PHYSICAL REVIEW LETTERS

VOLUME 71 | 15 NOVEMBER 1993 | NUMBER 20

Proposed Test for Temporal Bell Inequalities

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(Received 11 January 1993)

Temporal Bell inequalities can be violated for sequences of events (histories) for which probabilities satisfying consistent sum rules cannot be defined. We discuss possible experiments in which such violations, never observed so far, may indeed be seen. The basic scheme, which uses three optically driven and mutually interacting two-level systems, could be implemented in a variety of nanostructures. It could even be mapped onto the dynamics of a single electron four-level system thus allowing for a realization in atomic physics.

PACS numbers: 03.65.Bz

Quantum mechanics and common sense are frequently at odds. At the center of this conflict lie the superposition principle and the need for assigning complex amplitudes rather than probabilities to alternative events at different times. These simple principles have been the source of many paradoxes and of an ongoing debate during which some of the key ideas we discuss in this paper have been proposed.

In the context of the analysis of *macroscopic quantum effects*, Leggett and Garg [1] introduced temporal Bell inequalities, which specifically test the existence of interference effects and their persistence in time. The inequalities were originally proposed to confront quantum mechanics with theories based on the hypothesis of macrorealism and noninvasive measurability. This proposal met with strong criticisms [2,3] that questioned the possibility of performing noninvasive experiments with SQUIDs, even though the relevant variable is a collective mode involving some 10^{20} particles. A few years later, Tesche [4] discussed an interesting strategy that, avoiding back action effects, might be used to perform such experiments.

The aim of our paper is twofold: On the one hand, we hope to help in clarifying the significance of the temporal Bell's inequalities. Despite the fact that these inequalities can be used to exhibit fundamental features of quantum mechanics, they remain rather unknown even among specialists. On the other hand, we propose experiments in which optically driven systems are used to find quantum mechanical violations of these inequalities. It is noticeable that no experiments have even been done to test the inequalities. These experiments require a sequence of controlled measurements over identically prepared systems, which may be a rather serious challenge. The tests we propose here seem to be within the reach of modern technology now. They involve microscopic systems, which undoubtedly behave quantum mechanically, and would serve two purposes: First, they would constitute interesting demonstrations of basic quantum interference effects. Second, they would be a clean microscopic replica of the, still unfinished, SQUID experiment aimed at observing macroscopic quantum coherence [1,4]. Let us start with a discussion of the basic ideas behind the inequalities and then continue with an analysis of our basic model: an optically controlled three-particle system.

A simple way of deriving temporal Bell's inequalities is by using an argument, based on hidden variables, similar to the one usually invoked to obtain the ordinary Bell's inequalities [5]. However, in this case the hidden variables are rather peculiar: they are "histories"—trajectories $Q(t)$ in some generalized coordinate space. If trajectories exist (if they are "elements of reality") the results obtained measuring the coordinate Q at times t_1 and t_2 will be unequivocally determined by $Q(t_1)$ and

$Q(t_2)$, respectively. If we were able to perform measurements on a given ensemble of systems following different histories with probability $P[Q(t)]$, the resulting two-time correlation function $K(t_1,t_2)$ would be

$$K_{12}\equiv K(t_1,t_2)=\int [DQ]P[Q(t)]Q(t_1)Q(t_2)\,, \qquad (1)$$

where the integration is over all trajectories $Q(t)$. For a bounded coordinate Q (i.e., $|Q|\leq 1$), we get

$$-2\leq Q(t_1)Q(t_2)+Q(t_2)Q(t_3)+Q(t_3)Q(t_4) -Q(t_1)Q(t_4)\leq 2\,. \qquad (2)$$

Integrating over all trajectories, with weight $P[Q]$, we readily find one of the temporal Bell inequalities:

$$|K_{12}+K_{23}+K_{34}-K_{14}|\leq 2\,. \qquad (3)$$

This inequality, and others that can be derived in a similar way, are a consequence of just a few assumptions: the existence of an ensemble of histories (characterized by the probability $P[Q(t)]$), the noninvasive measurability of the histories [needed to write the correlation as in (1), where $Q(t_2)$ is independent of $Q(t_1)$], and the boundedness of Q (a condition satisfied by any spinlike variable and in other cases like a particle in a box, etc.). The role of noninvasiveness is (technically) similar to the one of locality in the ordinary Bell inequalities.

As quantum mechanics does not allow one to define consistent probabilities for general sets of histories, the above inequalities can be violated. The "consistent histories" approach to quantum mechanics [6] addresses the mathematical conditions that have to be satisfied for such probabilities to exist. Probabilities can be assigned to histories provided "consistency conditions" are satisfied. For such "consistent sets," temporal Bell inequalities cannot be violated. In this sense, they can be interpreted as indicating the consistency of the set of histories.

