Stability of Very Small Strangelets

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We study the stability of small strangelets by employing a simple model of strange matter as a gas of noninteracting fermions confined in a bag. We solve the Dirac equation and populate the energy levels of the bag one quark at a time. We find that for system parameters such that strange matter is unbound in bulk, there may still exist strangelets with $A < 100$ that are metastable. We cannot determine, however, whether the lifetime of these strangelets is sufficient to detect them in current accelerator experiments.

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With the advent of heavy ion colliders, it will soon be possible to search for stable or metastable lumps of quark matter with $S \sim A \sim 10$–30 [1] (for the sake of simplicity, throughout this Letter we assign the strange quark a strangeness of $+1$). The possible stability of strange quark matter ("strange matter") in bulk was pointed out by Witten in 1984 [2], and since then, there have been attempts to predict the properties of strange matter in bulk and in finite lumps ("strangelets") [3]. These studies generally only apply to A much larger than those accessible in heavy ion colliders. Here we present some qualitative information on very small strangelets obtained from an elementary model. Our model includes only quark kinetic energy, the Pauli principle, and confinement. It cannot tell us anything about important issues like the overall energy scale, or equivalently the bulk stability, of strange matter. It does, however, illustrate potentially interesting effects such as shell closures, "loading" and unloading of strangeness, and "islands of stability" in the (S,A) plane. None of the details of our predictions should be taken very seriously; they would undoubtedly be changed in more sophisticated models.

No one knows how to model quark matter in QCD accurately. Lattice simulations are as yet unable to cope with systems at nonzero chemical potential. Models of bulk strange matter have confined quarks in a bag and included residual gluon interactions perturbatively [4,5]. Surface effects were included for large strangelets by including surface modifications of the quark density of states as well as Coulomb effects [6]. The resulting Thomas-Fermi type model is only valid for strangelets with radii very large compared to the natural length scale of the system, $B^{-1/4} \sim 1$–2 fm. For typical strange-matter densities, a strangelet with baryon number $A \sim 200$ has a radius of only 5–6 fm, so only for such a large baryon number does the model of Ref. [6] become reliable. We model a small strangelet as a gas of noninteracting fermions confined in a bag. We determine the energy eigenvalues and fill the bag energy levels sequentially, obeying the exclusion principle, minimizing the energy (for each A) and adjusting the bag radius so the quark pressure balances the vacuum pressure, B. This is equivalent to minimizing the total energy at constant pressure, B. Values of B were taken from fits to bulk strange matter as described in Ref. [6]. We also include a phenomenological zero-point energy term $-1.84/R$. The free parameters we use are the energy per baryon in bulk, ϵ_b, the mass of the strange quark, m_s, and the baryon number, A. We ignore residual perturbative QCD interactions following Ref. [6]. We also ignore Coulomb corrections. This should be a good approximation because Z is very small for small A, $Z \ll A$.

The quantum numbers and energetics of small strangelets show regularities reminiscent of atomic physics. "Shell" effects are important when filling a bag with quarks: The rate of change of the energy per baryon with A changes dramatically near shell closures and leads to enhanced stability. We find that there exist regions of A in which strangelets are metastable even for parameters such that strange matter is not bound in bulk. We also observe that S for the most stable strangelet is an erratic function of A. Strange and nonstrange quark energy levels cross as a function of the bag radius R. When a nonstrange level dives below a strange level, the strange level "unloads" into the nonstrange one. This phenomenon is similar to the filling and emptying of inner d orbitals in the periodic table. Finally, we find that the spatial distri-

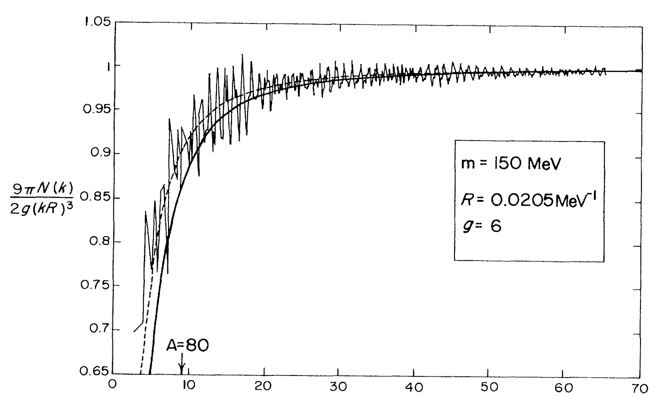

FIG. 1. The scaled number of states as a function of k compared with the asymptotic expansion including the surface correction.

