Ion-Neutral Collision Effect on an Alfvén Wave

Yoshimitsu Amagishi[1] and Masayoshi Tanaka[2]

[1]*Faculty of Liberal Arts, Shizuoka University, 836 Ohya, Shizuoka 422, Japan*

[2]*Department of High Energy Engineering Science, Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Kasuga, Fukuoka 816, Japan*

(Received 8 March 1993)

This paper reports that ion-neutral collisions in a magnetized plasma cause a drastic change in the dispersion relation of the shear Alfvén wave with poloidal mode number $m=0$, connecting to the branch of the $m=+1$ compressional Alfvén wave at frequencies below the ion-cyclotron frequency. An anomaly of the dispersion then appears on the refractive index curve and a wave packet in this frequency range undergoes strong amplitude damping and profile deformation. It is confirmed that the Kramers-Kronig relation holds for the dielectric function, estimated from both the measured refractive index and damping rate.

PACS numbers: 52.35.Hr, 52.25.Mq, 52.40.Db

In a magnetized but partially ionized plasma, the dispersion relation, field distribution, and polarization of the shear Alfvén wave (SAW) are strongly influenced by the ion-neutral collision as first presented by Woods [1] and after then extensively discussed [2–5]. Such collisional coupling of ions with neutrals leads to an increased Hall effect, which can be expressed in terms of the pseudo-ion-cyclotron frequency $\omega_{ci}^*=\omega_{ci}/S$, the complex factor S being given by

$$S=1+\frac{\rho_n}{\rho_0}\Bigg/\left(1-i\frac{\rho_n}{\rho_0}\frac{\omega}{\nu_{\text{in}}}\right), \qquad (1)$$

where ρ_0 and ρ_n are, respectively, the mass density of the ion and neutral, and ν_{in} the ion-neutral collision frequency. This is due to the fact that the neutrals are dragged along with the ions and the effective ion mass density $\rho_0^*=S\rho_0$ leads to damping of the SAW, resulting in an energy absorption around a critical frequency ω^*, at which the imaginary part of the parallel wave number reaches its maximum value, which does not exceed the corresponding real wave number.

Generally, a localized absorption in a certain frequency range produces a negative slope on the dispersion curve (anomalous dispersion), because the real and imaginary parts of the dielectric function are related to each other by the causality principle, i.e., the Kramers-Kronig relation. In such a region of the anomalous dispersion, the group velocity apparently exceeds the light velocity or even becomes negative, and hence the so-called "group" velocity is no longer valid for describing wave-packet propagation in anomalous dispersion media [6]. The physical mechanism for the breakdown of the group velocity is attributable to the fact that the Fourier components of the wave packet lying in the anomalous dispersion region (absorption band) decay faster than those in the normal dispersion region; the resultant change in the spectrum distribution after propagating a certain distance causes the deformation of the packet profile as well as the heavy damping of the amplitude [7,8], which violates the basic assumption for the group velocity.

In this Letter, we report the experiment concerned with an anomalous dispersion relation of Alfvén waves in a partially ionized plasma. The experiment was conducted in a linear device TPH [5,9] (test plasma of high density, 15 cm diam and 2 m long) at Shizuoka University with a maximum density of 4×10^{20} m^{-3} of a singly charged helium plasma and an ion temperature $T_i\leq20$ eV and an electron temperature $T_e\sim5$ eV around the plasma center. The ionization rate of the pulsed plasma (duration time ~1 ms) was not determined or controlled with accuracy, but should be less than the usual ionization rates of $\sim80\%$; in this experiment, the gas-puff pressure as well as the timing of the discharge operation was changed to make ionization rates lower. We did not measure precisely the neutral particle density in a plasma, because it is a formidable task to measure it as a function of time of microsecond as well as radial position. The axial magnetic field $B_0=0.30$ T gives an ion-cyclotron frequency $\omega_{ci}=7.2\times10^6$ rad/s. In previous papers [10,11], we used a pair of small-loop antennas to launch the $m=-1$ compressional Alfvén wave (CAW) which is mode converted to the SAW at the Alfvén resonance layer. As far as our knowledge goes, most shear Alfvén waves excited in laboratories have been generated indirectly by virtue of the mode conversion from CAW or MHD surface waves. In this experiment, however, we used another type of small antenna (Fig. 1) in order to directly excite the $m=0$ SAW near the plasma center.