To measure the correlation $K_{\alpha\beta}$ we must determine the joint probabilities. For this, one could be tempted to perform a sequence of ordinary measurements at t_α and t_β. However, any measurement implemented by dissipative interactions irreversibly perturbs the evolution of the system at these two times. To test the hypothesis behind the inequalities (3) one must perform a noninvasive measurement. The serious constraint this imposes on the measurement strategy can be satisfied by the following scheme (similar to the one proposed by Tesche in [4]). The complete measurement process can be split into a nondissipative filtering and a subsequent dissipative detection (recent related examples are memory devices recording "which-way-information" in proposed atomic beam double-slit experiments [7]). In our case, filters are created by controlled nondissipative interactions with auxiliary devices called "memories." The memories "record" the state of the system and may (or may not) change their individual state during the interaction. Finally, the records are retrieved by means of an ordinary (dissipative) measurement performed by "looking" at the memories after the time t_2. Before this observation, no real measurement has taken place: the unitary filtering process can be easily undone. Thus, this strategy allows us to obtain information about the state of the system at two times by making a single "real" measurement at a later instant. Noninvasiveness requires the obtained data to be purged by throwing away all the information coming from channels in which the state of the first memory changes. Repeating this with different system-memory couplings we can obtain $K_{\alpha\beta}$.

This strategy can be implemented in a simple model consisting of an array of three *physically* different two-level systems: the system S and the memories M_1 and M_2. The one-electron states of the three components $(j=M_1,S,M_2)$ are taken as eigenstates $|n_j\rangle$ of the pseudospin operator $\hat{\sigma}_j$ with eigenvalue $n_j=\pm 1$. For our idea to work, the states $|\pm\rangle$ must be localized in somewhat different spatial regions so that a transition between them not only involves a change of energy but also of charge distribution (charge-transfer excitations). The latter can be approximated as a change of (static) dipole moment, $\hat{\mathbf{d}}_j=\mathbf{e}_j d_j\hat{\sigma}_j$, pointing in the direction of the unit vector $\mathbf{e}_j$. Dipole-Dipole interaction then dominates the dynamics [8] and the total Hamiltonian is

$$\hat{H}=\sum_j \epsilon_j\hat{\sigma}_j+\sum_{j\neq k} C_{jk}\hat{\sigma}_j\hat{\sigma}_k\,. \qquad (4)$$

The eigenstates of $\hat{H}$ are the eight product states $|n_{M_1},n_S,n_{M_2}\rangle=|\pm,\pm,\pm\rangle$. Let the ground state be $|-,-,-\rangle$. The array described by (4) has twelve nondegenerate frequency channels connecting the eight possible energy eigenstates through single photon (dipole allowed) transitions (see Fig. 1). The minimal frequency splitting $\delta\nu$ is of the order of $|C_{jk}|$.

As a first step, we optically drive coherent (Rabi) oscil-

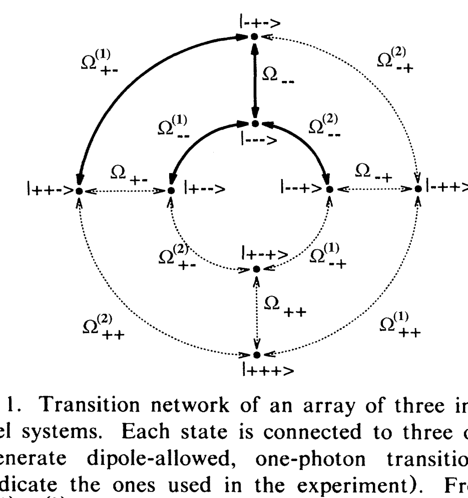

FIG. 1. Transition network of an array of three interacting two-level systems. Each state is connected to three others via nondegenerate dipole-allowed, one-photon transitions (bold lines indicate the ones used in the experiment). Frequencies $\Omega_{nn'},\Omega^{(1)}_{nn'},\Omega^{(2)}_{nn'}$ correspond to transitions involving a change of the state of S, M_1, and M_2, respectively. Subindices indicate the states of the unchanged components.

lations of the system S between its $|\pm\rangle$ states. For this purpose, after preparing the array on its ground state, we continuously apply a laser field with frequency Ω_{--} (see Fig. 1). For zero detuning, and time scales shorter than the decoherence time τ_c (a measure of how well isolated the system is), this coherently driven system would be theoretically characterized by the two-time correlation function

$$K_{12}=\tfrac{1}{2}\langle\hat{\sigma}_S(t_1)\hat{\sigma}_S(t_2)+\hat{\sigma}_S(t_2)\hat{\sigma}_S(t_1)\rangle$$
$$=\cos[2\Omega_R(t_1-t_2)]\,, \quad (5)$$

where Ω_R is the Rabi frequency (proportional to the intensity of the driving field). As noticed by Leggett and Garg [1], there is a complete analogy between the role played by the times t_i in (5) and the angle settings of the polarizers in EPR experiments. Violations of the inequalities (3) are a consequence of (5).

