Dissipation of the ^{3}He $A \rightarrow B$ Transition

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A rigorous hydrodynamic theory of the A-B transition is presented. All dissipative processes are considered. At low interface velocities, those occurring on hydrodynamic length scales, not considered hitherto, are most probably the dominant ones.

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The $A \rightarrow B$ transition of superfluid ^{3}He is rather remarkable. If undercooled sufficiently, it takes place with a spectacularly fast rate, and is accompanied by magnetic signals that can only be called bizarre [1]. However, no-one was left wondering about the damping mechanism, as Yip and Leggett [2] instantly identified it: The superfluid order parameter varies rapidly within the interface, transforming one phase into the other. This scatters quasiparticles (Andreev scattering) and constitutes a restoring force. The balance between this and the driving force $\Delta\mu$ (the difference in chemical potential of the two phases) yields a terminal interface velocity $\dot{u}$ that can be compared with experimental data [1]. Further and more detailed microscopic calculations [3] confirmed Andreev scattering as the source of damping; also, the magnetic signals were recently deciphered [4].

All this, one should think, holds for the hypercooled regime, with an undercooling $\varepsilon \equiv 1 - T/T_{AB} \gtrsim 0.5\,\%$. With ε smaller, the latent heat would warm up the B phase, and render it thermodynamically unstable again. So $\dot{u}$ is much slower and limited (instead of by Andreev scattering) by how efficiently the latent heat can be removed from the interface region (a difficult, nonlocal problem notorious from more mundane interfaces such as snow flakes). This is quite wrong: In superfluid ^{3}He, there is neither a transition regime limited by heat transfer, nor a sudden onset of hypercooled phase transition. Rather, it is the second-sound velocity c_2 that separates two different types of transitional behavior. For $\dot{u} \ll c_2$, or $\varepsilon \lesssim 2\,\%$, second sound is very efficient in removing the latent heat, which therefore cannot be the limiting factor. What is more, phase coherence across the interface equalizes the chemical potential, eliminating $\Delta\mu$ as the driving force. A hydrodynamic consideration [5] shows instead an interface driven by ΔT and damped by the Kapitza resistance. Curiously, the growing B phase is in this regime of "phase-coherent transition" colder than the receding A phase. When $\dot{u}$ greatly exceeds c_2, starting at $\varepsilon \approx (20\text{–}30)\%$, second sound is in comparison too slow to transfer appreciable amount of heat. Only then does the original scenario of hypercooling reemerge.

Following all the microscopic theories [2, 3], the hydrodynamic consideration [5] also contains the starting assumption that the dissipation accompanying the A-B transition occurs within the mean free path ξ_f of the interface, and that no dissipative temperature variations exist on hydrodynamic length scales. This universal assumption is most probably incorrect. To understand why, we first examine the case of a stationary interface between superfluid ^{3}He and a vessel wall, through which heat but no mass is transferred. Generally, the effective, measured resistance here is the sum of two contributions [6], $\kappa_e^{-1} = \kappa^{-1} + \kappa_{\rm sq}^{-1}$. The first accounts for the microscopically fast drop ΔT across the interface, the second stems from the "sq mode," a hydrodynamically slow variation $\delta T \exp(-|x|/\lambda_{\rm sq})$ in the superfluid. Because of the enormous extent of the decay length ($\lambda_{\rm sq} \sim T_f/T_c$ is at least 250 times the mean free path, usually much larger) the effective resistance κ_e^{-1} is dominated by $\kappa_{\rm sq}^{-1}$ [7]. Consequently, $\delta T \gg \Delta T$. Going back to the moving A-B interface, it is clear that something akin to the sq mode could also exist there. As we shall see, this is indeed the case. And since this (what we continue to call) sq mode has, for $\dot{u} \ll c_2$, essentially the same spatial extent, it is here probably also the dominant source of dissipation.

