Radiative Corrections to π_{l2} Decays

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Radiative corrections to π_{l2} ($l=e$ or μ) decays are examined. Higher order electroweak leading logarithms, short-distance QCD corrections, and structure dependent effects are incorporated. The results are employed to (1) test e-μ universality in $\Gamma(\pi\rightarrow e\bar{\nu}_e(\gamma))/\Gamma(\pi\rightarrow \mu\bar{\nu}_\mu(\gamma))$, (2) extract an f_π which is used to check the Goldberger-Treiman relation and PCAC-anomaly prediction for $\Gamma(\pi^0\rightarrow\gamma\gamma)$, and (3) determine the tau partial decay rate $\Gamma(\tau\rightarrow\pi\nu_\tau(\gamma))$.

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Calculations of electroweak radiative corrections and reliable estimates of their underlying theoretical uncertainties are crucial ingredients for precision tests of the standard model. An important case is provided by π_{l2} decays, $\pi\rightarrow l\bar{\nu}_l$, where $l=e$ or μ. Recently, experiments at TRIUMF [1] and PSI [2] have reported

$$R_{e/\mu}\equiv\frac{\Gamma(\pi\rightarrow e\bar{\nu}_e+\pi\rightarrow e\bar{\nu}_e\gamma)}{\Gamma(\pi\rightarrow \mu\bar{\nu}_\mu+\pi\rightarrow \mu\bar{\nu}_\mu\gamma)}$$
$$=1.2265\pm0.0034\pm0.0044\times10^{-4}\ (\text{TRIUMF})\,, \tag{1}$$
$$R_{e/\mu}=1.2346\pm0.0035\pm0.0036\times10^{-4}\ (\text{PSI})\,,$$

for the ratio of radiative inclusive decay rates. Those results represent about a factor of 3 (error) improvement when compared with the previous experimental value [3] $R_{e/\mu}=(1.218\pm0.014)\times10^{-4}$. Future measurements are expected to further reduce the uncertainty in $R_{e/\mu}$. However, already at the level in (1), e-μ universality is well tested and "new physics" scenarios are very constrained [4].

To fully utilize the results in (1), the theoretical prediction for $R_{e/\mu}$ must be known to at least the same level of precision and preferably much better. That entails the inclusion of electroweak radiative corrections which in the case of $R_{e/\mu}$ have long been known from the pioneering work of Berman [5] and Kinoshita [6] to be large, $\sim -4\%$. The main purpose of this Letter is to scrutinize the $O(\alpha)$ radiative corrections to π_{l2}, incorporate higher order effects, and most importantly, argue that the underlying theoretical uncertainties give rise to less than a $\pm0.05\%$ error in the standard model prediction for $R_{e/\mu}$.

Radiative corrections are also important for the extraction and application of electroweak parameters. In the case of $\pi_{\mu2}$ decays, one obtains the pion decay constant f_π, defined by the weak axial-current matrix element

$$\langle0|A_\mu(0)|\pi(p)\rangle=if_\pi p_\mu\,, \tag{2}$$

by comparing the experimental rate [7]

$$\Gamma(\pi\rightarrow\mu\bar{\nu}_\mu(\gamma))=(2.5284\pm0.0023)\times10^{-14}\ \text{MeV} \tag{3}$$

with theory. However, electroweak radiative corrections must be properly accounted for in extracting f_π [8,9].

After determining f_π, one can test the Goldberger-Treiman relation [10]

$$f_\pi g_{\pi pn}=\frac{1}{\sqrt{2}}(m_n+m_p)g_A\,, \tag{4}$$

and the PCAC (partially conserved axial-vector current) anomaly [11] prediction

$$\Gamma(\pi^0\rightarrow\gamma\gamma)=\frac{\alpha^2m_{\pi^0}^3}{32\pi^3f_\pi^2}\,, \tag{5}$$

both of which are expected to hold up to the (1–2)% level. In addition, one can employ f_π to predict the tau partial decay rate [12,13]

$$\Gamma(\tau\rightarrow\pi\nu_\tau(\gamma))=\frac{G_\mu^2f_\pi^2|V_{ud}|^2}{16\pi}m_\tau^3\left(1-\frac{m_\pi^2}{m_\tau^2}\right)^2[1+O(\alpha)]\,. \tag{6}$$

Of course, the full $O(\alpha)$ corrections to the decay $\tau\rightarrow\pi\nu_\tau(\gamma)$ as well as the parameters in (4) and (5) should be included for precise confrontations [8, 14,15].