According to Borg *et al.* [12], the Green function of the azimuthal magnetic component of the SAW for the delta function current along the external magnetic field (z direction) is, for our experimental parameters and $\omega/\omega_{ci}\ll1$, approximately given as follows:

$$G_\theta\sim\frac{1}{4\pi V_A r}\exp\left(i\frac{\omega}{V_A}z\right), \qquad (2)$$

where V_A is the Alfvén speed. This suggests the possibility of direct excitation of the $m=0$ SAW in our experi-

0031-9007/93/71(3)/360(4)$06.00

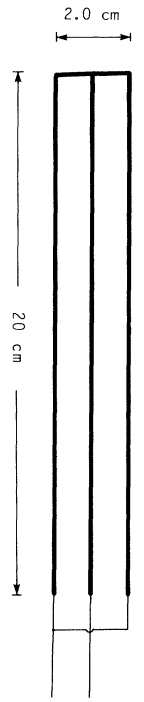

FIG. 1. An antenna to directly excite the $m=0$ shear Alfvén wave at the plasma center.

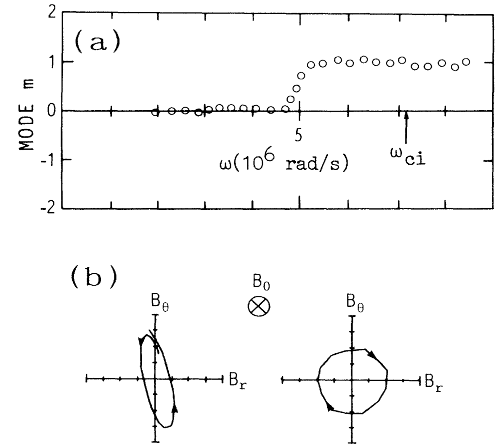

FIG. 2. (a) Measurement of poloidal mode numbers m vs frequency ω at $r=1$ cm. (b) Observation of polarization measured by using small b_θ and b_r probes: left-hand polarization of the $m=0$ mode at $\omega=3.5\times10^6$ rad/s (left) and right-hand polarization of the $m=+1$ mode at $\omega=5.5\times10^6$ rad/s (right).

ment by an external current along the magnetic field in the plasma center. Considering that there is a small deviation of the poloidal mode number from $m=0$ and that the $m=0$ CAW is cut off below ω_{ci} when boundary conditions exist, it may be possible to excite the $m=+1$ (right-hand rotation) CAW near the ion-cyclotron frequency; the dispersion relation of the $m=+1$ CAW is known to be influenced by the plasma density profile in the radial direction and boundary conditions, but not to have a cutoff as long as a vacuum annulus exists between the plasma and a conducting wall [13,14].

The poloidal mode number m was measured by using small magnetic probes azimuthally separated by 90° at $r=1$ cm and $z=23$ cm downstream from the antenna. The results are shown in Fig. 2(a), which suggests the azimuthal mode is changing from the $m=0$ to $m=+1$ at $\omega^*\sim5\times10^6$ rad/s. The polarization of the $m=0$ mode shown in Fig. 2(b) that was measured using b_θ and b_r probes at $r=1$ cm is left handed with the b_θ component much larger than the b_r component, while the $m=+1$ mode is right handed with almost circular polarization. These features in addition to the dispersion relations, which will be shown in Fig. 3, lead to the conclusion that the former is SAW and the latter CAW.

Figure 3 shows the measured frequency and the real wave number parallel to the magnetic field, which were determined from the cross-power spectra of the b_θ components between two wave packets propagating 20 cm along the magnetic field. The $m=0$ mode is obviously in good agreement with the best fit curve of SAW obtained by assuming $\rho_n/\rho_0=\frac{2}{3}$ corresponding to 60% ionization ($\nu_{in}\lesssim\omega^*$) and by taking a plasma density of 4×10^{20} m^{-3}; the curves in the figure are calculated from the dispersion relation of the SAW including ion-neutral collisions given by [5]

$$S\frac{\omega^2}{k_\parallel^2V_A^2}-\left(1-S^2\frac{\omega^2}{\omega_{ci}^2}\right)=0. \qquad (3)$$