What is more, there is some indication that, independently, $\Delta T \rightarrow 0$. Recently, Schopohl and Waxman [8] considered a moving interface, between the A and B phase that are in equilibrium otherwise. In contrast to all previous microscopic calculations [2,3] that are perturbative in essence, they have obtained an exact solution, in the ballistic limit, with an essential singularity at $\dot{u} = 0$. Amazingly, they found this motion to be (up to a fairly high critical velocity) little damped [9]. As will be shown below, the immediate consequence of this is a diverging Kapitza conductance, and $\Delta T \rightarrow 0$ for $\dot{u} \ll c_2$. In other words, if this finding can be verified, Andreev scattering as a dissipative source is eliminated altogether, while the hydrodynamic variation of temperature and counterflow becomes the only mechanism to prevent the transition rate $\dot{u}$ from diverging.

In this paper, we present the general hydrodynamic theory of the A-B transition. All dissipative mechanisms that may occur are considered. Despite a rather different language, they include collisions and scattering of quasiparticles, both among themselves and at the interface. More specifically, we derive the general boundary conditions connecting two strongly coupled superfluids and

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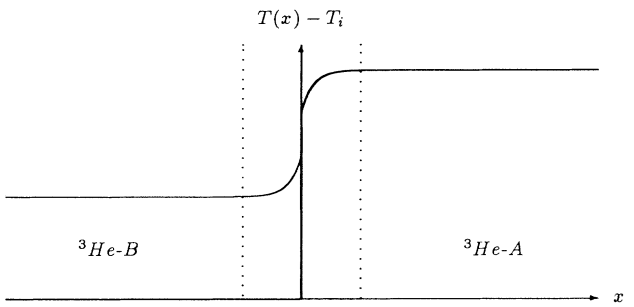

FIG. 1. The temperature field for $\dot{u} \ll c_2$, as in Eq. (1a).

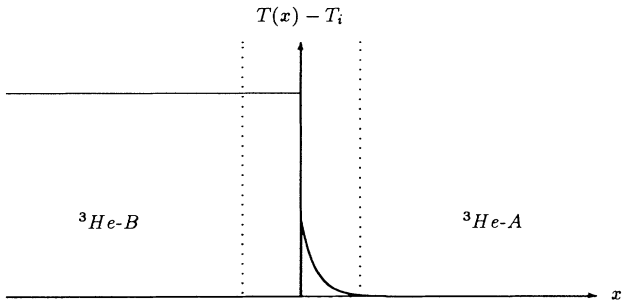

FIG. 2. The temperature field for $\dot{u} \gg c_2$, as in Eq. (2a).

calculate the temperature and counterflow fields. Although the hydrodynamic theory is never complete by itself, our results do provide a rigorous framework for the more detailed, and rather more complicated, microscopic theory. In fact, the latter is essentially reduced to the calculation of three Onsager coefficients.

Concrete results are obtained for the two limits $\dot{u} \ll c_2$ and $\dot{u} \gg c_2$. In the first case of slow, phase-coherent transition, the general temperature variation contains two exponential decays $\delta T_{\rm sq}^{A,B} \exp(-|x|/\lambda_{\rm sq})$ in the respective phase, and a discontinuity ΔT at the interface ($x = 0$); cf. Fig. 1. While $\delta T_{\rm sq}^{A,B}$ stem predominantly from collisions among quasiparticles, ΔT accounts for their scattering at the interface. (The counterflow is not independent, $\delta w^{A,B} \sim \delta T^{A,B}$.) The decay length $\lambda_{\rm sq}$ is a function of known bulk coefficients; to lowest order in $\dot{u}/c_2$ it is equal to the decay length, mentioned above, of ^{3}He close to a vessel wall, and hence large. The interface motion is damped by a total, effective Kapitza resistance, which is a series of three consecutive resistive elements, each causing one of the temperature drops. The amplitudes of these are determined by three Onsager coefficients, unknown in size. So it is these three numbers that need to be calculated, or measured. Until now, it was assumed that $\delta T_{\rm sq}^{A,B} = 0$, leaving ΔT to account for the total dissipation. If, conversely, ΔT is negligible as mentioned, one may (for lack of better knowledge) assume $\delta T_{\rm sq}^{A} \approx \delta T_{\rm sq}^{B}$. Then the total Kapitza resistance depends only on one parameter, which can be determined from the experimental data on $\dot{u}$, as we shall do.