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bution of strangeness is not uniform throughout a strangelet. Because they are less relativistic, strange quarks are concentrated in the interior. This phenomenon is related to the quark mass dependence of the surface tension in strange matter.

There have been previous attempts to use similar models to study very small strangelets. In Ref. [7], strangelets were constructed by filling the bag energy levels with the strangeness ratio held fixed. In the published work, S/A was fixed at 0.7 which is far from the optimal value for most A. Fixing S/A artificially prevents the system from finding a minimum energy configuration and obscures shell structure. The objects of greatest interest in heavy ion experiments are those of great stability, which cannot be found from Ref. [7].

More recently, Madsen [8] has studied very small strangelets using the asymptotic expansion of the density of states including the curvature term. Such expansions describe average properties of the density of states but are not sensitive to the shell structure which we expect to be important for the lightest strangelets. By using the exact density of states we automatically include the surface and curvature effects which were studied by Madsen. It is interesting to explore the magnitude of the shell effects relative to the average properties included in the asymptotic expansion used by Madsen. To study this issue we have plotted in Fig. 1 the actual integral of the density of states $N(k)$ for a light quark as a function of $kR = \sqrt{E^2-m^2}R$ in a rigid spherical cavity. We choose $m=150$ MeV and $R=0.0205$ MeV^{-1}. This value of R occurs for $A\approx 80$ in a one-flavor strangelet. $A=80$ is marked in the figure. For comparison we plot the asymptotic expansion for $N(k)$ with and without a curvature correction obtained by integrating the density of states, $\rho(k)$,

$$\rho(k)=\frac{g}{2\pi^2}\left[k^2V-\frac{\pi kS}{4}\left(1-\frac{2}{\pi}\arctan\frac{k}{m}\right)+\kappa_c\oint d^2s\frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\cdots\right], \tag{1}$$

where V and S are the volume and surface area of the bag, respectively, κ_c is the "curvature coefficient" which depends on the boundary condition and on k/m, and R_1 and R_2 are the principal curvatures at each point. The mass dependence of the curvature term, κ_c, is not known analytically. We follow Madsen [8] and use the form appropriate to the nonrelativistic limit, $m>k$, where $\kappa_c=1$. The dashed and solid curves in Fig. 1 correspond to Eq. (1) with and without the curvature correction, respectively. At $kR\approx 10$, which corresponds to $A\approx 80$ the actual $N(k)$ is a noisy function of k reflecting the eigenvalue spectrum of the Dirac operator in a cavity. At large k, $N(k)$ is well approximated by the integral of Eq. (1). The curvature term in Eq. (1) clearly improves the fit for small strangelets; however, the fluctuations are obviously important in this region, and as we shall see, lead to important variations in the stability of light strangelets.

Since we are dealing with light quarks confined to a small bag, we must consider them as relativistic particles. We write down the Dirac equation and the appropriate boundary condition,

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\Psi=E\Psi,\quad \text{for } r<R\,, \tag{2}$$

$$i\hat{\mathbf{r}}\cdot\boldsymbol{\gamma}\Psi=\Psi,\quad \text{for } r=R\,. \tag{3}$$

This boundary condition ensures that there is no probability flux leaving the bag ($\mathbf{j}\cdot\hat{\mathbf{r}}=0$). Note that the wave function need not go to zero on the boundary, whereas for the nonrelativistic case it must. This implies that the more massive, less relativistic, strange quarks will tend to shy away from the boundary of the bag.

Once the Dirac equation is solved with this boundary condition and geometry, we obtain expressions for the eigenfunctions and transcendental equations for the eigenvalues. We take the energy, momentum, and mass to be ω, k, and m, respectively. We define $\alpha=\omega R$, $x=kR$, and $\lambda=mR$, so that $\alpha^2=x^2+\lambda^2$. The bag eigenvalue equation which determines $\alpha(\lambda)$ can be found in Ref. [9], as can be normalized quark wave functions. For the u and d quarks, the mass is taken to be zero.

