Little-Parks and Aharonov-Bohm Oscillations in Fractional Hall Regime: Manifestation of Chern-Simons Gauge Flux

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We show that the recently developed Chern-Simons gauge theory for fractional quantum Hall effect leads naturally to a striking prediction that an applied gate voltage coupled to an isolated region in a two-dimensional electron gas can induce Little-Parks oscillations of the longitudinal conductance in the quantum Hall devices at odd denominator filling fractions. For even denominator filling fractions, the Chern-Simons fermions are in a Fermi liquid state, and the conductance fluctuations are similar to the Aharonov-Bohm conductance oscillations of a mesoscopic metal.

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Recently a new theoretical framework has been developed to describe the fractional quantum Hall effect (FQHE) in two-dimensional electron gas (2DEG) systems, based on a Chern-Simons gauge theory which can be shown to map exactly to the original interacting electron system [1, 2]. Although this new theoretical framework can be shown to be equivalent to the original Laughlin variational wave-function approach, it offers the distinct advantage of providing a simple, macroscopic Ginzburg-Landau description of this complex interacting electron problem, somewhat similar to the Ginzburg-Landau approach for the superconductivity problem. The order parameter of this theory is related to the hidden off-diagonal long-ranged order in the Laughlin's wave function [3]. Many interesting new theoretical concepts have been developed on the FQHE using this new theoretical insight, such as the prediction of the Hall insulator state, the mapping rules between the various different fractional Hall states as well as the integer quantum Hall states, and the global phase diagrams for the FQHE [4]. This Chern-Simons-Landau-Ginzburg (CSLG) theory has also been applied recently to the even denominator (e.g., $\nu = \frac{1}{2}$) regime with some success [5, 6], where the effects of dynamical fluctuations of the gauge field on the Fermi surface properties have been studied, in particular in relation to interpreting the acoustic data at the one-half filling factor [7]. The overall features of the magnetotransport data around even denominator filling fractions can also be explained within this model [6, 8].

The success of this new approach to quantum Hall effect is based in large part on the analogy with some known condensed matter systems that have been studied extensively in the past. In the case of the odd denominator filling fractions, the electrons are mapped to Chern-Simons (CS) bosons in a superfluid ground state, and in the case of even denominator filling fractions, they are mapped to Chern-Simons fermions in a Fermi liquid ground state. From this point of view, various experiments in the quantum Hall systems are manifestations of the known properties of these two well studied condensed matter systems [1–6]. In view of this success, it would be highly desirable to have a direct experimental test of this new picture for FQHE.

In this Letter, we propose a series of experiments that could provide such a direct test. In particular, we consider an inhomogeneous fractional Hall system with an external capacitive gate applied to an isolated region in the center of the device. We make a striking prediction that variations of the gate voltage should induce Little-Parks (LP) or Aharonov-Bohm (AB) type oscillations in the system's longitudinal conductance. For the case of odd denominator fractional Hall states, the Chern-Simons bosons are in a superfluid state, and they see the applied gate voltage as a source of magnetic flux. The requirement of flux quantization of a superfluid state leads to the quantized fractional charge of the quasiparticles, and the varying gate voltage leads to an effect similar to varying an external flux in a superconductor in the Little-Parks experiment: The resistance oscillates with the period of the fundamental flux quantum. In the case of even denominator filling fractions, the Chern-Simons fermions are in a Fermi liquid ground state. They too see the applied voltage as a source of magnetic flux, and the effect of varying the gate voltage is similar to the effect of varying the Bohm-Aharonov flux in a normal metal ring: If the sample size is smaller than the inelastic mean free path, there are well defined oscillations in conductance with a period of hc/e.