On the other hand, the $m=+1$ mode in Fig. 3 follows the dispersion relation of CAW [9] that is little subject to the ion-neutral collision. By taking into account the cross

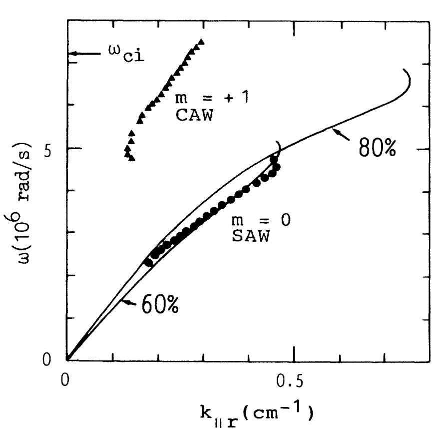

FIG. 3. Measured real values of dispersion relation of the $m=0$ shear Alfvén waves (SAW) and $m=+1$ compressional Alfvén wave (CAW). Solid curves are calculated from Eq. (3) for the ionization rates 60% and 80%.

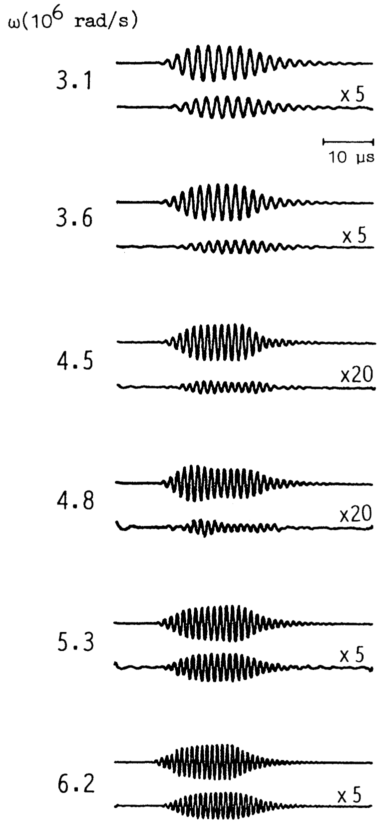

FIG. 4. Component b_θ of the propagating wave packets measured at $z = 3$ cm (top) and 23 cm (bottom) from the antenna. Note the ordinate scale is changed for the signals at 23 cm.

section of singly ionized helium with neutrals $\sigma_{in} \sim 5 \times 10^{-15}$ cm^2, the value of $(\rho_n/\rho_0)(\omega/\nu_{in})$ becomes of the order of unity at $\omega \sim \omega^*$ for our experimental parameters; then one may write the critical frequency from Eq. (3) approximately as

$$\omega^* \approx \omega_{ci} \left[1 + \left(\frac{\rho_n}{\rho_0} \right) \right]^{-1/2}, \tag{4}$$

which gives 5.6×10^6 rad/s for 60% ionization.

One can see in Fig. 3 an anomalous curve of the dispersion relation at $\omega^* \sim 5 \times 10^6$ rad/s, where the wave packet observed at the point $z = 23$ cm downstream from the antenna as shown in Fig. 4 exhibits heavy damping and deformation with anomalous time delay as expected from the theory [7,8]. The complex refractive index $n_\parallel = n_r + i n_i$ in the z direction is calculated from both the dispersion relation shown in Fig. 3 and the damping rate of the wave packet propagating along the z direction; Fig. 5 shows the refractive index as a function of frequency, from which one can obtain the complex dielectric function $\epsilon_\parallel = \epsilon_r + i\epsilon_i$, using the relations $\epsilon_r = n_r^2 - n_i^2$ and

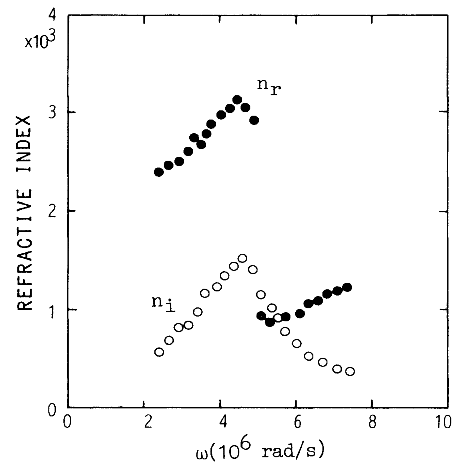

FIG. 5. Complex refractive indices which were obtained from the dispersion relation given in Fig. 3 and damping rate of the wave packet as shown in Fig. 4.