As described in Ref. [6], we determine the bag constant and an estimate of the radius of the bag as a function of ϵ_b, m_s, and A by studying the bulk limit, $A\to\infty$. In bulk equilibrium, the Fermi seas for the three quark species must have the same Fermi energy or chemical potential: $\mu=\mu_u=\mu_d=\mu_s$. μ is the change in total energy due to the addition of a quark. We obtain the number of particles/volume n_a for $a=u$, d, and s by integrating the k^2V term in Eq. (1). For the massless u and d quarks, $n_{u,d}=(\mu^3/\pi^2)$ for the s quark, $n_s=(\mu^3\cos^3\theta/\pi^2)$, where $\sin\theta=(m/\mu)$. In bulk, the baryon number is $A=(\frac{1}{3})\times\sum_a n_a V$. The total energy of the bag is $E=\sum_a\mu_a n_a V$. By using these two expressions, we find that $\epsilon_b=3\mu$. As noted in Ref. [6], the surface tension is positive. Thus, to minimize energy, the shape of the strangelet will be spherical. By inverting the relation between A and n, we obtain

$$R=\left\{\frac{9\pi A}{4}\left(2+\left[\frac{\sqrt{\epsilon_b^2-9m^2}}{\epsilon_b}\right]^3\right)^{-1}\right\}^{1/3}\frac{3}{\epsilon_b}\,. \tag{4}$$

This is the radius of a lump of strange matter in which all surface effects are ignored and is used as a first approximation to the actual radius that will balance the quark pressure against the vacuum pressure. The equilibrium condition on the volume gives us $B=-\sum\Omega_a\equiv$quark pressure. Using the Ω_a from Ref. [6], we get

$$B=\frac{2(\epsilon_b/3)^4}{4\pi^2}+\frac{1}{4\pi^2}\left\{\frac{\epsilon_b}{3}\left[\left(\frac{\epsilon_b}{3}\right)^2-m^2\right]^{1/2}\left[\left(\frac{\epsilon_b}{3}\right)^2-\frac{5}{2}m^2\right]+\frac{3}{2}m^4\ln(\epsilon_b/3+[(\epsilon_b/3)^2-m^2]^{1/2}/m)\right\}. \tag{5}$$

Once we choose m_s and determine a first approximation to the radius we calculate the energy levels by solving the transcendental equation numerically. We then adjust the radius, and recalculate the energy levels, until the total energy is minimized. Once a strangelet is thus created, we can read off its energy per baryon, strangeness, and radius directly.

We have performed a variety of checks on this calculation. First, we have calculated the number of states with "momentum" less than k. This function, $N(k)$, should be approximated by the integral of the asymptotic expansion of Eq. (1) for large k. This check is shown in Fig. 1 where it is clear that our model reproduces the asymptotic result and the surface correction. Second, we have checked that the energy per baryon and strangeness per baryon also converge to the bulk values as $A \to \infty$.

We now turn to issues of stability and composition of strangelets. Since all of the strangelets we consider have $\epsilon > 930$ MeV, they could eventually decay into $^{56}_{26}$Fe or other nuclei. Therefore, our strangelets can only be called metastable. If a strangelet is not in flavor equilibrium, it can decay via weak semileptonic decays, weak radiative decays, and electron capture all of which have $\Delta A = 0$. Other modes of decay such as fission, alpha decay, weak and strong neutron decays, and strong Λ, Σ, Ξ, Ω decays reduce the baryon number by one or more units. Our strangelets are already in equilibrium at a given A and thus are only subject to the decay modes which reduce the baryon number. We check the stability of our strangelets against these decay modes.

A strong (weak) neutron decay will occur if the difference in energy between two strangelets of $\Delta S = 0(-1)$ and $\Delta A = -1$ is greater than m_n. For the Λ, Σ, Ξ, Ω decays, we have $\Delta A = -1$ and $\Delta S = -1, -1, -2, -3$, respectively.

For small A, the dynamics are as follows. Given the choice between massive and massless particles, we fill the bag with less energetic massless particles first. We add massless particles until we build up a large enough Fermi sea so that it becomes energetically favorable to add an s quark to the system. Soon, it again becomes favorable to add nonstrange quarks to the system. One might expect that strange and nonstrange levels will fill in an alternating sequence. Figure 2(a) shows that this is not the case. This is because the massive quark energy levels change at a different rate with respect to the radius than the massless quark energy levels do. Energy levels can cross, and strange levels that have been filled may empty out into nonstrange levels. This can be seen in the data for $\epsilon_b = 950$ MeV, $m_s = 150$ MeV, where a level crossing occurs at $A = 30$ [see Fig. 2(a)].