Extensive studies of the $O(\alpha)$ radiative corrections to π_{l2} decays already exist [5,6,8,16–19]. Here, we summarize those calculations, describe how they should be utilized, and assess their level of theoretical uncertainty.

Combining the known short- and long-distance radiative corrections for the inclusive decays $\pi\rightarrow l\bar{\nu}_l(\gamma)=\pi\rightarrow l\bar{\nu}_l+\pi\rightarrow l\bar{\nu}_l\gamma$, ignoring for now pure structure dependent bremsstrahlung, we find

$$\Gamma(\pi\rightarrow l\bar{\nu}_l(\gamma))=\frac{G_\mu^2|V_{ud}|^2}{8\pi}f_\pi^2m_\pi m_l^2\left(1-\frac{m_l^2}{m_\pi^2}\right)^2\left[1+\frac{2\alpha}{\pi}\ln\left(\frac{m_Z}{m_\rho}\right)\right]$$
$$\times\left[1-\frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\left(\frac{m_\rho}{m_\pi}\right)+C_1+C_2\frac{m_l^2}{m_\rho^2}\ln\frac{m_\rho^2}{m_l^2}+C_3\frac{m_l^2}{m_\rho^2}+\cdots\right\}\right]\left[1+\frac{\alpha}{\pi}F(x)\right], \tag{7a}$$

where

$$F(x)=3\ln x+\frac{13-19x^2}{8(1-x^2)}-\frac{8-5x^2}{2(1-x^2)^2}x^2\ln x-2\left(\frac{1+x^2}{1-x^2}\ln x+1\right)\ln(1-x^2)+2\frac{1+x^2}{1-x^2}L(1-x^2)\,,$$

$$x=m_l/m_\pi,\quad L(z)=\int_0^z dt\frac{\ln(1-t)}{t},\quad G_\mu=(1.16639\pm0.00002)\times10^{-5}\,\mathrm{GeV}^{-2},\quad |V_{ud}|=0.9750\pm0.0007\,. \tag{7b}$$

The first factor in square brackets in (7a), with $m_Z=91.187$ GeV and $m_\rho=0.768$ GeV, represents the short-distance correction affecting all semileptonic charged current amplitudes when expressed in terms of G_μ [20]. [Both G_μ and $|V_{ud}|$ have been extracted from muon and β decay experiments after correcting for $O(\alpha)$ effects.] The low-frequency cutoff at m_ρ in the log is somewhat arbitrary. It represents a typical hadronic mass scale used as a demarcation between short- and long-distance loop corrections. The second and third bracketed corrections correspond (for $C_1=C_2=C_3=0$) to Kinoshita's calculation [6,21] of the QED corrections to the decay of a pointlike (structureless) pion. We have also employed m_ρ in place of his ultraviolet cutoff as a means of crudely matching short- and long-distance loop effects. Hadronic structure and matching uncertainties are parametrized in terms of an unknown constant C_1 [nominally of $O(1)$] which is m_l independent. The leading lepton mass dependent structure effects (the C_2 term) were calculated by Terent'ev using PCAC [16]:

$$C_2\simeq3+\frac{2}{3}\left(1-\frac{7}{4}\gamma\right)\left(\frac{m_\rho^2}{4\pi^2f_\pi^2}\right), \tag{8}$$

where γ ($\gamma_{\text{expt}}\simeq0.45$) is the ratio of axial and vector form factors in radiative pion decay. (We have verified that result.) The smaller C_3 terms are model dependent and represent the main theoretical uncertainty in $R_{e/\mu}$. The $\cdots$ indicate more suppressed lepton mass dependent effects which are of no consequence.

Equation (7) is expected to provide an accurate prediction for $R_{e/\mu}$ because of the following: (i) As proven in Ref. [18], the dominant m_l dependent term, $3\ln x$ in (7b), has a coefficient which is not affected by strong interactions. (ii) The m_l dependent terms of order $(\alpha/\pi)x^2$, which are potentially significant in the μ channel, arise from the pion pole amplitudes and are properly incorporated in the calculation of Ref. [6]. Therefore, the leading unknown or uncertain $O(\alpha)$ correction to $R_{e/\mu}$ is of the form $C_3(\alpha/\pi)m_\mu^2/m_\rho^2\simeq4.4\times10^{-5}C_3$, which should be very small as long as C_3 is not too large.

