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Nondispersive Phase of the Aharonov-Bohm Effect

Gerald Badurek,[1] Harald Weinfurter,[2] Roland Gähler,[3] Achim Kollmar,[4] Stefan Wehinger,[2] and Anton Zeilinger[2]

[1]*Institut für Kernphysik, Technische Universität Wien, A-1020 Wien, Austria*
[2]*Institut für Experimentalphysik, Universität Innsbruck, A-6020 Innsbruck, Austria*
[3]*Reaktorstation Garching d. Technische Universität München, D-8046 Garching, Germany*
[4]*Institut für Festkörperforschung, Forschungszentrum Jülich, D-5170 Jülich, Germany*
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An essential signature of the topological nature of all Aharonov-Bohm type phases is that they are nondispersive, i.e., independent of the velocity (wavelength) of the interfering particles. This implies that an Aharonov-Bohm phase shift can greatly exceed the usual limit given by the coherence length of the interfering beams. We report the results of a polarized neutron experiment demonstrating this property for a spin-rotation analog of the scalar Aharonov-Bohm effect.

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In 1959 Aharonov and Bohm [1] published their famous proposals on the influence of electromagnetic potentials in electron interference experiments, known since then as the Aharonov-Bohm (AB) effects. Unlike classical physics where potentials are considered merely as convenient mathematical tools to calculate electromagnetic fields of force by solving Maxwell's equations, the AB effects reveal the much deeper physical significance of potentials in quantum mechanics. They are illustrious examples of quantum nonlocality, because they predict an observable phase shift of the electron's de Broglie wave packet which depends on fields in regions of space-time *not* accessible to the interfering electron. Hence there is *no force* acting on the particle and the phase shift is entirely due to nonzero potentials, namely, the vector potential $\mathbf{A}(\mathbf{r})$ in the so-called magnetic (or "vector") AB effect and the scalar potential φ in the less often cited electric (or "scalar") AB effect (Fig. 1). An essential feature of both Aharonov-Bohm effects is their nondispersivity, which implies that none of the AB effects should lead to any measurable positional shift or spread of the electron wave packet. This is a consequence of the topological nature of the effect. Moreover, such a positional shift would provide a means to detect, by observing just one of the interfering beams, the presence of an electromagnetic field without having to invoke the topology of the whole arrangement [2]. Therefore an AB phase shows up only as an overall phase factor of the wave packet and thus it is only observable in an interference experiment. We emphasize that in contrast a phase shift due to, say, transmission through a static potential well is

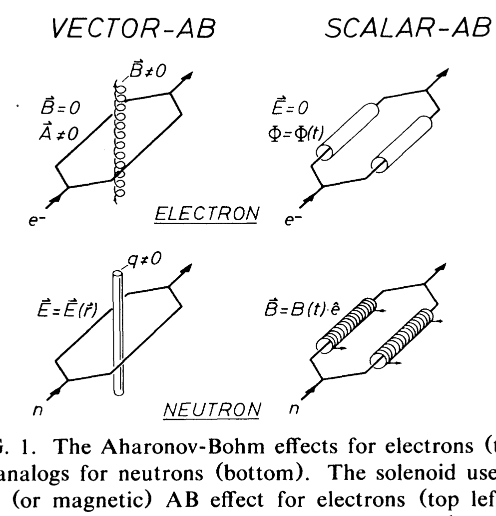

FIG. 1. The Aharonov-Bohm effects for electrons (top) and their analogs for neutrons (bottom). The solenoid used in the vector (or magnetic) AB effect for electrons (top left) is replaced in its neutron counterpart by a line charge (bottom left). In the scalar AB effect a time-dependent Hamiltonian is provided either by properly switched Faraday cages in the electron case (top right) or by switched magnetic fields in the neutron analog (bottom right).

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dispersive and therefore leads to a lag or advancement of the wave packet depending on the sign of the potential. In view of the significance of the Aharonov-Bohm concept it is certainly worthwhile and of general interest to experimentally demonstrate the nondispersivity of the effect. A striking consequence of the nondispersivity of the AB phase and hence of the topological nature of the AB effect is the property that in an AB experiment phase shifts can be achieved which greatly exceed the usual limit given by the coherence length of the interfering beams. It is the purpose of the present Letter to report the first explicit experimental demonstration of that effect. For practical reasons this was done for the specific case of a neutron analog of the scalar AB phase.

