PHYSICAL REVIEW LETTERS

VOLUME 71 | 22 NOVEMBER 1993 | NUMBER 21

Black Hole Statistics

Andrew Strominger
Institute for Theoretical Physics and Department of Physics, University of California, Santa Barbara, California 93106
(Received 16 July 1993)

The quantum statistics of charged, extremal black holes is investigated beginning with the hypothesis that the quantum state is a functional on the space of closed three-geometries, with each black hole connected to an oppositely charged black hole through a spatial wormhole. From this starting point a simple argument is given that a collection of extremal black holes obeys neither Bose nor Fermi statistics. Rather, they obey an exotic variety of particle statistics known as "infinite statistics" which resembles that of distinguishable particles and is realized by a q deformation of the quantum commutation relations.

PACS numbers: 04.60.+n, 05.30.−d, 97.60.Lf

Semiclassical calculations [1] indicate that the mass of a charge Q black hole will decrease via Hawking radiation until it reaches a critical value proportional to Q. The resulting "extremal" black hole appears to be a quantum mechanically stable object (in a world in which there are no elementary particles with mass less than QM_{Planck}). Extremal black holes have been found to be a useful item in the gedanken laboratory for studying the quantum mechanics of black holes. Their utility derives from the fact that, for large Q, they are macroscopic objects whose behavior should be governed by (hopefully) well-understood laws of low-energy physics. Thus the short-distance problems of quantum gravity might be divorced from the quantum puzzles of black holes. In the last several years there has accordingly been much progress in understanding the scattering of low-energy particles by extremal black holes [2].

In this paper we shall consider a different process: the scattering of two extremal black holes. There are many interesting aspects of this problem, but we shall consider here only the zeroth-order question, "Do they scatter as bosons, fermions, or something else?" We shall argue that at least in some cases the answer is "something else."

In nongravitational field theories, it was shown long ago by Finkelstein and Rubenstein [3] that soliton statistics can be determined from the fact that the quantum state is a functional (or more generally "sectional") on the space of field configurations. For example, double exchange of a pair of Skyrmions is an operation which is continuously deformable to the identity, while single exchange is an operation which is continuously deformable to a 2π rotation of a single Skyrmion. Thus the exchange operator $\mathcal{E}$ must have eigenvalues either plus or minus 1 (since it squares to the identity) and the solitons may correspondingly be either bosons or fermions. Since $\mathcal{E}$ is continuously deformable to a 2π rotation of a single Skyrmion, the eigenvalue is connected with the angular momentum of the Skyrmion wave function, in accord with the expected spin-statistics relation [4].

If the soliton has no internal states, then the exchange operator $\mathcal{E}$ will commute with every operator in the theory simply because it does not change the physical field configuration. This has the consequence that the eigenspaces of $\mathcal{E}$ are superselection sectors: Bosons are forever bosons and fermions are forever fermions. If the soliton does have internal states then $\mathcal{E}$ may or may not change the field configuration, depending on whether or not the solitons are in the same state. $\mathcal{E}$ will then obey a composition law with those operators which act on the internal state reflecting the statistics of identical particles with internal degrees of freedom.

The application of these ideas to quantum gravity is developed in a series of beautiful papers by Friedman and Sorkin [5]. In quantum gravity, the wave function must be invariant under all diffeomorphisms which are asymptotically trivial and can be continuously deformed to the identity. The square $\mathcal{E}^2$ of the exchange operator acting on two identical solitons is in this category, but $\mathcal{E}$ itself is in general (depending on the soliton in question) a nontrivial "large" Z_2 diffeomorphism. These considerations alone may allow either Bose or Fermi statistics but, because observables must commute with large diffeomorphisms, the statistics form superselection sectors. In some cases the statistics can be fixed by consideration of the soliton creation process [6].

We wish to apply these ideas to the problem of charged extremal black hole statistics. The first issue is the relevant space of three-geometries.

Perhaps the first possibility which comes to mind is to consider three-geometries with a boundary at each black hole horizon. One must then decide whether or not these boundaries are distinguishable, which is completely equivalent to deciding the statistics. So nothing is learned in this approach.