At this point, we stress that the $O(\alpha)$ corrections in (7) correspond to the definition of f_π given in (2) and parts could, in principle, be absorbed into a redefinition of f_π. They would then come back as induced $O(\alpha)$ effects in applications of f_π. We have chosen to factor out the short-distance corrections induced by the use of $G_\mu^2|V_{ud}|^2$ as well as *all* m_l dependent corrections which are clearly specific to $\pi\to l\bar\nu_l(\gamma)$. Neither of the two f_π definitions given by the Particle Data Group [7] have the latter property. In fact, both absorb $\ln(m_\pi/m_\mu)$ terms.

The dominant unknown contribution in (7) resides in C_1. A reasonable range for C_1 can be estimated by equating it with the effect of varying the cutoff m_ρ by a factor of 2. In that way, we estimate

$$C_1=0\pm2.4 \tag{9}$$

for assessing uncertainties associated with C_1 and applications of f_π. Fortunately, in the ratio $R_{e/\mu}$, the C_1 dependence cancels [18].

Having reviewed the $O(\alpha)$ corrections to π_{l2} decays, we next examine the dominant higher order effects. The first such corrections are leading short-distance logs of the form $[(\alpha/\pi)\ln(m_Z/m_\rho)]^n$, $n\geq2$. Employing the renormalization group to sum up all such contributions [13,22], we find that $1+2(\alpha/\pi)\ln(m_Z/m_\rho)$ is replaced by

$$S(m_\rho,m_Z)=\left(\frac{\alpha(m_c)}{\alpha(m_\rho)}\right)^{3/4}\left(\frac{\alpha(m_\tau)}{\alpha(m_c)}\right)^{9/16}\left(\frac{\alpha(m_b)}{\alpha(m_\tau)}\right)^{9/19}\left(\frac{\alpha(m_W)}{\alpha(m_b)}\right)^{9/20}\left(\frac{\alpha(m_Z)}{\alpha(m_W)}\right)^{36/17}, \tag{10}$$

where $\alpha(\mu)$ is a running $\overline{\text{MS}}$ (modified minimal subtraction) coupling with the following values:

$$\alpha^{-1}(m_Z)=127.90,\quad \alpha^{-1}(m_W)=127.94,\quad \alpha^{-1}(m_b)=132.01,\quad \alpha^{-1}(m_\tau)=133.26\,,$$
$$\alpha^{-1}(m_c)=133.57,\quad \alpha^{-1}(m_\rho)=134.05\,. \tag{11}$$

Using Eqs. (10) and (11), one finds

$$S(m_\rho,m_Z)=1.0235\,. \tag{12}$$

QCD corrections to the short-distance contribution in (7) can also be calculated [23]. Employing the formalism developed in Refs. [20] and [22], one finds that a part of the short-distance correction $[\frac{1}{2}(\alpha/\pi)\ln(m_Z/m_\rho)]$ is reduced by the QCD factor $(1-\alpha_S/\pi)$. That effect reduces the correction by about 0.00033. Therefore, we find a total short-distance enhancement factor

$$1.0232\ \text{(short-distance enhancement factor)} \tag{13}$$

in place of $1+(2\alpha/\pi)\ln(m_Z/m_\rho)$. Part of that correction could be absorbed into f_π; however, we choose not to do so because similar corrections were separated out in the extraction of $|V_{ud}|$ from superallowed β decays. Other unknown two-loop short-distance corrections are presumably much smaller and can be safely neglected.

In the case of higher order long-distance corrections of the form $[(\alpha/\pi)\ln(m_\pi/m_l)]^n$, $n\geq 2$, we can also estimate their effect using the renormalization group. However, such terms are only important for $l=e$; so, we will examine them separately in our discussion of $R_{e/\mu}$. (Corrections of the form $[(\alpha/\pi)\ln(m_\rho/m_\pi)]^n$, $n\geq 2$ are small $\simeq 0.000\,02$ and can be lumped into the unknown constant C_1.)

Taking the ratio of e and μ decay rates in (7), the short-distance corrections and most uncertainties cancel. One finds (still neglecting pure structure dependent bremsstrahlung)

$$R_{e/\mu}=R_0\left\{1+\frac{\alpha}{\pi}\left[F\left(\frac{m_e}{m_\pi}\right)-F\left(\frac{m_\mu}{m_\pi}\right)+C_2\frac{m_\mu^2}{m_\rho^2}\ln\frac{m_\rho^2}{m_\mu^2}+C_3\frac{m_\mu^2}{m_\rho^2}\right]\right\}, \tag{14}$$

where

$$R_0=\frac{m_e^2}{m_\mu^2}\left(\frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2}\right)^2=1.28347\times 10^{-4},\quad C_2\simeq 3.1\,, \tag{15}$$

and $F(x)$ is defined in (7b). Inserting mass values into those expressions, the ratio is reduced to $R_{e/\mu}=(1.2337+0.000\,05C_3)\times 10^{-4}$ [6].