To measure the joint probabilities we induce the interaction between S and the memories by applying a sequence of π pulses at specific times. The frequency of each pulse is such that a memory can change its state (the transition is resonant) only if S is in a specific dipole eigenstate. For example, the following is the recipe we should follow to measure $p(+,t_1;+,t_2)$: At time t_1 we apply a π pulse with frequency $\omega_1=\Omega^{(1)}_{--}$ which induces a change in the state of M_1 only if the system S is in the state $|-\rangle$. Later, at t_2, we apply a π pulse with frequency $\omega_2=\Omega^{(2)}_{--}$. The evolution of the wave function of the array is displayed in Fig. 2(a). After the application of the two pulses we can read the states of the memories (see below) and obtain some of the joint probabilities: $p(+,t_1;+,t_2)$ is just the relative frequency of finding the two memories in their ground (unchanged) state. In this way one can also measure $p(+,t_1;-,t_2)$. However, the above sequence cannot be used to determine the probabilities $p(-,t_1;\pm,t_2)$. In fact, as the dipole-dipole interaction is symmetrical, the system S is kicked out of resonance if the state of M_1 is changed by the π pulse. This is the notorious back action effect, present even in this purely coherent evolution. The remaining probabilities can be analogously measured by choosing the pulse at t_1 so that M_1 switches its state only if the state of S is $|+\rangle$. When this new sequence of pulses is applied, the array wave function evolves as shown in Fig. 2(b).

The final measurement of the state of the memories might be done by an excitation luminescence experiment on S, exploiting the dependence of its transition frequencies on the states of M_i. Thus to obtain the probability for a configuration of M_1 and M_2, we should apply a laser which is resonant with a transition of S only if both memories are in such configuration. The intensity of the luminescence is proportional to the probability. This strategy requires a specific time scale hierarchy: the decay time of the excited state of S must be much shorter than the lifetime of both memories. If this constraint is satisfied by the $|\pm\rangle$ states (i.e., if the $|+\rangle$ state of S has a longer lifetime than the $|+\rangle$ states of both M_i), the frequency of the laser should be one of those appearing in Fig. 1. Otherwise, a convenient alternative is to excite S to a third level with localization properties such that it rapidly decays into $|+\rangle$ (see [8]). Any of these strategies would enable us to perform the experiment with ensembles of arrays.

In order to accurately measure the joint probabilities we must make sure that the error caused by each π pulse is small enough: to detect a violation of (3), the error in determining $K_{\alpha\beta}$ must be smaller than 10% since the left hand side of this inequality is bounded by $2\sqrt{2}$. A simple numerical estimate implies that this requires the pulse

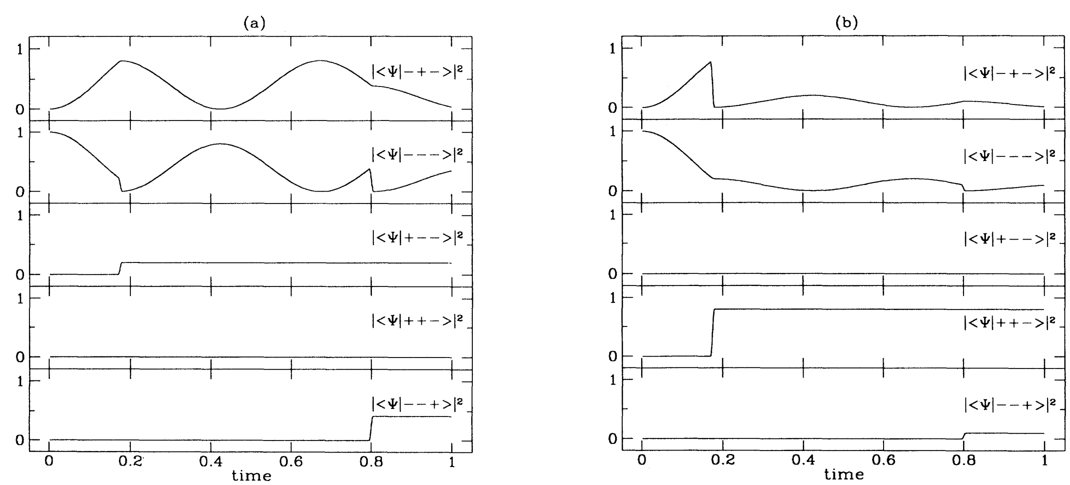

FIG. 2. Evolution of the array wave function (time in units of the Rabi period of system S). The initial state is $|---\rangle$. A driving laser (frequency Ω_{--}) is continuously applied. π pulses are applied at $t_1=0.175$ (frequency ω_1, $\Delta T=10^{-2}$) and $t_2=0.8$ (same duration, frequency ω_2). (a) $\omega_1=\Omega^{(1)}_{--}$, $\omega_2=\Omega^{(2)}_{--}$; (b) $\omega_1=\Omega^{(1)}_{+-}$, $\omega_2=\Omega^{(2)}_{--}$.

