Identical Bands in Superdeformed Nuclei: A Relativistic Description

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Relativistic mean field theory in the rotating frame is used to describe superdeformed nuclei. Nuclear currents and the resulting spatial components of the vector meson fields are fully taken into account. Identical bands in neighboring rare earth nuclei are investigated and excellent agreement with recent experimental data is observed.

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Since the experimental discovery of superdeformed bands in rapidly rotating nuclei, many unexpected features of these highly excited configurations have been observed (for a recent review see Ref. [1]). One of the most striking properties is the existence of so-called *identical bands* or *twin bands*, i.e., nearly identical transition energies E_γ of the emitted γ radiation in bands belonging to neighboring nuclei with different mass numbers. In a considerable number of nuclei in the Dy region as well as in the Hg region one has found differences in E_γ of only 1–3 keV; i.e., there exist sequences of bands in neighboring nuclei, which are virtually identical, $\Delta E_\gamma / E_\gamma \sim 10^{-3}$. Since these transition energies are directly related to the corresponding dynamical moments of inertia obeying on the average a simple $A^{5/3}$ dependence, one would have expected changes of 1 order of magnitude larger.

Several groups have tried to understand this phenomenon by means of conventional investigations within the framework of the semiphenomenological Strutinski method in connection with a rotating Nilsson or Saxon-Woods potential [2,3]. It has been pointed out that the different single particle orbits can give rather different contributions to the moment of inertia. This is most clearly seen in the simple oscillator model, where orbits with oscillator quanta along the rotational axis have vanishing angular momentum operator matrix elements. Other groups [4] realized that a new coupling scheme in nuclei exists, the so-called pseudospin scheme, which to a very large extent decouples the pseudo-orbital motion from the pseudospin degrees of freedom and favors the strong coupling limit. In all these investigations, however, polarization effects, which are expected to produce much larger changes of the moments of inertia than those observed in identical bands, are either neglected completely [4] or taken into account only partially by minimizing the rotating energy surface with respect to a few deformation parameters.

We therefore feel that it is very important to carry out fully self-consistent microscopic calculations, where all the degrees of freedom are taken into account. Such calculations are not simple, but they are nowadays feasible. A first investigation of this type using the density dependent Skyrme III force has very recently been carried out in the Hg region [5]. It has been found that

full self-consistency has indeed a considerable influence on the details of the moment of inertia in neighboring nuclei. There are, however, many important questions still open. In these nuclei identical bands occur in a spin region where pairing plays an important role. Since particle-particle correlations are not very well described within the Skyrme scheme, it is not at all clear to what extent the present deviations of the theoretical from the experimental results can be understood by such deficiencies.

The present investigation is therefore devoted to the Dy region, where superdeformed bands are observed up to very high angular momenta. At these very large rotational frequencies the Coriolis-antipairing effect reduces these correlations considerably, such that they have little influence on the moment of inertia. We use a relativistic field theory which includes σ, ω, and ρ mesons as well as the electromagnetic field. In addition we consider a nonlinear self-coupling of the σ field. We use the parameter set NL1, which has been adjusted [6] to nuclear matter and a few spherical nuclei. This parameter set has turned out to be very successful for the description of many ground state properties over the entire periodic table [7]. In particular one has found excellent agreement with ground state deformations in open shell nuclei. The rotation is treated within the cranking approach, in accordance with the concept of a mean field description. This leads us to a relativistic self-consistent cranking theory (RSCC) as developed in Ref. [8].

