Quantitative Theory of Richtmyer-Meshkov Instability

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The acceleration of a material interface by a shock wave generates an interface instability known as the Richtmyer-Meshkov instability. Previous attempts to model the growth rate of the instability have produced values that are almost twice that of the experimental measurements. This Letter presents numerical simulations using front tracking that for the first time are in quantitative agreement with experiments of a shocked air-SF_6 interface. Moreover, the failure of the impulsive model, and the linear theory from which it is derived, to model experiments correctly is understood in terms of time limits on the validity of the linear model.

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When a shock wave collides with the interface between two different materials, small perturbations of this interface grow into nonlinear structures having the form of "bubbles" and "spikes." The occurrence of this shock-induced instability was predicted by Richtmyer [1] and confirmed experimentally by Meshkov [2]. The Richtmyer-Meshkov instability is similar to the more familiar Rayleigh-Taylor instability and is important in both natural phenomena (supernovae) and technological applications (inertial confinement fusion).

Theory and computation have so far failed to provide an understanding of the Richtmyer-Meshkov instability that is in quantitative agreement with existing experiments [3–7]. Computations of the Richtmyer-Meshkov instability for singly shocked, sinusoidally perturbed interfaces have overpredicted growth rates by factors from 40% to 100% [5] as compared to experiments. The main theoretical model used in this area, Richtmyer's impulsive model [1], also consistently predicts a growth rate that is too large.

This paper presents results from numerical simulations of the Richtmyer-Meshkov instability that for the first time agree with experimentally measured growth rates of interface perturbations. Our computations are further validated by a comparison of small amplitude perturbation, early time, simulations with solutions to a linearized set of equations of motion. An analysis of the time interval for the validity of the linearized model provides an explanation of the failure of the linearized and impulsive models to agree with experiment.

We focus on the simplest case of the shock tube experiments of the Richtmyer-Meshkov instability where a sine shaped material interface is accelerated by a single shock wave, as in the experiments of Meshkov [2], Benjamin [3,4], and others. The general configuration of the computation and experiments is shown in Fig. 1. A thin membrane was used in the experiments to separate the two gases at the material interface. Quantitative agreement was achieved between our computational results and the experimental measurements of Benjamin [4] for the rate of growth of a shocked air-SF_6 interface. The collision results in a transmitted shock and a reflected wave that can be either a shock or a rarefaction depending on the values of the fluid parameters. The experiments considered in this paper are of the reflected shock type. Viscosity and heat conduction are negligible here, and the fluid motion is described by the Euler equations.

The key new feature of these computations is the use of

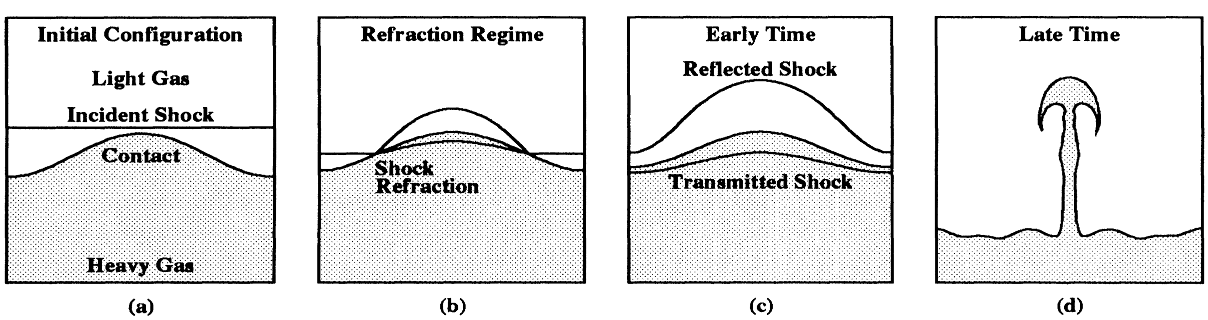

FIG. 1. A schematic representation of the geometry of the Richtmyer-Meshkov instability modeled in this paper. The interaction consists of the collision of a shock wave with a material interface. The refraction of the shock by the interface produces reflected and transmitted waves. The instability consists of the growth of perturbations of the material interface with time.

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front tracking [8,9]. Front tracking is a numerical method for the sharp resolution of waves. It eliminates numerical diffusion across the tracked waves and achieves highly resolved solutions on relatively coarse grids. The method combines a standard finite difference method computed on a rectangular grid with a set of lower dimensional moving grids that follow selected wave fronts. Here the tracked waves include the incident shock, the material interface, and the transmitted and reflected shocks. The position and states on the tracked fronts are updated using Riemann solutions. The states and locations of the tracked fronts are then used as "internal" boundary conditions for the computation of the flow away from the fronts. The solution on the finite difference grid can be computed using any of several different methods. The computations shown in this paper use a second order Godunov method [10,11].