It is preferable to formulate the problem in terms of three-geometries without boundaries. Because of flux conservation, the two-sphere surrounding the black hole cannot be topologically trivial (assuming there are no charged sources). In this paper we will investigate the consequences of the assumption that the flux reemerges elsewhere in our (or possibly another) universe out of an oppositely charged extremal black hole. The relevant space of three-geometries is then R^3 with N handles whose cross sections are two-spheres, corresponding to N positively charged and N negatively charged black holes. It is worth noting that this formulation is consistent with semiclassical instanton calculations [7,8], which describe creation of charged black hole pairs connected by a wormhole in an electromagnetic field.

Yet another possibility—briefly discussed below—is to allow the flux to end at a 't Hooft-Polyakov monopole or an electric charge inside the horizon. This formulation is inherently more complex as it necessarily involves time-dependent three-geometries. However, because these descriptions differ only behind a horizon, one may hope that they lead to the same observable consequences.

We further want to consider only extremal black holes with a unique ground state, to avoid complications of the exchange operator mixing up internal states. [A finite degeneracy (e.g., as suggested by the Bekenstein-Hawking entropy) would not qualitatively affect the discussion.] This is quite different from (though of course relevant to) the situation considered in [8], wherein extremal black holes were excited in various ways by low-energy scattering of massless fermions. In a theory with no massless fermions or scalars, there is an energy barrier to throwing matter into the black hole, and it should be possible to scatter black holes at sufficiently low energies without exciting any internal degrees of freedom.

The basic point is now very simple. Black hole exchange is the exchange of two wormhole ends. This results in a new three-geometry: It is not a diffeomorphism. Quantum mechanics does *not* require that the wave function should have any particular symmetry properties under this operation. The wave function of N like-charge black holes will be a general function $\psi(x_1,\ldots,x_N)$ of their N positions. Extremal black hole scattering will resemble that of distinguishable particles: It is also the case that the wave function for N identical particles with more than N internal states is a general function $\psi(x_1,\ldots,x_N)$ if none of the N particles are in the same internal state. The low-energy experimentalist who discovers the extremal black holes might believe that she has found a new type of massive particle with a large number of degenerate internal states. However, no matter how long she searches, two particles in the same internal state (i.e., that scatter like identical particles) will never be found.

While black hole exchange is not a large diffeomorphism, wormhole exchange is. This involves the simultaneous exchange of a negatively charged and a positively charged black hole pair. *A priori* the eigenvalue of the wormhole exchange operator $\mathcal{E}_w$ may have either sign. However, as in previously studied examples [6], the sign is fixed by consistency with the rules for pair creation. In [7] an instanton was found describing single wormhole creation in a magnetic field. To get the creation rate one must exponentiate the single instanton. In so doing one counts only once the four-geometries which differ by wormhole exchange on the final three-geometry. Thus it is implicit in this calculation—which led to reasonable results—that $\mathcal{E}_w$ has eigenvalue $+1$. We shall henceforth assume the wormholes are bosons. (This is also consistent with the spin-statistics relation, since the single instanton creates the wormhole in a state invariant under 2π rotations.)

One might expect that the Bose nature of black hole pairs would lead to observable consequences. In order to scatter two wormholes one must first locate the two charge $-Q$ partners of two charge $+Q$ black holes. One might attempt to do this by shining a flashlight in one end of the wormhole and seeing where the light reemerges. Of course classically this is prohibited by causality: There is a horizon in the middle of the wormhole (although the spacelike slice containing the wormhole may cross a pair of horizons without entering into a region of trapped surfaces, as in the $t=0$ slice of maximally extended Schwarzchild). This is a special case of more general "topological censorship" theorems [9]. The gist of these theorems is that classically it is impossible for an external observer to detect nontrivial topology.

If topological censorship were not generally valid, many pathologies would arise. For example, wormhole traversal might be used for time travel [10]. Thus it is natural to assume that *quantum* topological censorship is valid as well. In this case one can never locate the other end of a given extremal black hole and observe Bose scattering of wormholes.

One might be concerned that if extremal black holes are quantum mechanically similar to elementary particles with an infinite number of internal states they will share with the latter object an infinite pair-production rate. Our description of extremal black holes is consistent with—indeed was inspired by—the description of quantum gravity transition amplitudes as sums over *inequivalent* four-geometries. Euclidean instanton methods were used in [7,8] to argue that this prescription leads to a finite (and semiclassically computable) pair-production rate. Thus—at least in this description—the quantum mechanics of extremal black holes differs in this regard from that of an object with an infinite number of internal states. Indeed, the experimentalist who tires in her search for two objects that interfere like identical particles might learn that she is on the wrong track by measuring the pair-production rate.