Our starting point is the CSLG Hamiltonian for

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the FQHE:

$$H = \frac{1}{2m^*}\int d^2x\,\phi^\dagger(\mathbf{x})\left[\left(\frac{1}{i}\boldsymbol{\nabla} + \frac{e}{c}\mathbf{A}(\mathbf{x}) - \frac{e}{c}\mathbf{a}(\mathbf{x})\right)^2 - eU(\mathbf{x})\right]\phi(\mathbf{x}) + \frac{1}{2}\int d^2x\,d^2y\,\delta\rho(\mathbf{x})V(\mathbf{x}-\mathbf{y})\delta\rho(\mathbf{y})\,, \quad (1)$$

where $U(\mathbf{x})$ is the electrostatic potential acting on the electron gas, either from random impurity scattering potential or from an applied gate voltage, and $\mathbf{A}(\mathbf{x})$ is the vector potential due to the applied magnetic field B (in the z direction perpendicular to the 2DEG). $V(\mathbf{x}-\mathbf{y})$ is the (Coulomb) interaction potential among electrons, and $\delta\rho(x) = \rho(x) - \bar{\rho}$ is the deviation from uniform density. $\mathbf{a}(\mathbf{x})$ is the Chern-Simons gauge field, defined as

$$b(\mathbf{x}) = \nabla \times \mathbf{a}(\mathbf{x}) = \phi_0 m\rho(\mathbf{x}), \quad (2)$$

where $\phi_0 = hc/e$ is the flux quantum. When $m = 2k+1$ is an *odd* integer, the matter field $\phi(\mathbf{x})$ corresponds to a *boson* field. Whereas for *even* integer $m = 2k$, $\phi(\mathbf{x})$ corresponds to a fermion field. Since the filling factor $\nu = \bar{\rho}\phi_0/B$, we see that when $\nu = 1/m$, the new CS particles see no net magnetic field, except a fluctuating one due to inhomogeneities in the self-consistent particle density, $\delta\rho(\mathbf{x})$.

To illustrate our idea of gate voltage induced Little-Parks oscillations due to the CS gauge field, let us consider a high mobility 2DEG sample where a capacitive gate of mesoscopic dimension (e.g., with an area of order $A \sim 10^{-10}$ cm^2) is coupled to an isolated region of the sample, as depicted in Fig. 1. Assume the external magnetic field is such that $\nu = 1/(2k+1)$. The new CS particles [with m in Eq. (2) chosen to be $2k+1$] described by operators $\phi(\mathbf{x})$ obey Bose statistics, and these particles on average do not see an average magnetic field, except a fluctuating one due to $\delta\rho(\mathbf{x})$. It is by now well established that these CS bosons condense into a Bose-Einstein superfluid, which is responsible for the observed fractional quantum Hall effect in the system's transport properties ($\sigma_{xx} \to 0$, and $\sigma_{xy} = \frac{1}{2k+1}\frac{e^2}{h}$) [1, 2]. Suppose we now turn on a gate voltage V_g which is coupled capacitively to the central "hole" region in Fig. 1. The application of this voltage leads to a change in the local density $\delta\rho(x) = \rho(x) - \bar{\rho}$, which in turn results in a net magnetic field $\delta b(x) = B - b(x)$ localized inside the hole region. However, the net flux inside a multiply connected superfluid system Φ is not arbitrary, but quantized in units of $\phi_0 = hc/e$, chosen in such a way as to minimized the "kinetic" energy