In the rotating frame time reversal invariance is broken. This leads to nucleonic currents in the interior of the nucleus, which form the source of magnetic potentials in the Dirac equation (*nuclear magnetism*). In this way the charge current $\mathbf{j}_c$ is the source of the normal magnetic potential $\mathbf{A}$, the isoscalar baryon current $\mathbf{j}_B$ is the source of the spatial components $\boldsymbol{\omega}$ of the ω mesons, and the isovector baryon current $\mathbf{j}_3$ is the source of the spatial component $\boldsymbol{\rho}_3$ of the ρ mesons. In contrast to the Maxwellian magnetic field $\mathbf{A}$ having a small electromagnetic coupling, the large coupling constants of the strong interaction cause the fields $\boldsymbol{\omega}$ and $\boldsymbol{\rho}$ to be important in all cases, where they are not forbidden by symmetries, such as time reversal. They have a strong influence on the magnetic moments [9] in odd mass nuclei, where time

reversal is broken by the odd particle, as well as the moment of inertia in rotating nuclei, where time reversal is broken by the Coriolis field. In an early investigation of rapidly rotating superdeformed nuclei within the framework of cranked relativistic mean field theory [10] these components were not taken into account for reasons of numerical simplicity. Strong deviations from the experimentally observed moments of inertia were found. Only the size of the quadrupole deformation was reproduced properly. Semiclassical corrections turned out to be large, but could not reproduce the proper experimental values of the moment of inertia.

In this investigation nuclear magnetism is taken fully into account in a self-consistent way. Starting from the Lagrangian

$$\mathcal{L} = \bar{\psi}\left(\not{p} - g_\omega\not{\omega} + g_\rho\not{\rho}\cdot\boldsymbol{\tau} - \frac{1}{2}e(1+\tau_3)\not{A} - g_\sigma\sigma - M_N\right)\psi$$
$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$
$$-\frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} , \quad (1)$$

where M_N is the bare nucleon mass and ψ is its Dirac spinor. We have in addition the scalar meson (σ), isoscalar vector mesons (ω^μ), isovector vector mesons (ρ^μ), and the photons A^μ, with the masses m_σ, m_ω, and m_ρ and the coupling constants g_σ, g_ω, g_ρ. For simplicity in the following equations we neglect the ρ meson and the photon. In the calculations these contributions are, however, taken into account. The field tensors for the vector mesons are given as

$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu . \quad (2)$$

For a realistic description of nuclear properties a nonlinear self-coupling for the scalar mesons has turned out to be crucial [11]:

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma_+^2\frac{g_2}{3}\sigma_+^3\frac{g_3}{4}\sigma^4 . \quad (3)$$

Starting from this Lagrangian and transforming to a frame rotating with a uniform velocity Ω_x around the x axis perpendicular to the symmetry axis of the deformed nucleus in its ground state, we obtain the classical equations of motion

$$\{\boldsymbol{\alpha}\cdot(\mathbf{p}+g_\omega\boldsymbol{\omega}) + g_\omega\omega_0 + \beta(M+g_\sigma\sigma) - \Omega_x J_x\}\psi_\alpha$$
$$= \epsilon_\alpha\psi_\alpha \quad (4)$$

for the nucleon spinors and

$$\{-\Delta + (\Omega_x L_x)^2\}\sigma + U'(\sigma) = -g_\sigma\rho_s , \quad (5a)$$
$$\{-\Delta + (\Omega_x L_x)^2 + m_\omega^2\}\omega^0 = g_\omega\rho_v , \quad (5b)$$
$$\{-\Delta + (\Omega_x J_x)^2 + m_\omega^2\}\boldsymbol{\omega} = g_\omega\mathbf{j} , \quad (5c)$$

where $J_x = L_x + S_x$ and the spin operator S_x is a 4×4 matrix for the spinor fields with spin $\frac{1}{2}$ and a 3×3 matrix

for vector fields with spin 1. For details see Ref. [8].

These equations are solved self-consistently by expanding the Dirac spinors as well as the meson fields in terms of eigenfunctions of a deformed oscillator, as discussed in detail in Refs. [7,8]. Up to $N_F = 13$ major oscillator shells were taken into account for the large components of the fermion fields and up to $N_B = 13$ shells for the meson fields. Because of the large number of configurations in this space high-lying orbitals with a deformed oscillator energy larger than $10.3 \times \hbar\omega^0$ for the fermions and larger than $10.5 \times \hbar\omega^0$ for the bosons have been neglected. In general one has to allow for complex expansion coefficients in this basis. However, assuming mirror symmetry at the three planes (x, y), (x, z), and (y, z) for the densities and rotational symmetry for the currents it is possible to restrict oneself to real coefficients. Since, in the rotating frame, there is no a priori reason for these symmetries, a complex code was used to show that, even with initial conditions strongly violating these symmetries, we find after many iterations final self-consistent solutions which obey these symmetries. All the following calculations have therefore been carried out with the real code only.