The arguments given herein mesh well with, and are in a sense an extension of, those given in [8]. In [8] we considered (in a theory with massless fermions) the possibility that information lost in low-energy particle-hole scattering might be stored in the form of an infinite degeneracy of zero-energy black hole states in which the matter fields are excited behind the horizon. One might expect that this information could be recovered by quantum interference experiments. It was argued [8] that in fact causality dooms such experiments to failure, and black holes will always scatter as distinguishable particles, even if they are initially in the same zero-energy state. In this paper we have argued that even extremal black holes with no internal excitations scatter as distinguishable particles. Adding the possibility of internal excitations would not alter this conclusion, so our results are consistent with the causality requirements of [8].

Let us now briefly consider a description of extremal magnetic black holes in which there are no wormholes, and magnetic flux is terminated at a 't Hooft–Polyakov monopole. In a theory with the grand unification mass $M_{\mathrm{GUT}} > M_{\mathrm{Planck}}$ (although this makes sense classically, it is far from clear that there is any quantum mechanical meaning to a particle with Compton wavelength less than its Schwarzchild radius), a smooth, slowly varying field configuration with nonzero magnetic charge will in general collapse to form a magnetic black hole, which will then Hawking radiate down to extremality. The exterior of the black hole configuration will then be static, but the interior will continue to evolve.

In this system the topology of a spacelike slice with a collection of black holes is simply R^3, provided we continue the spacelike slice inside the horizon in such a way so as to avoid any singularities. An extremal black hole is therefore represented as an object with many internal states, associated with the degrees of freedom inside the horizon. In the scattering of two like-charge extremal black holes, those internal states cannot be observed and should therefore be traced over. In general the scattering will therefore resemble nonidentical particles. However, a better understanding of the dynamics is required to understand to what extent this description of black hole scattering is consistent with the previous one.

One might attempt to obtain a quantum field theoretic description of extremal black holes in which they are treated as point particles. (However, we wish to stress that since black holes are gravitational solitons rather than fundamental particles, it is not clear how far such a description can be pushed. For example, second-quantized descriptions of grand unified theory monopoles have only a limited validity.) A classification of all possible particle statistics consistent with general principles of quantum field theory was obtained by Doplicher, Haag, and Roberts [11]. In addition to recovering Bose statistics, Fermi statistics, and parastatistics (which is related to internal color) they found an additional, less familiar, possibility: "infinite statistics." A collection of particles which obey infinite statistics can be in any representation of the particle permutation group. These are evidently the statistics obeyed by extremal black holes.

More recently it was realized [12] that a Fock-like realization of infinite statistics can be obtained from a q deformation of the commutation relations:

$$a_k a_l^\dagger - q a_l^\dagger a_k = \delta_{kl} , \tag{1}$$

where k,l may be viewed as labeling spatial momenta. (This should not be confused from the q-deformed quantum theory of [13] which, after second quantization, leads to negative norm states [14].) $q=\pm 1$ corresponds to bosons or fermions, while other cases correspond to "quons." States are built by acting on a vacuum which obeys

$$a_k|0\rangle = 0 . \tag{2}$$

It follows from (1) and (2) that for $-1<q<1$ one can form $N!$ linearly independent states from N oscillators $a_{k_1}^\dagger \cdots a_{k_N}^\dagger$ if the k_i are all different. (In the Bose or Fermi case there is only one independent state.) This is easy to see for the special case $q=0$ for which (1) reduces to

$$a_k a_l^\dagger = \delta_{kl} . \tag{3}$$

It follows that the inner product of two N-particle states is

$$\langle 0| a_{k_N} \cdots a_{k_1} a_{l_1}^\dagger \cdots a_{l_N}^\dagger |0\rangle = \delta_{k_1 l_1} \cdots \delta_{k_N l_N} . \tag{4}$$

Thus two states obtained by acting with the N oscillators in different orders are orthogonal. Thus—as for extremal black holes—the states may be in any representation of the permutation group.

