field near-resonantly excites a coherent superposition of the states $(\gamma,-)$ and $(\gamma,+)$ with roughly equal amplitudes. This loop is effectively the beam splitter of our interferometer, providing the required coherent superposition of F magnetic moments $\pm\mu\hat{\boldsymbol{\sigma}}$. The molecules travel in this state for a distance of $l=2.05$ m before reaching the second rf loop which plays the role of the recombining beam splitter. The rest of the apparatus then determines what fraction P of the molecules made the transition to $(\gamma,+)$. Close to resonance, the Ramsey fringe pattern has the usual form [8]

$$P=\tfrac{1}{2}\left[1+\cos\left[(\omega-\omega_0)\frac{l}{v}+\delta+\Delta\Phi\right]\right], \quad (4)$$

where ω is the rf frequency, ω_0 is the resonance frequency, v is the beam velocity, δ is the phase difference between the two rf fields, and $\Delta\Phi$ is any additional phase shift between the two states in the interferometer, such as the AC phase.

Since the molecule in an external electric field is cylindrically symmetric around the field direction z, the expectation value of the F nuclear spin in the states $(\gamma,\pm)$ lies along the positive and negative z axes. Hence $\sigma_y=0$ in both the states $(\gamma,\pm)$ and, in accordance with Eq. (2), the AC effect is completely suppressed. While this is a great advantage in the search for an EDM, where the AC effect is a potential source of systematic error, it is obviously an obstacle to be overcome in the present context. In order to study the AC phase, we must rotate the F moment so that it acquires a projection $\mu\sigma_y$, and this is done by applying a uniform magnetic field B along the y axis, as shown in Fig. 2(a). The new states corresponding to $(\gamma,\pm)$ in this magnetic field have the F moment rotated by an angle θ in the y-z plane [Fig. 2(a)] and σ_y takes on the values $\pm\sin\theta$. When our Ramsey interferometer prepares a coherent superposition of these two states, an AC phase evolves between them in accordance with Eq. (2), and that is what we measure.

We digress for a moment to note that in this system, the magnetic moment is a tensor, with different values μ_{perp} and μ_{para} for components perpendicular and parallel to the z axis. The AC phase in our case is determined by the difference between the perpendicular moments $\mu^{\pm}_{\text{perp}}$ of the two states $(\gamma,\pm)$. Thus the quantity μ in Eq. (2) stands for $(\mu^{+}_{\text{perp}}-\mu^{-}_{\text{perp}})/2$. This is very close to the magnetic moment of the free F nucleus, having the value 1.929 kHz/G, which is 3.7% less than the free nuclear moment [12] because the applied field is slightly shielded by the presence of the Tl nucleus. Although this shielding is completely understood, it is not of much significance here because our AC phase measurements are barely at that level of accuracy and we therefore refrain

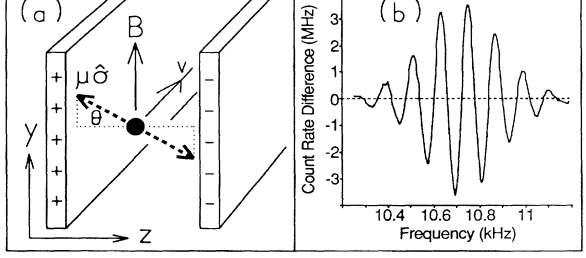

FIG. 2. (a) Configuration of this experiment. Thallium fluoride molecules prepared in a coherent superposition of opposite spin states travel through an electric field. The natural alignment of the spins is along the electric field (z axis), so a magnetic field B is used to give them a component along the y axis. (b) Typical experimental Ramsey pattern for the fluorine magnetic resonance transition.

from a discussion of the details.

Our first experiment checks the magnitude of the AC phase and demonstrates that it is independent of the velocity of the molecules. The electric field strength is set to $E=29.5$ kV/cm and initially the magnetic field is turned off. We begin by sweeping the frequency of the rf oscillator to produce a Ramsey resonance pattern. In order to separate the interference fringes from the more slowly varying background, we employ the standard technique of switching the relative rf phase δ [Eq. (4)] between $+\pi/2$ and $-\pi/2$ and taking the difference to produce the line profile shown in Fig. 2(b). This tells us that the resonance frequency for the transition $(\gamma,-)\rightarrow(\gamma,+)$ is $\omega_0=2\pi\times 10.72$ kHz. Next, we turn on the magnetic field ($B\approx 0.4$ G) and scan through the line again to find the shifted frequency ω_0', which is higher than ω_0 by $2\pi\times 96$ Hz for this particular field. Assuming that B is exactly perpendicular to the z axis, a knowledge of both frequencies amounts to a measurement of B since the three are related by Pythagoras' theorem $(\omega_0')^2=\omega_0^2+(2\mu B/\hbar)^2$. This allows us to express the projection factor σ_y as

$$\sigma_y=\sin\theta=\pm\sqrt{1-(\omega_0/\omega_0')^2} \tag{5}$$

and gives $\sigma_y=0.133$ in our case. The method is not valid if the magnetic field has a component B_z along the electric field direction, producing a linear Zeeman shift. In setting up the apparatus, however, we suppressed the linear shift by rotating the direction of B until it was accurately perpendicular to E. In this experiment, the linear contribution to the shift is less than 1 Hz and therefore our value of σ_y is accurate to better than 1%. Using this value, Eq. (2) predicts an AC phase shift of $\Delta\Phi_{AC}=2.18$ mrad, which is shown as the straight line in Fig. 3.