In Fig. 1 we show the static and the dynamic moment of inertia for the lowest superdeformed band in the nucleus ^{152}Dy as a function of the angular momentum. It is clearly seen that a calculation without nuclear magnetism, i.e., without the spatial contributions of the vector meson fields, which is in good agreement with the experimental quadrupole moments (see Ref. [10]) produces much too small moments of inertia. A semiclassical correction where these contributions, derived in
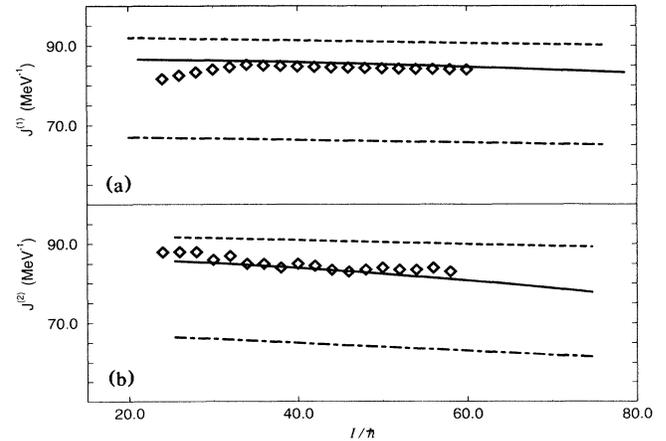

FIG. 1. (a) Static ($\mathcal{J}^{(1)}$) and (b) dynamic ($\mathcal{J}^{(2)}$) moment of inertia for the lowest superdeformed band in the nucleus ^{152}Dy. The dashed line corresponds to the calculation without the spatial contributions of the vector mesons. In the dotted line such contributions are taken into account in a semiclassical way and the full line represents the self-consistent solution including these contributions fully.
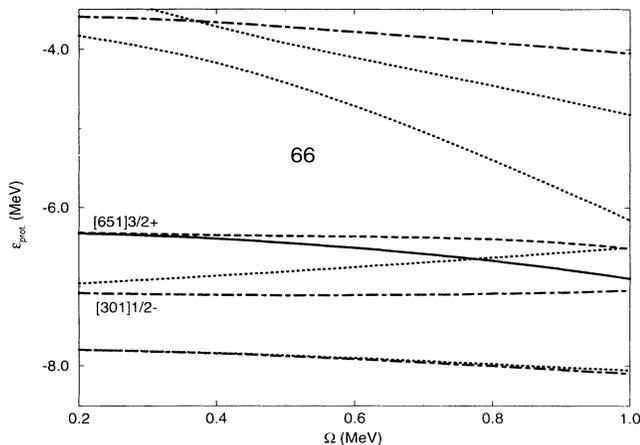
FIG. 2. Single-proton spectra in the self-consistent rotating potential of the superdeformed band in ^{152}Dy as a function of the cranking frequency Ω_x. For small values of Ω_x, where the two signatures are nearly degenerate, we indicate the approximate Nilsson quantum numbers.

TABLE I. Binding energy E, mass quadrupole moment Q_0, dynamic $(\mathcal{J}^2)$, static $(\mathcal{J}^1)$, and rigid body $(\mathcal{J}_{\text{rig}})$ moment of inertia at the angular momentum $I = 50\hbar$ for the superdeformed band in ^{152}Dy and relative changes of these values (given in %) several bands in the neighboring nucleus ^{151}Tb at the same angular momentum.