duration ΔT_i to be bounded by $\Delta T_i < 10^{-2}\Omega_R^{-1}$ (remember that we must also consider Rabi oscillations shorter than τ_c beyond which dissipation becomes important, i.e., $\Omega_R^{-1} \ll \tau_c$). The effect of the off resonant terms can be made small by having a large minimal frequency splitting $\delta\nu \gg 1/\Delta T_i$. Finally, in order to have approximately monocromatic and selective π pulses, we must require $\Delta T_i \gg 1/\omega_i$.

Our analysis has been based on the transition network sketched in Fig. 1. Possible physical realizations of such network could be obtained using heterogeneous molecular systems (possibly on substrate) and specific semiconductor quantum dot arrays. Their characteristic feature would be a set of charge-transfer excitations embedded in a material matrix. The most critical parameters are the minimal frequency shift $\delta\nu$ (which depends upon the local change in dipole moment and the distance between components) and the decoherence time τ_c. Realistic estimates could be $\delta\nu = 10^{12}$ Hz (for a typical distance of 20 nm, which produces an energy splitting of 1 meV), $\tau_c = 10^{-6}$ sec. A parameter window satisfying the constraints discussed in the previous paragraph could be reached in the near future.

It is worth noticing that we only need four out of the eight states of the array (including their selective connectivity). A minimal model, constrained to the states $|1\rangle = |---\rangle$, $|2\rangle = |-+-\rangle$, $|3\rangle = |+--\rangle$, and $|4\rangle = |-++\rangle$, might thus act as a "quantum simulator" of the above measurement series. In such a simple system we can continuously drive 1-2 Rabi oscillations and use two short π pulses to induce a 1-3 transition at time t_1 and a 2-4 transition at time t_2. The joint probabilities are obtained by measuring the population of the levels after t_2. After a second run with the pulses in reverse order, we can obtain K_{12}. This scheme could be tested on properly chosen atoms (such as the trapped $^9Be^+$ ions used in the demonstration of the quantum Zeno effect [9]).

Knowing how the states of the memories must have been reached in time, we can reconstruct a history for each individual system. Common sense would make us believe that this history cannot have been brought into existence at the time of the observation t_f but should rather have existed before even though we did not know. Between actual measurements events (set at will by the initial preparation and the final reading of the state) histories are not an element of reality: coherent evolution cannot consistently be decomposed into "intermediate steps" (thus a system with a two-dimensional Hilbert space must *not* be described as being always in one of its alternative states, a bivalued telegraph signal, as in [10]). It is remarkable that this striking quantum feature can be made to show up experimentally: This possibility rests upon a strategy which exploits so-called noninvasive measurements to infer two-time correlation functions which enter temporal Bell inequalities. Their violation proves the above statement. The reverse, however, is not true: If the inequalities are not violated, consistent histories may or may not be assigned.

We could use the same measurement setup (especially in the atomic context mentioned above) to examine the transition from quantum to classical regime (where histories can be considered as "real"). This process, known as decoherence [11], is induced by the interaction with an external environment. In our case, such interaction can be controlled by optically coupling the relevant states of the network to other unstable states. The strength of the system-environment coupling can be controlled by the intensity of the driving fields (a similar situation as in the partial Zeno effect [12]). Violations of temporal Bell inequalities can be made to disappear by increasing the strength of the interaction with the environment (formed by the modes of the luminescence field).

Our work has at least two other interesting motivations: The above arrays of optically driven two-level systems, can be used for creating many particle coherent states like those involved in GHZ versions of Bell theorem [13]. Furthermore, the selective coherent switching we introduced may become the basic ingredient for a quantum computer model [14]. In this context, temporal Bell inequalities not only could be used as a way of testing the existence of quantum coherent effects but also impose constraints on the interpretation of a quantum computation [15] in terms of histories.

We thank Carlton Caves, Steth Lloyd, and Wojciech Zurek for valuable discussions. One of us (G.M.) thanks the Sante Fe Institute for its hospitality during his sabbatical visit and the Deutsche Forschungsgemeinschaft for a travel grant.

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