$$E(\Phi) = (\Phi - \Phi_g)^2/2L\,. \quad (3)$$

Here $\Phi_g = (2k+1)CV_g\phi_0/e$ is the flux induced by the gate voltage and can be changed continuously, C is the capacitance of the hole region with respect to the gate, and L is a parameter which can be related to the quasiparticle creation energy. The quantization condition is accurate as long as the sample width is large compared with the magnetic length. For $\Phi_g = N\phi_0$, $E(\Phi)$ is depicted in Fig. 2(a). In this case we obtain $\Phi = N\phi_0$, and we see that there is a gap to adding one more flux quantum to the system, $\Delta E = \phi_0^2/2L$. Integrating Eq. (2) over two-dimensional space we obtain $\Phi = (2k+1)\phi_0 Q/e$, where Q is the induced charge on the hole. Thus adding one unit of flux is equivalent to adding $Q = e/(2k+1)$ unit of fractional charge, and this energy can therefore be identified with the creation energy of a Laughlin quasiparticle, or the incompressibility gap. Denoting this gap by $\Delta(2k+1)$, this allows us to identify the parameter L through $\phi_0^2/2L = \Delta(2k+1)$. However, when $\Phi_g = (N+1/2)\phi_0$, $E(\Phi)$ is shown in Fig. 2(b), and we see that $\Phi = (N+1)\phi_0$ is degenerate with $\Phi = N\phi_0$. The incompressibility gap vanishes and dissipation could occur in this case. Equations (2) and (3) therefore lead to a highly nonlinear, in fact stepwise dependence of the screening charge Q on the applied gate voltage [9], as shown in Fig. 2(c). Whenever $\Phi_g = (N+1/2)\phi_0$, Q changes by $e/(2k+1)$. At this value of Φ_g, there is no gap to add a fluxon to the hole region, and the change in Q, or equivalently Φ, is accomplished by the spontaneous creation of a quasiparticle and quasihole pair (equivalent to a fluxon antifluxon pair) in the bulk followed by a tunneling process in which the quasihole goes to the center hole region and the quasiparticle goes to the edge of the sample. Since the tunneling motion of fluxons across the superfluid region is associated with a longitudinal voltage drop (due to the Faraday effect), one

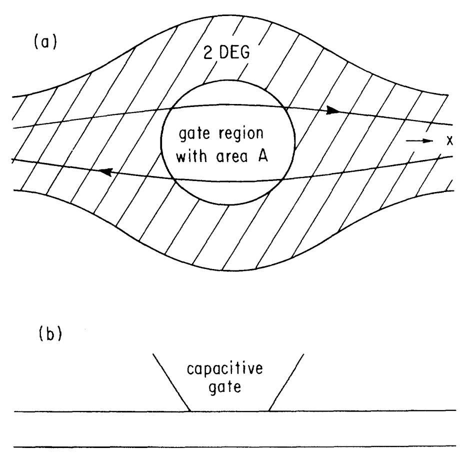

FIG. 1. Geometry of the inhomogeneous two-dimensional electron gas device under our study. It has an Aharonov-Bohm ring type geometry. The central (isolated) region of area A is coupled to a capacitive gate. The two solid lines indicate the forward and backward Feynman paths which are necessary for calculating the Aharonov-Bohm conductance oscillations.

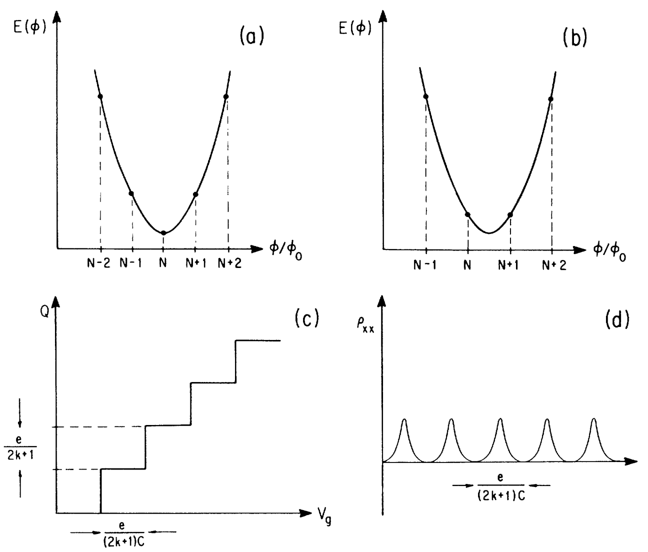

FIG. 2. For odd denominator states: (a) Energy as a function of Φ for $\Phi_g = N\phi_0$. (b) Energy as a function of Φ for $\Phi_g = (N+1/2)\phi_0$. (c) Induced charge Q on a capacitive gate as a function of gate voltage, Q jumps by $e/(2k+1)$ whenever $\Phi_g = (N+1/2)\phi_0$. (d) Oscillations of the longitudinal resistance as a function of gate voltage. Resistance peaks at $\Phi_g = (N+1/2)\phi_0$, similar to the peaks in the Little-Parks experiment of a superconductor. Note the period in the gate voltage is proportional to $\nu = 1/(2k+1)$.