The impulsive model proposed by Richtmyer [1] is commonly used to estimate the growth rate of a shock accelerated interface. This model is derived by assuming that the shock acceleration can be treated as being impulsive, and that the interaction is nearly incompressible once the shock wave has passed through the material interface. It is also assumed that the flow can be observed in a frame where the average position of the material interface is at rest, and the position, $y(x,t)$, of the material interface at time t is given by $y(x,t) = a(t)\sin kx$, where k is the wave number of the perturbation. Richtmyer's formula gives the growth rate of $a(t)$ as

$$\dot{a}(t) = k\Delta u \,\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\, a(0+), \tag{1}$$

where Δu is the difference between the shocked and unshocked mean interface velocities, ρ_i are the postshocked densities on the two sides of the interface (the incident shock moves from material "2" to material "1"), and $a(0+)$ is the perturbation amplitude immediately after the collision of the shock with the material interface. This formula implicitly assumes the initial preshocked amplitude, $a(0-)$, is small.

Assuming explicitly that $a(0-)$ is small so that $ka(0-) \ll 1$, a more exact calculation of the amplitude growth rate can be made. The Euler equations are linearized around the solution of a one dimensional Riemann problem defined by the head-on collision of a planar shock with a zero amplitude (planar) material interface, using the initial amplitude of the sinusoidal perturbation as a small expansion parameter. The result of the linearization is a system of partial differential equations in one spatial dimension with associated boundary conditions. This system can be solved numerically for the growth rate of the perturbed interface. This approach, following Richtmyer [1], has recently been generalized to include reflected rarefactions as well as reflected shocks [12]. Simple order of magnitude estimates limit the validity of the linearized equations to the dimensionless time interval

$$t_{*\,\mathrm{min}} \equiv ka(0-) \ll t_* \ll 1/[ka(0-)] \equiv t_{*\,\mathrm{max}}. \tag{2}$$

Here the dimensionless time $t_* = kc_0M_0t$, where M_0 is the incident shock Mach number and c_0 is the sound speed of the fluid ahead of the incident shock. The limits $t_{*\,\mathrm{min}}$ and $t_{*\,\mathrm{max}}$ represent, respectively, the transit time of the incident shock through the perturbed interface and the time required for the perturbation to grow to unit amplitude. Necessarily, these time limits apply to the derivation of the impulsive model as well, since it is an approximation to the linear theory. Recent systematic comparisons of the impulsive model and the linear theory have revealed both regions of agreement and of disagreement in parameter space [12].

Other models for the growth rate of the interface in the linear regime have also been proposed. Mikaelian constructed extensions of the impulsive model to multiple fluid layers [13], and Fraley [14] performed an asymptotic analysis for a small amplitude Richtmyer-Meshkov instability with a reflected shock. Fraley's analysis uses Laplace transform methods to solve the linearized Euler equations. Recently Mikaelian [15] compared the results of direct numerical simulations of small amplitude interfaces with both Fraley's theory and the impulsive model. He found that Fraley's model better predicted the results of the direct numerical simulations than did the impulsive model. He found that Fraley's model was in closer agreement with these nonlinear simulations than was the impulsive model.

We compared our simulations of a singly shocked air-SF_6 interface to the experiments of Benjamin [4]. The material interface is accelerated by a shock wave with Mach number 1.2 moving from air into SF_6. The initial amplitude, $a(0-)$, was 0.006 37 times the period of the sinusoidal perturbation. For these experiments, $t_{*\,\mathrm{max}} \approx 2.5$, while the observational time interval is $15 \le t_{*\mathrm{obs}} \le 50$. The observational times and the validity of the linear theory fail to overlap by a factor of about 6. We conclude that the linear theory has no relationship to this experiment.

Figure 2 shows plots of the amplitude and amplitude growth rate of the material interface as obtained from experiment, the front tracking simulation, the linearized theory, and Richtmyer's impulsive model. The time axis in these figures is shifted so that $t = 0$ corresponds to the time at which the shock wave has completed its refraction through the interface.

As can be seen from these figures the front tracking results are in substantial agreement with the experimental results in the sense that the growth rate derived by a least squares analysis of the front tracking amplitude data, 9.2 m/s, is identical to the same quantity derived from the experimental data. Note that for late (i.e., experimentally observed) times the linearized theory and the impulsive model growth rates are a factor of 2 larger