If $\dot{u} \gg c_2$, the varying fields of the temperature and counterflow $\sim \delta T_d^A$, δw_d^A are independent, diffusive, and decay only into the A phase; cf. Fig. 2. The decay length is smaller by the factor $c_2/\dot{u}$. Neither the temperature nor the chemical potential is continuous across the interface; ΔT, $\Delta\mu \neq 0$. There is no special reason why ΔT should be much larger or smaller than δT_d^A. The interface motion is damped by a total growth coefficient which, however, contains additive as well as multiplicative contributions.

It is noteworthy that all the results of Ref. [5] remain asymptotically valid (i.e., for distances from the interface that are large compared to all decay lengths), if one substitutes the respective resistance with the total Kapitza and growth resistance obtained here. As will be explained in detail below, this is connected to the fact that one can consider an effective interface, hydrodynamically wide, that includes all the temperature and counterflow variations; cf. the dotted lines of Figs. 1 and 2. Then, of course, the original assumption that dissipation takes place only within the interface is again correct. In this work, for lack of space, we do not consider the effects of lateral walls, which lead to an R dependence of the terminal velocity $\dot{u}$, as observed [1].

An interface in motion can be viewed as condensate and quasiparticles traversing the interface. It is plausible that the condensate should not be damped. But the Schopohl-Waxman solution [8] shows that even the quasiparticles are little damped in equilibrium, despite considerable Andreev scattering. This is a surprising result, and as the following arguments show, has direct bearing on the nonequilibrium properties of the interface: Usually, the temperature establishes itself on the scale of the mean free path ξ_f, and the temperature gradient ∇T has a hydrodynamic scale much larger. However, across a strongly resistive obstacle of microscopic dimension $\xi \ll \xi_f$, the change in the temperature will be on the same scale ξ and can be hydrodynamically accounted for as a discontinuity ΔT. The A-B interface, with a width of order correlation length $\xi_c \ll \xi_f$, was taken as just such a microscopic obstacle [2,3]. And its resistivity (outside a very narrow range next to the normal-superfluid transition) would come mainly from Andreev scattering of ballistic quasiparticles. If this is indeed inoperative in equilibrium, it cannot turn into a strongly resistive mechanism ever so slightly off equilibrium. The temperature gradient will therefore have normal, hydrodynamic values, and $\Delta T \approx \xi_f \nabla T$ vanishes. A more formal line of arguments that shall be published elsewhere leads to the same conclusion. Further away from equilibrium, when $\dot{u}$ becomes comparable to, or much larger than, the second sound velocity c_2, $\Delta\mu$ builds up across the interface [5]. This would constitute the microscopic obstacle lacking at $\dot{u} \ll c_2$, and an accompanying ΔT can no longer be ruled out by the same argument.

We start our hydrodynamic consideration with the

general solution that is stationary in the rest frame of the interface. For both $\dot{u} \ll c_2$ and $\dot{u} \gg c_2$, we may linearize the hydrodynamic equations [10] as in Ref. [5], with respect to the variables (i) $w \equiv \rho_s(v_n - v_s)/\rho_n \equiv v_n - g/\rho$ and (ii) $T^{A,B} - T_i$, the deviation of the temperature in the respective phase from the initial temperature T_i. Retaining terms of first order in $\dot{u}/c_2$, the solution (in both phases) for $\dot{u} \ll c_2$ is

$$T^{A,B} = T_i + \delta T_2^{A,B} + \delta T_{\rm sq}^{A,B} \exp(\mp x/\lambda_{\rm sq}) , \tag{1a}$$

$$w^{A,B} = \pm \frac{c_2 \sigma_T}{\sigma} \left(\delta T_2^{A,B} - \sqrt{\frac{\lambda_T}{\lambda_w}} \delta T_{\rm sq}^{A,B} \exp\left(\mp \frac{x}{\lambda_{\rm sq}}\right) \right) . \tag{1b}$$