Band	E (MeV)	Q_0 (fm^2)	$J^{(2)}$	$J^{(1)}$	J_{rig}
^{152}Dy	-1228.358	4287.7	82.544	86.41	93.35
^{151}Tb*$(+,+)$	0.53	-2.91	-1.554	-1.25	-1.69
^{151}Tb $(+,-)$	0.51	-3.17	0.654	0.35	-1.80
^{151}Tb*$(-,+)$	0.56	1.30	-0.001	0.10	-0.45
^{151}Tb*$(-,-)$	0.54	1.19	0.145	1.48	0.39

Thomas-Fermi approximation using a rigid rotor current, are taken into account in first order perturbation theory overemphasized the moments of inertia by roughly 10%. Only if one takes these contributions into account in a fully self-consistent way is perfect agreement with experimental data achieved. In the region of small angular momenta one still observes very small deviations, which could possibly be understood as the influence of remaining pairing correlations in this region of intermediate spins.

We also find that nuclear magnetism has practically no influence on the shape of the nucleus. The mass quadrupole moments decrease in the spin range from $20\hbar$ to $60\hbar$ only very little, running from 4350 to 4260 fm^2 and the corresponding hexadecupole moments change for the same region from 20 600 to 19 600 fm^4. The changes induced by nuclear magnetism are of the order of a few per mille. The average charge quadrupole moment is found to be 18.6 e b, which is in good agreement with a value of 18 e b obtained in a nonrelativistic calculation [12], and the experimental value of 19 e b [13]. From the quadrupole moments we can derive the Hill-Wheeler parameters $\beta = 0.72$ and $\gamma = 0.7°$ for the quadrupole deformations, which corresponds closely to a nearly prolate deformed nucleus with an axis ratio of 1:1.9, close to the standard value 1:2 of the harmonic oscillator model.

Let us now investigate the problem of identical bands. For this purpose we calculate, in a self-consistent way, bands in the neighboring nucleus ^{151}Tb by removing one proton from the ^{152}Dy core. In Fig. 2 we show the single particle spectrum for protons in the rotating potential formed by the ^{152}Dy core. The large gap at $Z = 66$ is clearly recognized. Taking particles out of the orbits directly below this gap, we can produce different bands

in the nucleus ^{151}Tb. They have the quantum numbers (PS) of parity (P) and signature (S), namely, $(+-)$ for the dashed line, $(++)$ for the full line, $(-+)$ for the dotted line, and $(-+)$ for the dash-dotted line.

The proton hole induces a polarization of the ^{152}Dy core, which has two effects: it leads to changes of deformation and in addition to changes in the current distribution. In Table I we show the values obtained after solving in a fully self-consistent fashion the relativistic mean field equations for the odd system in the four lowest configurations. We show the values for several observables for the lowest superdeformed band in ^{152}Dy. The relative changes with respect to this reference band in the four bands of ^{151}Tb are given in per mille. According to the simple $A^{5/3}$ rule we expect changes in the moment of inertia by ≈ 11 per mille. The calculated values for the moments of inertia $\mathcal{J}^1$ and $\mathcal{J}^2$ for the band with the quantum numbers $(-+)$, which we shall in the following call the identical band, are, however, at least an order of magnitude smaller. This is by no means trivial, because we find considerably larger changes in the quadrupole moments and in the rigid body moments of inertia. In fact in most of the other bands the changes are also much larger.

In order to have a direct comparison with the experiment we show in Fig. 3 the differences $\Delta E_\gamma = E_\gamma(\text{Tb}) - E_\gamma(\text{Dy})$ between the transition energies in several bands in the nucleus ^{151}Tb and in the lowest superdeformed band in ^{152}Dy. The agreement with the experimental value is excellent for the band with the quantum numbers $(-+)$, where the energy differences are of order of 1 keV. As we see in Fig. 2, this band corresponds to a hole in the orbit with the approximate Nilsson quantum numbers $[301]\frac{1}{2}-$. This orbit has a very small number of oscillator quanta along the z axis (the symmetry axis), which yields nearly vanishing contributions to the moment of inertia. We are therefore in agreement with the qualitative argument put forward in Ref. [14]