Within our model, there exist metastable strangelets for various system parameter values (see Fig. 3). Those species noted in the figure are stable against single baryon emission. We find that for $\epsilon_b < 930$ MeV, which is the value of ϵ for $^{56}_{26}$Fe, there exist many metastable strangelets (see Fig. 3). Whenever the slope of the $\epsilon(A)$ curve is negative enough, the decrease in energy due to emission of a particle is not enough to offset the increase in energy due to the slope. Thus, it is frequently energetically unfavorable for a strangelet to decay via baryon emission. Many of the smaller strangelets, however, are subject to fissioning into several Λ hyperons and a nucleus, or dissolving into Λ hyperons and neutrons. This mode is a strong decay, but its rate will be suppressed by several orders of magnitude due to the unlikeliness of the quarks simultaneously arranging themselves into the decay products. The suppression is difficult to estimate, however, because we are dealing with a collective, many-particle effect. Some quasistable species occur in regions where the slope of $\epsilon(A)$ is positive [e.g., Fig. 3(d) near $A = 60$]. In this region, the change in strangeness between most stable species with A and $A-1$ is $\Delta S = -3$ [see Fig. 2(a)] requiring Ω emission which is energetically forbid-

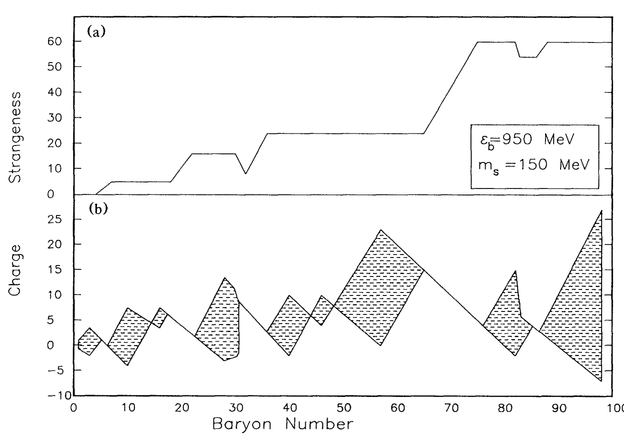

FIG. 2. (a) Strangeness as a function of A for the most stable species. (b) Strangelet charges as a function of A.

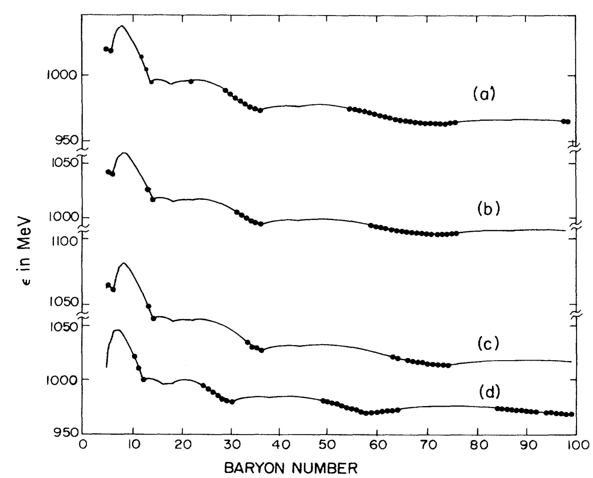

FIG. 3. Energy per baryon as a function of A. (a) $\epsilon_b = 930$ MeV, $m_s = 150$ MeV, $B^{1/4} = 154.64$ MeV. (b) $\epsilon_b = 950$ MeV, $m_s = 150$ MeV, $B^{1/4} = 158.71$ MeV. (c) $\epsilon_b = 970$ MeV, $m_s = 150$ MeV, $B^{1/4} = 162.31$ MeV. (d) $\epsilon_b = 950$ MeV, $m_s = 250$ MeV, $B^{1/4} = 151.60$ MeV.

den. Neutron emission requires $\partial\epsilon/\partial A$ *at fixed strangeness* to be positive. Direct calculation shows $\partial E/\partial A|_s$ to be negative here.