Notations and explanations: Upper sign refers to the A phase, here and below. $\delta T_2^{A,B}$: amplitude of the second-sound step function in the respective phase [5]. Although the steps are at $\pm c_2 t$, $t \to \infty$ must be set, since Eqs. (1) display the stationary solution. (Here and below, if the context is clear, the superscripts A and B, e.g., in c_2^A, will be suppressed.) $\delta T_{\rm sq}^{A,B}$: amplitude of the sq mode, source of hydrodynamic dissipation and resistance. σ: entropy per unit mass, $\sigma_T \equiv \partial\sigma/\partial T$. $\lambda_{\rm sq}^{A,B} = 2(\lambda_T \lambda_w)^{1/2} \mp (\lambda_T + \lambda_w)\dot{u}/c_2$: the sq decay length for a moving interface, $\lambda_T \equiv k/2c_2\rho T\sigma_T$, $\lambda_w \equiv [(4/3)\eta - \rho(\zeta_1 + \zeta_4) + \zeta_2 + \rho^2\zeta_3]\rho_s/2\rho\rho_n c_2$, where the heat conductance k and the viscosities η, ζ_{1-4} are defined in the usual way [10], neglecting the anisotropy.

The solution for $\dot{u} \gg c_2$ is, to lowest order in $c_2/\dot{u}$,

$$T^B = T_i + \delta T^B , \quad T^A = T_i + \delta T_d^A \exp(-\dot{u}x/2c_2\lambda_T) , \tag{2a}$$

$$w^B = \delta w^B , \quad w^A = \delta w_d^A \exp(-\dot{u}x/2c_2\lambda_w) . \tag{2b}$$

δT_d^A and δw_d^A are, respectively, the diffusive modes of a moving interface [11]. Next order terms in $c_2/\dot{u}$ mix these two modes.

Each of the four amplitudes of Eqs. (1,2) is to be determined in conjunction with $\dot{u}$ from boundary conditions, better: connecting conditions (CCs). The general structure of the CCs depends, as do bulk hydrodynamic theories, only on the conserved quantities and the spontaneously broken symmetries on both sides of the interface [6]. In our case, the CCs are given by the continuity of the fluxes for energy, mass and momentum, the phase coherence across the A-B interface, and the surface entropy production rate R_s. These are, respectively,

$$\Delta Q = \Delta g = \Delta(p + \pi + \pi^D) = 0 , \tag{3a}$$

$$\Delta\dot{\varphi} \equiv -\Delta(\mu + v_n v_s + z^D) = 0 , \tag{3b}$$

$$R_s = \langle f \rangle \Delta T + g\Delta(\mu + z^D) + \Delta(v_n(\pi + \pi^D - \rho z^D)) . \tag{3c}$$

(p: pressure; π: the nonlinear part of the stress tensor, π^D its dissipative part; z^D: dissipative part of the Josephson equation.) Equations (3) reduce to the expressions of Ref. [5] if one excludes dissipative terms (with superscript D). All quantities are defined in the interface system; $\langle\ \rangle$ and Δ denote average and difference across the interface, and all suppressed indices point along the interface normal. Neglecting $\Delta\rho/\rho \sim 10^{-8}$ and for time scales slow compared to first sound velocity, $g = -\rho\dot{u}$ holds and $\Delta g = 0$ is always satisfied. Linearizing the other CCs, for the weakly supercooled case $\dot{u} \ll c_2$, with respect to w, $\dot{u}$, and ΔT, we obtain

$$\Delta f = 0 , \quad \Delta(p + \pi^D) = 0 , \ \Delta(\mu + z^D) = 0 , \tag{4a}$$

$$\langle f \rangle = \kappa \Delta T , \quad v_n^{A,B} = \mp \alpha_{A,B} (\pi^D - \rho z^D)^{A,B} . \tag{4b}$$

Equations (4b) are the Onsager relations that follow from R_s of Eq. (3c). The last two CCs are new: Neglecting dissipative terms, they would vanish (first in R_s and hence altogether). Positivity of entropy production requires $\alpha_{A,B} > 0$; the cross terms, such as $v_n^A \Delta T$ in R_s, are neglected for simplicity. The values of $\alpha_{A,B}$ determine the rate of dissipation both within the interface (contribution to R_s) and outside (contribution from the sq mode). The latter with a vastly larger width $\sim \lambda_{\rm sq}$, dominates.