should observe a resistance peak at this value of Φ_g. The fundamental periodicity in Φ_g is ϕ_0. This implies a voltage periodicity of $\Delta V_g = \frac{e}{(2k+1)C}$, i.e., proportional to $\nu = 1/(2k+1)$. These resistance oscillations are similar to the Little-Parks resistance oscillations of a superconductor in an applied magnetic field, where resistance peaks at $\phi = (N+1/2)\phi_0/2$; the factor of 2 difference with our case comes from the charge $2e$ of the Cooper pairs. The resistance in between the peaks should vanish exponentially as temperature approaches zero. We suggest that in addition to measuring the resistance oscillations at a given temperature, one also measure the activation gap from the temperature dependence, to verify that it vanishes at $\Phi_g = (N+1/2)\phi_0$.

However, it has been argued by Thouless and Gefan [10] that at zero temperature the process of quasiparticle and quasihole tunneling proceeds with a rate that depends exponentially on the width of the Hall bar. Therefore, the gate potential has to be varied at a rate slower compared to the quasiparticle quasihole tunneling rate in order to observe the above mentioned voltage periodicity. A much faster rate is the tunneling of a real electron from the center hole region to the edge, since the tunneling barrier for this process is that between the 2D electron gas and the surrounding reservoir. If the rate for varying the gate potential is faster than the quasiparticle quasihole tunneling rate but slower than the tunneling rate of real electrons, then the resistance oscillation with a voltage periodicity of $\Delta V_g = e/C$ is expected. This periodicity does not contain any information about the fractional charge.

This phenomenology is very similar to that of the Coulomb blockade of a quantum dot coupled to capacitive gate [11]. Indeed, the Coulomb blockade mechanism for quasiparticles with charge $e^* = e/(2k+1)$ has been recently invoked [12] to interpret the resistance oscillations experimental data by Simmons *et al.* [13]. In this picture, the scattering of edge states gives rise to the resistance peaks mentioned earlier. In the absence of the applied gate voltage, resonance tunneling is prohibited since the intermediate state has a Coulomb energy of $(e^*)^2/2C$ above the Fermi energy. However, in the presence of an external gate voltage, this is reduced by an amount of e^*V_g; thus at $V_g = e^*/2C$, the energy of the central Coulomb island is degenerate with the Fermi energy, and the resonant scattering between the two edge states gives rise to a peak in the longitudinal resistance.

In a series of experiments by Simmons *et al.* [13], the conductance fluctuation is measured by both varying the external magnetic field and the uniform back gate voltage. The observed periodicity in back gate voltage is the same for $\nu = 1$ and $\nu = \frac{1}{3}$. This experiment is originally interpreted in terms of the theory by Kivelson and Pokrovsky [14], in which a magnetic flux period of hc/e^* is assumed. This observed period is different from the prediction we made above, in which the voltage periodicity of $\Delta V_g = e^*/C$ is expected. We have two general comments concerning this point. First of all, this experiment is performed with a variation of the back gate voltage that acts on all electrons, unlike the case discussed here where a local potential is varied. The local potential in our case can be changed continuously in a controlled way, whereas the impurity potential from the donors arises from quantized positive charge, which preferably attracts integral numbers of electrons. Second, even in the case where the local potential can be varied continuously, one has to be sure that the rate of the variation is slow compared with the tunneling rate of the quasiparticles, as discussed above.

We now consider a device with the same multiply connected geometry, but for magnetic field at an *even* denominator state $\nu = \frac{1}{2k}$. For concreteness, let us first consider the fraction $\nu = \frac{1}{2}$. The new CS particles in this case are still fermions, but they do not on average see a net field in the absence of a gate voltage. Thus one can still speak of a Fermi surface for these CS particles, with a Fermi wave vector $k_F = (4\pi n_e)^{1/2} = 1/l_B$, where $l_B = (hc/eB)^{1/2}$ is the magnetic length at this filling factor [5]. The transport properties of these particles are approximately the same as electrons in the presence of random magnetic flux with a zero average field [5, 6, 8]. When a gate voltage is applied, the effective magnetic field for the CS particles in the hole region becomes nonzero, with a value

$$\delta b = -2\phi_0 \delta\rho = \frac{2C}{eA}\phi_0 V_g. \tag{4}$$

But here since we are still dealing with interacting fermions, they do not have superfluid properties, but instead have metallic (Fermi liquid) behavior.