There are pure structure dependent (SD) bremsstrahlung corrections to $\pi\to l\bar{\nu}_l\gamma$ which are not helicity suppressed by m_l^2/m_π^2 and, therefore, potentially important for $l=e$. Such effects are suppressed by m_π^4/m_ρ^4 and hence small. Calling those contributions $\Delta R_{e/\mu}^{\rm SD}$, one finds [16,24]

$$\frac{\Delta R_{e/\mu}^{\rm SD}}{R_{e/\mu}}\simeq 5.4\times 10^{-4}(1+\gamma^2)\,, \tag{16}$$

Employing [7] $\gamma_{\rm exp}\simeq 0.45$ leads to

$$\Delta R_{e/\mu}^{\rm SD}\simeq 8\times 10^{-8}\,, \tag{17}$$

The final contributions to $R_{e/\mu}$ that we need consider are corrections of the form $[(\alpha/\pi)\ln(m_\mu/m_e)]^n$, $n\geq 2$. Indeed, since the $-3(\alpha/\pi)\ln(m_\mu/m_e)\simeq -3.7\%$ correction dominates the $O(\alpha)$ terms in (14), one expects its higher order counterparts to similarly dominate their respective orders. Summing all such logs via the renormalization group gives the enhancement

$$\frac{[1-(2\alpha/3\pi)\ln(m_\mu/m_e)]^{9/2}}{1-3(\alpha/\pi)\ln(m_\mu/m_e)}=1.000\,55\,, \tag{18}$$

which increases (14) to 1.2344×10^{-4}. Including pure SD bremsstrahlung in (17),

$$R_{e/\mu}^{\rm theory}=(1.2352\pm 0.0005)\times 10^{-4}\,, \tag{19}$$

where a conservative range of $C_3=0\pm 10$ has been employed for the hadronic structure uncertainties.

Comparing the theoretical prediction in (19) with the experimental results in (1), we find

$$\frac{R_{e/\mu}^{\rm expt}}{R_{e/\mu}^{\rm theory}}=0.9930\pm 0.0045\pm 0.0004\ \ (\text{TRIUMF})\,, \tag{20a}$$

$$\frac{R_{e/\mu}^{\rm expt}}{R_{e/\mu}^{\rm theory}}=0.9995\pm 0.0041\pm 0.0004\ \ (\text{PSI})\,. \tag{20b}$$

Averaging the two results, one finds

$$\frac{R_{e/\mu}^{\rm expt}}{R_{e/\mu}^{\rm theory}}=0.9966\pm 0.0030\pm 0.0004\,. \tag{20c}$$

The level of agreement between theory and experiment is impressive. It constrains all sorts of new physics scenarios [4]. A further significant reduction in the experimental uncertainties would provide a stringent test of the standard model and could unveil, rather than merely restrict new physics.

The radiative corrections in (7) are also necessary for extracting f_π from $\pi_{\mu 2}$ decays. Including the short-distance enhancement in (12), we find by comparing (7) with (3)

$$f_\pi=130.7\pm 0.1+0.15C_1\ \text{MeV}\,. \tag{21}$$

The uncertainty in (21) comes from $|V_{ud}|$ while the second term illustrates the dependence of f_π on C_1. For applications of f_π we allow $C_1=0\pm 2.4$ as suggested by (9), and then an additional $\pm 0.28\%$ uncertainty is implied. That uncertainty is not particularly large. Nevertheless, one would like to see a calculation of C_1 in a model of hadronic structure.

As our first application of f_π, we consider the Goldberger-Treiman relation [10] in (4), which should be exact in the chiral limit $m_u=m_d=0$ (modulo radiative corrections). Employing [7] $g_A=1.257\pm 0.003$ and [25] $g_{\pi pn}=13.04\pm 0.06$, one finds [15]

$$\Delta_\pi=1-\frac{(m_n+m_p)g_A}{\sqrt{2}f_\pi g_{\pi pn}}=0.021\pm 0.005+0.0011C_1\,, \tag{22}$$