Since the original proposal the magnetic Aharonov-Bohm effect has found many experimental realizations [3], culminating in an elegant electron holography experiment by Tonomura *et al.* [4]. A common feature of all these experiments is that the coherence length of the electrons is orders of magnitude larger than their wavelength, typically $10^5\lambda$ for the experiment of Ref. [4]. This would require phase shifts which are also many orders of magnitude larger than 2π. Since in the experiments the flux enclosed between the beams was only a few flux quanta, the existing AB experiments are far from proving the nondispersivity of the effect.

On the other hand, the scalar AB effect for electrons, though it is conceptually simpler than the magnetic one, has not even found experimental verification so far. This is because its realization would necessarily require that the electric potential of one of the Faraday cages placed along the beams in an electron interferometer is raised and subsequently lowered again while the electron wave packet propagates just inside this cage. Since the particle velocities in electron interference experiments commonly are of the order of a few percent of the speed of light this implies that the Faraday cage would have to be switched extremely rapidly with frequencies of typically about 10 GHz. Besides the fact that such an ultrafast charging and discharging procedure would hardly be realizable in practice it would imply temporal fields inside the Faraday cage and thus grossly violate adiabaticity.

In order to analyze the possibility of demonstrating the nondispersivity, it is therefore advisable to turn to the generalized Aharonov-Bohm topologies which arise if the electrons are replaced by neutral particles with a magnetic moment, for instance, neutrons [5]. The neutron analog of the magnetic AB effect, the often called Aharonov-Casher (AC) or Anandan-Aharonov-Casher (AAC) effect [6], predicts a phase shift for a magnetic dipole diffracted around a charged electrode. The existence of this very small phase shift has been demonstrated experimentally by Cimmino *et al.* [7] using a three-crystal neutron interferometer, but the topological character of this quantum effect also remains to be checked.

In the present Letter we show how the neutron analog of the scalar Aharonov-Bohm effect easily provides the possibility of proving the nondispersivity of the AB phase and we report experimental evidence pertaining to that fact. The latter effect has been proposed explicitly many years ago [5,8] and the existence of the phase shift predicted has recently been shown by Allman *et al.* [9]. In that experiment, the time-dependent potential is realized by properly switched homogenous magnetic fields. However, again, the experiment is far from being able to show the nondispersivity of the effect [10]. It is evident that a simultaneous verification of both the nonlocality and the topologic nature of the scalar Aharonov-Bohm (SAB) effect could most easily be accomplished with the diffraction grating interferometer for very cold neutrons at the Institut Laue-Langevin, Grenoble [11]. For that instrument the monochromaticity requirements are less stringent than with perfect crystal neutron interferometers and, because of the small neutron velocity of only 40 ms^{-1}, tiny magnetic fields are sufficient to achieve large phase shifts.

Before turning to our experiment we briefly review the physics of the neutron scalar AB experiment. Consider the situation where in one beam of the neutron interferometer a purely time-dependent magnetic field $\mathbf{B}(t)$ interacts with the neutron. The corresponding Hamiltonian then reads as

$$H = -\mu\sigma\mathbf{B}(t)\,, \tag{1}$$

where μ is the neutron's magnetic moment. In order for the phase to be topological it is necessary to perform the experiment such that the neutron wave packet does not sense any spatial dependence of the magnetic field. Therefore the field has to be homogenous and switched on and off in such a way that it is zero both at the time when the neutron enters the magnetic field region and when it leaves that region again. This automatically ensures that no force will act on the particle. Then the two spin eigenstates will experience just a different change in frequency $\Delta\omega = \pm\mu B/\hbar$, and the particle momentum $\mathbf{p} = \hbar\mathbf{k}$ ($\mathbf{k}$ neutron wave vector) remains a constant of motion ($[\mathbf{p},H]=0$). For simplicity we assume that only the magnitude of the field but not its direction varies as a function of time. Evidently such a magnetic field leads to a phase shift

$$\Delta\Phi_{\mathrm{SAB}} = \pm\mu/\hbar\int B(t)dt \tag{2}$$

of the neutron wave function which is nondispersive as required for a topological effect and of different sign for the two spin eigenstates. Because of this difference the phase shift can also be detected through the coherent superposition of the two eigenstates. A general initial polarization can be viewed as being split into the two eigenstates parallel and antiparallel to the direction of the magnetic field. These two eigenstates acquire different phase shifts through the interaction with the magnetic field, which after superposition at the end of the field region results in an overall spin rotation. This polarized neutron interferometry scheme was already used earlier to measure Berry's phase for neutrons [12]. On the other hand, the spin rotation has led to some discussion in the literature as to what extent the resulting phase shift is an analog of the original Aharonov-Bohm concept [5,13]. Here we do not want to analyze the details of this discussion. We rather point out that the property of nondispersivity of the effect is shared also by the neutron SAB effect.