This is, however, not the full story. In order to investigate the very good quantitative agreement, we have carried out two additional calculations in Fig. 4 for the *iden-*
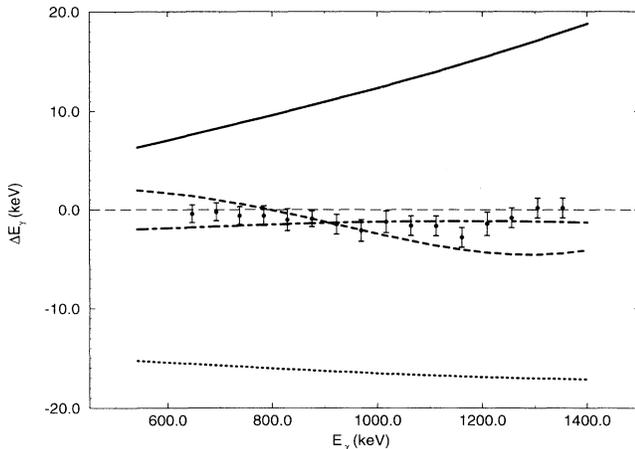
FIG. 3. Differences ΔE_γ in the transitional energies $E_\gamma = E(I) - E(I - 1)$ between the lowest bands for each pair of quantum numbers (P, S) in the nucleus ^{151}Tb and the superdeformed band in ^{152}Dy are compared with experimental values for the excited superdeformed band with negative parity in ^{151}Tb.

FIG. 4. Differences ΔE_γ for the identical band with the quantum numbers $(-+)$. The fully self-consistent solution (full line) and solutions neglecting nuclear magnetism (dashed line) or polarization induced by the proton hole (dash-dotted line) are compared with the experiment (full dots).

tical band with the quantum numbers $(-+)$: First we neglected the polarization induced by the proton hole; i.e., we calculated the energy differences for wave functions for the nucleus ^{151}Tb obtained from the ^{152}Dy core by just removing one proton, without requiring self-consistency for the odd mass configuration. In this case we find the dash-dotted line in Fig. 4, which is in sharp disagreement with the experimental data. Next we took into account the polarization, but we neglected nuclear magnetism, i.e., the contributions of the spatial components of the vector meson fields, and find the dashed line in Fig. 4, which is also in disagreement with experiment.

We therefore conclude that a very delicate cancellation process occurs in identical bands in superdeformed nuclei. Polarization of the quadrupole moments and of the density alone would induce changes of the order of 5–10 per mille. Neglecting nuclear magnetism would also lead to changes of this order of magnitude. Obviously both act in opposite directions, such that the final differences are only in the order of 1 per mille. So far the precise mechanism for this cancellation is not fully understood. It requires definitely much more systematic investigation. Nonetheless it seems to us a very satisfying and surprising result that without any free parameter, and simply using the set NL1 adjusted to nuclear matter and a few spherical nuclei, long before identical bands had been identified, we can obtain this degree of accuracy in the relatively simple minded relativistic mean field approach. We have to emphasize, however, that full self-consistency as well as the inclusion of the nuclear currents are very important in this context.
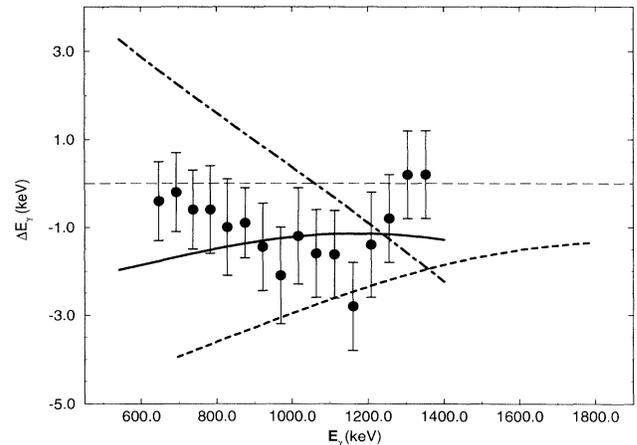
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