Thus we can think of the present situation as an Aharonov-Bohm geometry, where a small isolated "hole" region has a magnetic field, whereas outside this region, the CS electrons do not feel any magnetic field. The amplitude transmission coefficient for the single CS electron wave function from one current terminal to the other can be written for a sufficiently clean sample as

$$t \sim \sum_p A_p \exp\left(i\int_p \delta\mathbf{a}(\mathbf{x})\cdot d\mathbf{x}\right), \tag{5}$$

where p denotes Feynman paths for a single CS electron wave function, A_p denotes the amplitude for this path, and $\delta\mathbf{a}(\mathbf{x})$ is a vector potential for the $\delta b(\mathbf{x})$ field which is nonvanishing only inside the hole region.

Using the Landauer formula $\sigma_{xx} \propto |t|^2$, we find, assuming that A_p is a smooth function which we take as approximately constant (semiclassical approximation) [15, 16],

$$\sigma_{xx} \sim \sum_\phi \delta\sigma_\phi \cos\left(2\pi\frac{\phi}{\phi_0}\right) \approx \sigma_{xx}^0 \frac{\sum_\phi \cos(2\pi\phi/\phi_0)}{\sum_\phi 1}, \tag{6}$$

where ϕ denotes the flux from the δb field enclosed by a loop made from a forward Feynman path and a backward one (see Fig. 1), and σ_{xx}^0 denotes the conductance of the sample at zero gate voltage. Here we have assumed the sample size $L \leq L_\phi$, where L_ϕ is the phase coherence length for the CS electrons, and we have made the reasonable approximation that $\delta\sigma_\phi$ is a smooth function of ϕ that we may treat as a constant. Since the field δb that the CS electrons see is confined to an area A in the hole region, the distribution of ϕ is centered around zero and $\phi = 2\phi_0 C V_g/e$. Therefore, when one plots the Fourier spectrum of the resistance fluctuation as a function of the voltage, one should see one peak centered around zero, and another one centered around $\Delta V_g = e/2C$. We remark that the use of semiclassical approximation here is justified by the high mobility ($> 10^6$ cm^2/V sec) of the 2DEG samples usually used in such experiments. When scattering from impurities must be included, the form of the Feynman paths becomes more complicated, but the qualitative features of the AB oscillations remain unchanged, provided that the sample size $L \leq L_\phi$, and the area of the ring region is chosen to be small compared to the area of the hole region, such that the "universal conductance fluctuations" occur at a much larger field scale [15].

We may estimate the value of this characteristic period ΔV_g as follows. It is known experimentally that the change in 2DEG gas density is proportional to the gate voltage by a constant of order 10^{11} cm^{-2}/V. Thus the constant C/eA is of order 10^{11} cm^{-2}/V. This gives the characteristic cutoff Aharonov-Bohm period $\Delta V_g \sim \frac{1}{2} \times 0.1$ V, for a hole region of size of order 1000 Å×1000 Å. With the availability of the modern electron lithography technique, a mesoscopic gate of this dimension is not difficult to achieve experimentally. The condition that the sample dimension as a whole be within the phase coherence length is also not hard to satisfy, since at 100 mK type temperatures the phase coherence length L_ϕ for clean samples can be longer than 5 μm.

We can readily generalize our picture above to any other even denominators $\nu = \frac{1}{2k}$. It is important to notice that when one goes from one even denominator state to another, e.g., from $\nu = \frac{1}{2}$ to $\nu' = \frac{1}{4}$, the entire oscillatory function $\sigma_{xx}(V_g)$ (for a given sample) should map onto each other, with a simple rescaling of the horizontal axis $V_g \rightarrow \frac{\nu'}{\nu} V_g$. Thus we see that the Aharonov-Bohm oscillations at various even denominator fractional quantum states should reveal intimately the Chern-Simons flux parameter $m = 1/\nu$.

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