where the ± 0.005 uncertainty stems mainly from $g_{\pi pn}$. The effect of C_1 is not very significant, unless C_1 is well outside the range in (9). The deviation from zero in (22) is in accord with theoretical expectations [26] (if $C_1\approx 0$ or not too large), which roughly suggest $|\Delta_\pi|\simeq(m_u+m_d)/2m_p\simeq 1\%$. We note, however, that an earlier [27] $g_{\pi pn}=13.4\pm 0.1$ value gives a less acceptable 4.7% deviation in (22). The situation regarding the value of $g_{\pi pn}$ is still not completely settled and deserves continued scrutiny. (A small discrepancy also exists between the directly measured g_A we employ and the value $g_A = 1.264$ implied by the neutron lifetime and superallowed Fermi transitions.) In addition, the effect of $O(\alpha)$ radiative corrections on $g_{\pi pn}$ should be examined. If $g_{\pi pn}$ should return to its former value, it might be suggestive of a large negative $C_1 \simeq -(20\text{–}30)$. However, as we shall see, such a range would be inconsistent with other tests of f_π.

Another test of f_π is provided by the PCAC—anomaly prediction [11] for $\pi^0 \to \gamma\gamma$ in (5). Employing (21),

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.73 \pm 0.01 - 0.018 C_1 \,\text{eV}. \tag{23}$$

That prediction is to be compared with the particle data group value $\Gamma(\pi^0 \to \gamma\gamma)_{\text{expt}} = 7.74 \pm 0.55$ eV where the error has been scaled by a factor of 3 due to experimental inconsistencies [7]. The good agreement is consistent with chiral symmetry breaking which could easily accommodate a 1% or 2% difference [14]. We note, however, that the single best π^0 lifetime experiment [28] suggests $\Gamma(\pi^0 \to \gamma\gamma) = 7.25 \pm 0.23$ which implies a much harder to explain [14] $(6.3 \pm 3.0 - 0.23 C_1)\%$ deviation from (23). That potential discrepancy also needs further experiental study. If confirmed, a very large positive $C_1 \approx +20$ would bring theory and experiment together, but at the expense of weakening the Goldberger-Treiman relation (particularly if $g_{\pi pn}$ reverts back towards its earlier value). It therefore seems that at present $C_1 \simeq 0$ is a good central value in applications of f_π, but not well tested.

As a final application of f_π, we consider the decay $\tau \to \pi\nu_\tau(\gamma)$. Including only the leading short-distance radiative corrections [13] gives

$$\Gamma(\tau \to \pi\nu_\tau(\gamma)) = \frac{G_\mu^2 |V_{ud}|^2 f_\pi^2}{16\pi} m_\tau^3 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \times \left[1 + \frac{2\alpha}{\pi} \ln\frac{m_Z}{m_\tau} + \cdots\right], \tag{24}$$

where $\cdots$ now represent uncalculated $O(\alpha)$ corrections. The full $O(\alpha)$ corrections depend on structure dependent and independent contributions. Employing $f_\pi |V_{ud}| = 127.44$ MeV, $m_\tau = 1777$ MeV, and including leading logs and short-distance QCD corrections [18] to (24) we find

$$\Gamma(\tau \to \pi\nu_\tau(\gamma)) = (2.48 \pm 0.025) \times 10^{-13}\,\text{GeV}, \tag{25}$$

or normalizing in terms of the τ lifetime

$$B(\tau \to \pi\nu_\tau(\gamma)) = (0.1113 \pm 0.0011) \left[\frac{\tau_{\text{tau}}}{2.95 \times 10^{-13}\,\text{s}}\right]. \tag{26}$$

The unknown $O(\alpha)$ corrections have been crudely estimated [29] to give a $\pm 1\%$ uncertainty in (25) and (26). At present, the Particle Data Group gives [7] $B(\tau \to \pi\nu_\tau(\gamma)) \simeq 0.116 \pm 0.004$ which is in rough accord with (26). An interesting confrontation between theory and experiment will be realized when the experimental error on $B(\tau \to \pi\nu_\tau(\gamma))$ reaches the ± 0.001 level. At that point, the full $O(\alpha)$ corrections must be included.

In summary, we have argued that the theoretical uncertainty in $R_{e/\mu}$ is less than $\pm 0.05\%$ and hence presently negligible in the comparison of theory and experiment. Experiments could be pushed another order of magnitude before further theoretical refinements become necessary. We also found $f_\pi = 130.7 \pm 0.1 + 0.15 C_1$ MeV and then employed $C_1 = 0 \pm 2.4$, in applications of f_π. There are, however, at present no precise tests of that uncertainty range. In the future, continued scrutiny of $g_{\pi pn}$, $\Gamma(\pi^0 \to \gamma\gamma)$, and particularly $B(\tau \to \pi\nu_\tau(\gamma))$ should provide consistency checks on C_1 and tests of the standard model.

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