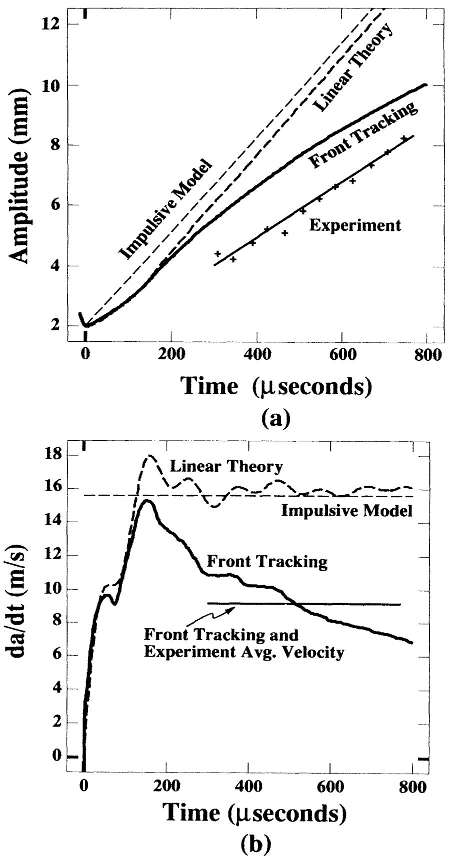

FIG. 2. Perturbation amplitude, $a(t)$, and amplitude growth rate, $\dot{a}(t)$, of a shocked air-SF_6 interface. This graph compares the results of experiment, front tracking simulation, linear theory, and Richtmyer's impulsive model. Also shown are results of a least squares fit to the front tracking and experimental amplitude data over the period of experimental observation. The difference between the least squares average velocity for experiment and simulation is indistinguishable in this graph.

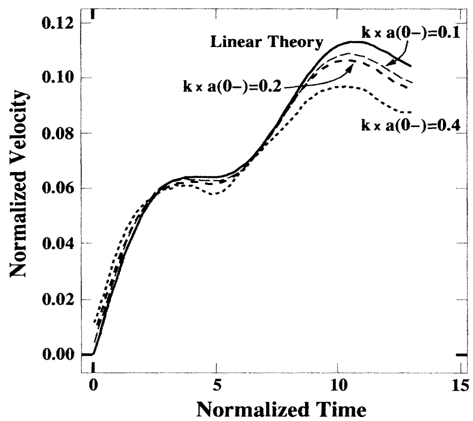

FIG. 3. A comparison of three separate calculations of the normalized perturbation growth rate, $\dot{a}(t)/[kc_0M_0a(0-)]$, of a shocked air-$SF_6$ interface with three different initial amplitudes where k is the wave number, c_0 is the sound speed ahead of the incident shock, and M_0 is the incident shock Mach number. We see that the linear theory agrees with the nonlinear computations for sufficiently small amplitudes. The horizontal axis is in dimensionless time units kc_0M_0t.

than those found in experiment or in our simulation. This may be due to the fact that this particular configuration has a relatively large initial amplitude and quickly leaves the region of validity of the linearized theory and impulsive model. The displacement of the experimental curve with respect to the front tracking curve is possibly due to membrane effects; i.e., the material strength of the membrane or the influence of its fragmentation may effect the fluid flow.

The front tracking results indicate a decay in amplitude growth rates while Benjamin [4] finds a fairly constant growth rate during the measurement period. Other experiments, however, have shown a decaying growth rate [16,17]. The figures shown in this paper used a resolution of 125 zones per wavelength, and mesh refinement studies in the range 125–208 zones per wavelength showed very little change in the amplitude growth rate. We also tested our simulation against changes in other numerical parameters and found that the value of $\dot{a}(t)$ was insensitive to these changes. We conclude that this decay is a real effect and not due simply to numerical dissipation as has been suggested [4].

A further validation of the nonlinear simulations can be accomplished by comparison to the small amplitude theory (Fig. 3). This serves both to determine the range of validity of the linear theory and to validate the solution of the full Euler equations at small amplitudes. As can be seen in Fig. 3, the front tracking calculation is converging to the linear result as we reduce the amplitude. We note that the interval of convergence of the nonlinear simulations to the linear theory appears to be finite. This is in contrast to formula (2) which suggests that the domain of validity of the linearized equations should increase with decreasing initial amplitude. This point deserves further study.

Of interest is the question of why our results agree with experiment while results found through other numerical methods do not. Prior disagreement between the growth rates measured in experiments and those predicted by numerical simulation has led to the suggestion that mass diffusion and membrane effects may have an important role in the behavior of the interface instabilities. For example, recent computations of Mikaelian [15] of earlier experiments by Benjamin showed much better agreement with the experiments than the results from the impulsive model, the theory of Fraley [14], or the solution of the linearized equations. However, his results are still 50% larger than the experimental value. He attributed this remaining difference to possible membrane effects. Our work does not exclude this possibility, but the agreement of our computations with experiment suggests that a proper numerical resolution of the material interface is essential to obtain agreement with experiment, and also that if other effects are important, they may be offsetting one another.

It is clear that there is still much to learn about the highly nonlinear aspects of the Richtmyer-Meshkov instability. These effects include the possible coupling between nonlinear modes, and their study will require experiments on singly shocked interfaces as well as computations with random interfaces which have been run to late times. Similarly, understanding the effects of reshocking remains an important theoretical challenge. For the single mode case, a systematic study of mass diffusion, membrane effects, and a detailed comparison to earlier calculations of others would be desirable.

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