$\epsilon_i = 2n_r n_i$. Figure 6(a) represents the real and imaginary parts of the dielectric function versus real ω. Even though the number of data might not be enough to numerically apply the Kramers-Kronig relation, let us try to estimate the imaginary part of the dielectric function from the real part of the dielectric function ϵ_r given in Fig. 6(a), by calculating the principal integral of the Kramers-Kronig relation. The result, which is shown in Fig. 6(b), indicates a fairly good agreement with experi-

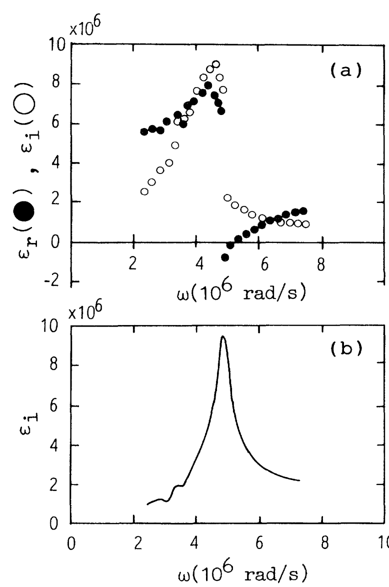

FIG. 6. (a) Complex dielectric function obtained from the refractive index given in Fig. 5. (b) Imaginary part of the dielectric function calculated from the real part of the measured dielectric function of (a), by applying the Kramers-Kronig relation.

mental results shown in Fig. 6(a), except for the frequency width in the absorption region.

In conclusion, by measuring the propagation delay and damping rate of the Alfvén waves transmitted through a partially ionized plasma along the magnetic field, we have presented the first experiment to prove the Kramers-Kronig relation for the magnetized plasma with the anomalous dispersion, where the ion-neutral collision effect gives rise to the anomaly of dispersion and localized strong absorption. Hence, the SAW with a frequency close to ν_{in} suffers from tremendous decay and deformation in the wave packet during the propagation. The packet velocity in the anomalous dispersion region decreases in contrast to the behavior of the group velocity defined in the normal region, the details of which will be presented in a subsequent paper. For further study we plan to control and measure the neutral particle density, which will reveal more clearly the physical mechanism in such an absorption region of magnetized plasmas.

This work was supported in part by a Grant-in-Aid for Science Research from the Ministry of Education, Science and Culture of Japan.

[1] L. C. Woods, J. Fluid Mech. **13**, 570 (1962).
[2] D. G. Swanson, R. W. Gould, and R. H. Hertel, Phys. Fluids **7**, 269 (1964).
[3] D. A. McPherson and D. C. Pridmore-Brown, Phys. Fluids **9**, 2033 (1966).
[4] G. Müller, Plasma Phys. **16**, 813 (1974).
[5] Y. Amagishi and A. Tsushima, Plasma Phys. Controlled Fusion **26**, 1489 (1984).
[6] L. Brillouin, *Wave Propagation and Group Velocity* (Academic, New York, 1960).
[7] M. Tanaka, Plasma Phys. Controlled Fusion **31**, 1049 (1989).
[8] G. G. Borg, M. H. Brennan, R. C. Cross, L. Giannone, and I. J. Donnelly, Plasma Phys. Controlled Fusion **27**, 1125 (1985).
[9] Y. Amagishi, K. Saeki, and I. J. Donnelly, Plasma Phys. Controlled Fusion **31**, 675 (1989).
[10] Y. Amagishi, Phys. Rev. Lett. **64**, 1247 (1990).
[11] Y. Amagishi, J. Phys. Soc. Jpn. **59**, 2374 (1990).
[12] C. Cross and J. A. Lehane, Phys. Fluids **11**, 2621 (1968).
[13] K. Appert, J. Vaclavik, and L. Villard, Phys. Fluids **27**, 432 (1984).
[14] G. A. Collins and N. F. Cramer, Plasma Phys. Controlled Fusion **26**, 273 (1984).