It was noted in [12] that the theory of quons contains some mild forms of nonlocality. For example, the expression for the Hamiltonian is both nonlocal and nonpolynomial in the field operators. Nevertheless cluster decomposition, CPT, and an analog of Wicks theorem were found to hold. It is not clear if quons lead to physically unacceptable forms of nonlocality. Perhaps the nonlocality discussed in [12] is related to the fact that wormholes connect spacelike separated points, and can be eliminated by superselection rules arising from topological censorship.

In summary, we believe that understanding the quantum statistics of extremal black holes is a key ingredient for unraveling quantum black hole puzzles. We have argued that, at least in one description, extremal black holes behave very differently from elementary particles. It is far from obvious that this description is appropriate to the real world. Nevertheless we feel it is a logical possibility—apparently consistent with quantum mechanics—that our Universe contains fundamental entities which are neither fermions nor bosons, but rather are quons.

I wish to thank S. Giddings, J. Harvey, A. Srivistava, S. Trivedi, and especially T. Banks and J. Friedman for useful discussions. This work was supported in part by DOE Grants No. DEAC-03-8ER4050 and No. NSF PHY89-04035.

[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); Phys. Rev. D **14**, 2460 (1976); G. W. Gibbons, Commun. Math. Phys. **44**, 245 (1975).

[2] J. Preskill, P. Schwarz, A. Shapere, S. Trivedi, and F. Wilczek, Mod. Phys. Lett. A **6**, 2353 (1991); C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. D **45**, R1005 (1992); for recent reviews see J. A. Harvey and A. Strominger, in Proceedings of the 1992 TASI Summer School, Boulder, Colorado (Report No. EFI-92-41, hep-th@xxx/9209055); S. B. Giddings, in Proceedings of the International Workshop of Theoretical Physics, 6th Session, June 1992, Erice, Italy (Report No. UCSBTH-92-36, hep-th@xxx/9209113).

[3] D. Finkelstein and H. Rubinstein, J. Math. Phys. **9**, 1762 (1968).

[4] For a more complete discussion, and an introduction to these ideas and their generalizations see A. P. Balachandran, G. Marmo, B. S. Skagerstam, and A. Stern, *Classical Topology and Quantum States* (World Scientific, Singapore, 1991) and R. D. Sorkin, Commun. Math. Phys. **115**, 421 (1988).

[5] J. L. Friedman and R. D. Sorkin, Phys. Rev. Lett. **44**, 1100 (1980); Gen. Rel. Grav. **14**, 615 (1982). For a review see J. L. Friedman, in *Proceedings of the Osgood Hill Conference on Conceptual Problems in Quantum Gravity*, edited by A. Ashketar and J. Stachel (Birkhauser, Boston, 1991).

[6] R. Sorkin and F. Dowker (to be published).

[7] D. Garfinkle and A. Strominger, Phys. Lett. B **256**, 146 (1991); see also G. W. Gibbons, in *Fields and Geometry*, Proceedings of the 22nd Karpacz Winter School of Theoretical Physics, Karpacz, Poland, 17 February–1 March 1986, edited by A. Jadczyk (World Scientific, Singapore, 1986).

[8] T. Banks, M. O'Loughlin, and A. Strominger, Phys. Rev. D **47**, 4476 (1993).

[9] J. Friedman, K. Schleich, and D. Witt, Phys. Rev. Lett. **71**, 1486 (1993).

[10] M. S. Morris, K. S. Thorne, and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).

[11] S. Doplicher, R. Haag, and J. Roberts, Commun. Math. Phys. **23**, 199 (1971); **35**, 49 (1974).

[12] O. W. Greenberg, Phys. Rev. Lett. **64**, 705 (1990); Phys. Rev. D **43**, 4111 (1991); J. Cuntz, Commun. Math. Phys. **57**, 173 (1977). A recent summary can be found in O. W. Greenberg, D. M. Greenberger, and T. V. Greenbergest, in Proceedings of the International Conference on Fundamental Aspects of Quantum Theory, 1993 (Report No. hep-ph/9306225).

[13] A. Yu. Ignatiev and V. A. Kuzmin, Yad. Fiz.. **46**, 786 (1987) [Sov. J. Nucl. Phys. **46**, 444 (1987)].

[14] O. W. Greenberg and R. N. Mohapatra, Phys. Rev. Lett. **59**, 2507 (1987).