Another interesting effect that can be seen is the phenomenon of shell closures. The first level to fill is a $1s_{1/2}$ level. At every occurrence of a shell closure, the $\epsilon(A)$ curve takes a noticeable dip. This generates a large slope for $\epsilon(A)$ and thus, metastable regions in the neighborhood of a closed shell. This is similar to atomic physics where shell closures produce more stable, less chemically reactive elements. Shell closures can be seen at $A=6,14,18,22,\ldots$ (see Fig. 3). These particular values correspond to a strange $s_{1/2}$ shell, a nonstrange $p_{3/2}$ shell, a nonstrange $p_{1/2}$ shell, and a strange $p_{3/2}$ shell, respectively. The locations of these shell closures are a function of the self-consistent "potential," in which the quarks are bound—in this case, the bag. Therefore, the precise values should not be taken too seriously.

The most surprising results are uncovered when we examine values of $\epsilon_b > 930$ MeV. Looking at $\epsilon_b=950$, 970 MeV, $m_s=150$ MeV, we see that there still exist islands of metastability against single baryon decay (see Fig. 3). This is interesting because the failure of terrestrial searches to find stable strange matter suggests that strange matter in bulk may well be unstable [10]. Our results indicate that even though this may be the case, there is still a chance of detecting small strangelets in the laboratory provided the strong decay into light nuclei and several hyperons or complete dissolution does not proceed too rapidly. These islands of metastability persist until $\epsilon_b \approx 1000$ MeV, $m_s=150$ MeV.

The charge systematics of light strangelets are important for experimenters. In bulk, we expect roughly equal numbers of u, d, and s quarks, thus $Z/A \ll 1$. Even for nuclei, where $Z\sim A$, Coulomb effects are not important for small A. The possible charges of small strangelets are determined by which shells are filled, and which one is currently filling. Figure 2(b) shows that the allowed charges for strangelets as a function of A is a complex function that reflects the nature of the shell filling process. Our model possesses exact isospin symmetry since we have ignored both quark masses and electromagnetic effects. Thus, the most stable configuration at each A is an isospin multiplet. In actuality, small isospin violating effects will select a single species at each A. However, transitions among states of the same A are mediated by slow processes like β decay, so the species shown in Fig. 2(b) are equally stable from the experimental standpoint. For $A<100$, the charge remains relatively small (in magnitude) in comparison to A, so we are justified in neglecting the Coulomb energy contribution.

We also plotted the spatial density for the quarks in the strangelets. The heavier s quarks have a distribution that is concentrated closer to the center of the strangelet than the u and d quarks (see Fig. 4). By requiring that no probability flux leave the bag rather than requiring that $\Psi=0$, a relativistic quark may have a nonzero density, $\Psi^\dagger\Psi$, at the boundary. As the mass of the particle increases, we approach the nonrelativistic limit where the boundary condition becomes $\Psi=0$.

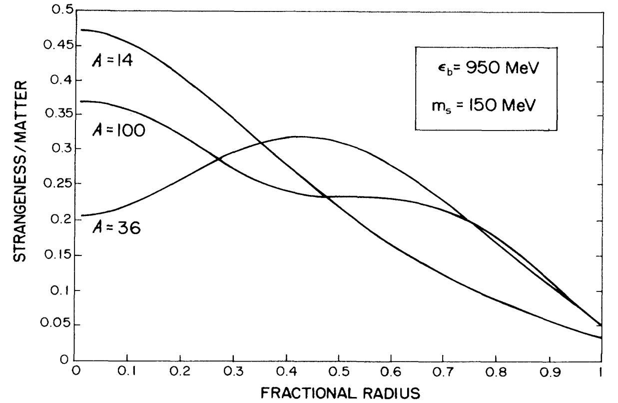

FIG. 4. The ratio of radial strangeness density to radial total matter density for small A.

We have shown that the energetics associated with shell closures are likely to be important in the study of very small strangelets. Our admittedly crude method brings out this aspect of the system that is not seen when the smoothed density of states is employed. Our results are consistent with those obtained for large A. We therefore conclude that metastable strange matter may be found in small lumps. The suppressed strong decay into a nucleus (or many neutrons) and Λ hyperons might render it difficult to detect, however. One characteristic that would identify a strangelet is its unusual charge/mass ratio. The charge is typically small as described above.

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