We expand Eqs. (4) around T_i and denote all thermodynamic quantities at that temperature. To distinguish, a square bracket with index i is added, e.g., $[\Delta\mu]_i \equiv \mu_B(T_i, p_i) - \mu_A(T_i, p_i)$; while in Eqs. (4a) $\Delta\mu \equiv \mu_B(T_B, p_B) - \mu_A(T_A, p_A)$. With $\tilde{\alpha} \equiv \alpha c_2 \rho\rho_n/\rho_s + (\lambda_T/\lambda_w)^{1/2}$, the results are

$$\delta T_2^{A,B} = \tfrac{1}{2}[\mp \Delta\mu/\langle\sigma\rangle - (\dot{u}/c_2)\Delta\sigma/\langle\sigma_T\rangle]_i , \tag{5a}$$

$$\delta T_{\rm sq}^{A,B} = \mp \tilde{\alpha}_{A,B}^{-1} [\tfrac{1}{2}\Delta\mu/\langle\sigma\rangle + (\dot{u}/c_2)\langle\sigma\rangle/\langle\sigma_T\rangle]_i , \tag{5b}$$

$$\dot{u} = \left(\frac{-2\kappa}{(\tilde{\alpha}_A^{-1} + \tilde{\alpha}_B^{-1})\kappa + \rho c_2 \langle\sigma_T\rangle_i} - 1 \right) \left[\frac{c_2 \langle\sigma_T\rangle \Delta\mu}{2\langle\sigma\rangle^2} \right]_i . \tag{5c}$$

The extended part $\delta T_2^{A,B}$ of the temperature field agrees with that of Ref. [5], in which the dissipative terms were neglected. To understand why this is not an accident and what the essence of the new information here is, we need to address the concept of the effective CC. Since the CCs are, as emphasized, quite generally valid, we have a certain discretion towards the choice of the interface width: It can be either microscopic, of order ξ_f, or it can be hydrodynamic, somewhat larger than $\lambda_{\rm sq}$. Equations (4), such as they stand, are the proper CCs for the microscopic interface. It provides complete information on $\dot{u}$, the hydrodynamic fields from $x = \pm 0$ to $\pm\infty$, and their discontinuities across the interface, e.g., $\Delta T = \delta T_2^B + \delta T_{\rm sq}^B - \delta T_2^A - \delta T_{\rm sq}^A$; cf. Fig. 1 and Eq. (1a) for $x = 0$. The CCs of the macroscopic interface (dotted lines in Fig. 1) are simpler in three aspects: First, since it is thicker than the sq decay length $\lambda_{\rm sq}$, it ends in a region where the dissipative terms are small and can be neglected. Second, eliminating dissipative terms especially simplifies R_s and reduces the number of CCs, commensu-

rate with the fact that only $\delta T_2^{A,B}$ need to be determined. Third, the effective discontinuities across the wider interface include the sq decay, e.g., $\Delta_e T = \delta T_2^B - \delta T_2^A$; cf. Eq. (1a) for $|x| \gg \lambda_{sq}$. Equations (4) with these three modifications incorporated reduce to [12] $\Delta_e f = 0$, $\Delta_e \mu = 0$, $\langle f \rangle_e = \kappa_e \Delta_e T$, with an effective Kapitza conductance κ_e. They constitute the effective CCs for the hydrodynamically wide interface, and are in fact the very CCs employed in Ref. [5] to obtain $\delta T_2^{A,B}$ and

$$-\rho \dot{u} = \kappa_e \left[\Delta\mu/\langle\sigma\rangle^2\right]_i . \tag{6}$$