In order to measure the AC phase, we set the oscillator to the central zero crossing, where the Ramsey pattern is most sensitive to small phase shifts, and use the fact that $\Delta\Phi_{AC}$ changes sign when either E or B is reversed. The change of signal that we observe in synchronism with these field reversals is first determined as an equivalent frequency shift, calibrated against a known 1.25 Hz shift of the rf oscillator (a computer-controlled frequency synthesizer). Since we also know the frequency interval between zero crossings of the Ramsey pattern, this result can finally be converted to a phase.

The molecular beam is focused by two electrostatic quadrupole lenses, whose focal lengths depend upon the strength of the quadrupole field and on the velocity of the molecules. Thus our resonance signal is derived from a narrow slice ($\sim$20%) of the full Maxwell-Boltzmann distribution, which we are free to choose by adjusting the voltages on the quadrupole lenses. The velocity is measured by the fringe spacing of the Ramsey pattern, which goes through zero each time the quantity $(\omega-\omega_0)l/v$ increases by π, as can be seen from Eq. (4). The data points in Fig. 3 are our AC phase measurements, taken at various beam velocities, with error bars based on the observed signal fluctuations. The weighted mean of these measurements yields a measured phase shift $\Delta\Phi=2.22\pm 0.11$ mrad, which is consistent with the predicted value, $\Delta\Phi_{AC}=2.18$ mrad. There is no evidence for any variation with velocity.

Our second experiment demonstrates that the AC phase is proportional to the electric field. For this measurement we choose a convenient beam velocity of 250 m/s, and the same magnetic field as before. The experimental points in Fig. 4 show the phase shifts measured at five different electric fields between 10 and 30 kV/cm. At each point we also measured σ_y (which varies weakly through a slight dependence of ω on E) in order to know the theoretical prediction of Eq. (2). The continuous curve in Fig. 4 is an interpolation of these theoretical values. It is almost straight because the AC phase is proportional to E, but not perfectly so because it incorporates the slight variation of σ_y. Without any fitting parameters we find excellent agreement between theory and

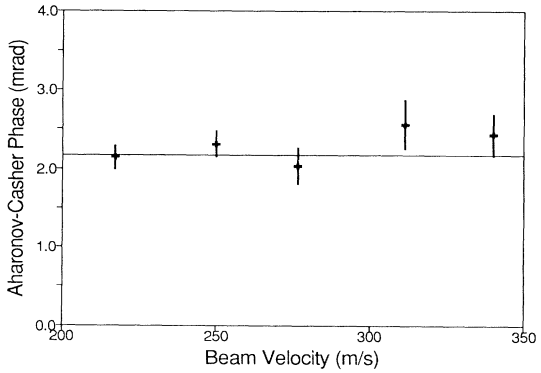

FIG. 3. Aharonov-Casher phase versus beam velocity. The experimental points are in good agreement with the theoretical expectation. There are no free parameters.

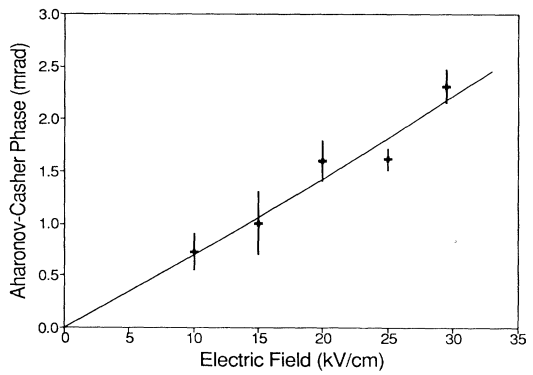

FIG. 4. Aharonov-Casher phase versus electric field strength. The experimental points are in good agreement with the theoretical expectation. There are no free parameters.

experiment, the goodness of fit being $\chi^2=5.0$ with 5 degrees of freedom.

These measurements have verified the presence of the Aharonov-Casher phase in an atomic system and have demonstrated explicitly that it is proportional to the electric field and independent of the velocity. If each phase shift we have measured is divided by the corresponding theoretical prediction, we obtain an average value (weighted mean) for the ratio of experiment to theory of

$$\frac{\Delta\Phi_{\mathrm{EXPT}}}{\Delta\Phi_{\mathrm{AC}}}=0.99\pm0.04\,, \tag{6}$$

which verifies the absolute magnitude of the AC phase shift at the level of 4%. In a future experiment, it would be interesting to extend these measurements to an electron spin flip transition, where the magnetic moment is 3 orders of magnitude larger, leading to a possible AC phase shift in excess of 2π.

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