The sq decay was hence implicitly included as a source of interface dissipation. These previous results therefore remain valid, and the new information provided by the CCs, Eqs. (4), can be seen by comparing these results with Eqs. (5), yielding an expression for κ_e

$$(\kappa_e - \tfrac{1}{2}\rho c_2 \langle\sigma_T\rangle_i)^{-1} = \kappa^{-1} + \sum_{A,B} \left(\tilde{\alpha}\rho c_2 \langle\sigma_T\rangle_i\right)^{-1} . \tag{7}$$

The total effective resistance κ_e^{-1} has four constituting elements: The three in series are on the right hand side: one microscopic and two sq contributions. The latter become maximal for $\alpha_{A,B} = 0$, i.e., if the sq amplitudes are maximal. The fourth resistive element, on the left of Eq. (7), is circuited in parallel to the other three. It stems from emission of second sound which rids the interface of latent heat independent from heat transfer across the interface. Therefore, this term enables phase transition even if the actual conductance κ vanishes. (Since its contribution is numerically small, it was not, but should have been, displayed in Eq. (5) of Ref. [5].) As discussed above, the actual resistance $1/\kappa$ is most probably negligible. The experimental data [1] on $\dot{u}$ then imply $\tilde{\alpha}_{A,B} \approx 8 \times 10^2$, if we take $\alpha_A = \alpha_B$ for lack of better knowledge.

For $\dot{u} \gg c_2$, the same double approach of actual and effective CCs applies. From R_s of Eq. (3c), we obtain (each to the lowest order of $w/\dot{u}$ and neglecting cross terms)

$$g = K[\langle\sigma\rangle \Delta T + \Delta(\mu_0 + z^D)] , \tag{8a}$$

$$\langle f^D \rangle = \beta \Delta T , \; v_n^A = -\alpha_A (\pi^D - \rho z^D)^A , \tag{8b}$$

where μ_0 is the chemical potential for a given temperature and pressure in a system with $v_n = v_s = 0$, and f^D the dissipative part of the entropy current. The effective CCs are given by $\Delta_e Q = 0$, $\Delta_e \dot{\varphi} = 0$, and $g = K_e(\langle\sigma\rangle_e \Delta_e T + \Delta_e \mu_0)$. As partly reported in Ref. [5], the latter lead (again via an expansion around T_i) to

$$\delta T^B = -[(\Delta\mu_0 + T\Delta\sigma)/T\sigma_T^B]_i , \tag{9a}$$

$$\rho \dot{u} = K_e \left[\Delta\mu_0 - \tfrac{1}{2}\Delta\sigma \delta T^B\right]_i , \tag{9b}$$

$$\delta w^B = -(\rho_s/\rho_n \dot{u}) \left[\Delta\mu_0 - \sigma^B \delta T^B\right]_i . \tag{9c}$$

The second equation is valid including $(\delta T^B)^2$. The proper CCs for the microscopic interface, Eqs. (3a,b),(8), provide the additional information

$$\frac{1}{K_e} = \left(\frac{1}{K} + \frac{C}{\beta} + \frac{1}{\rho^2 \alpha}\right) \left[1 + \frac{1}{2}\frac{\Delta\sigma \delta T^B}{\Delta\mu_0}\right]_i , \tag{10a}$$

$$\delta w_d^A = \frac{\rho_s}{\rho\rho_n}\frac{1}{\alpha} \; , \; \delta T_d^A = \delta T^B \left[1 - \frac{\rho \dot{u} \sigma_T^A}{2}\frac{1}{\beta}\right]_i , \tag{10b}$$

if α and β are such that $\delta w_d^A \ll \dot{u}$, $\delta T^B - \delta T_d^A \ll \delta T^B$. (Otherwise, the hydrodynamic dissipation would be too large for the experimental data [1].) $C \equiv -\frac{1}{2}\{[\sigma\lambda_w/(\lambda_T - \lambda_w) - \frac{1}{2}\Delta\sigma]\sigma_T\}_i^A \delta T^B$. In the first factor of K_e^{-1}, three resistive elements are in series: the first two are microscopic in origin, from $\Delta\mu$ and ΔT, respectively; the third is from w diffusion; temperature diffusion gives rise to the second factor.

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