Let us recall the goal of the experiment, whose setup is depicted in Fig. 2, that is to show the nondispersivity of the SAB phase shift, i.e., its independence on the neutron wave number

$$\frac{\partial}{\partial k}(\Delta\varphi_{\mathrm{SAB}})=0\,. \qquad (3)$$

Contrary to the AB case, interaction with a static potential leads to a dispersive phase shift $\Delta\varphi_{\mathrm{stat}}(k)$. It is well known and has been demonstrated, e.g., by neutron spin echo techniques [14] that precession of a neutron spin in a static magnetic field leads to a loss of polarization. This feature is due to the fact that upon entering the static field the wave packets of the two spin eigenstates are accelerated and decelerated, respectively, since they experience a different change in momentum $\Delta k/k = \pm\mu B/2E$ with the energy E of the neutron being constant. The change of the neutron momentum upon entering a static field leads to a separation of the spin eigenstates along the propagation direction and is therefore called the longitudinal Stern-Gerlach effect. This has been demonstrated directly by exploiting the extremely high momentum resolution of ray propagation in perfect crystals [15]. After leaving the field region of length L the centers of the two wave packets will be separated by the distance $s\approx L\mu B/E$. Their reduced overlap then results in a corresponding decrease of polarization. The polarization disappears completely if the displacement is larger than the coherence length $l_{\mathrm{coh}}\approx\lambda^2/\delta\lambda$ of a beam with spectral width $\delta\lambda$ [16]. In contrast to the static field case just analyzed, in an SAB experiment with properly chosen time dependence of the magnetic field Eq. (2) holds and no decrease of the polarization should occur even if the phase shift of the two spin states greatly exceeds the coherence length of the beam. This is because in such an experiment no momentum change occurs and thus the overlap between the wave packets of the two spin eigenstates is not reduced.

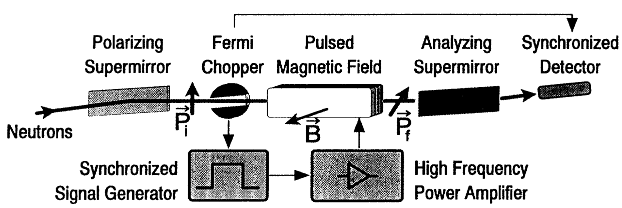

FIG. 2. Sketch of the experimental setup: A rotating Fermi chopper imposes a time structure on the neutrons polarized through reflection from a supermirror. The subsequent magnetic field oriented in a direction orthogonal to the neutron polarization may either be constant in time or switched on and off while the neutrons are inside the field region. The detector then measures the intensity of the neutrons reflected by the analyzing supermirror.

The experiment was performed at the external neutron guide laboratory of the Garching research reactor facility. A white thermal beam of neutrons polarized by a supermirror (average degree of polarization $\bar{P}_0\approx 0.9$) and confined to a height of 11 mm and a width of 12 mm by a B_4C diaphragm is periodically pulsed by a small horizontally rotating Fermi chopper at a frequency $\nu=687\pm 2$ Hz. The opening time of the chopper slits is $\Delta t_{\mathrm{CH}}=37$ μs. A time-of-flight measurement of the beam transmitted through the chopper yields a spectral distribution with significant intensity contributions only in a wavelength band between 2 and 6 Å (Fig. 3). The coherence length of the beam was about 3.3 Å [17]. The incident polarization vector $\mathbf{P}_0$ is perpendicular to the beam direction and defines the $+\hat{z}$ axis. After a flight path of several centimeters the neutrons enter a magnetic coil (length $L=56$ cm, width 5 cm) which is connected to a power amplifier and serves to establish a homogeneous time-dependent magnetic field $\mathbf{B}(t)$ of variable strength along the $+\hat{x}$ direction. The rise and fall times of this field, which can rapidly be switched on and off at presettable times controlled by a quartz-stabilized digital timer, are of the order of $\Delta t_{\mathrm{SW}}\approx 10$ μs. The maximum field amplitude is $B_{\max}\approx 3.5$ mT; the reverse stray field at the entrance and exit face of this field coil is found to be negligibly small. Behind the coil and in front of the neu-

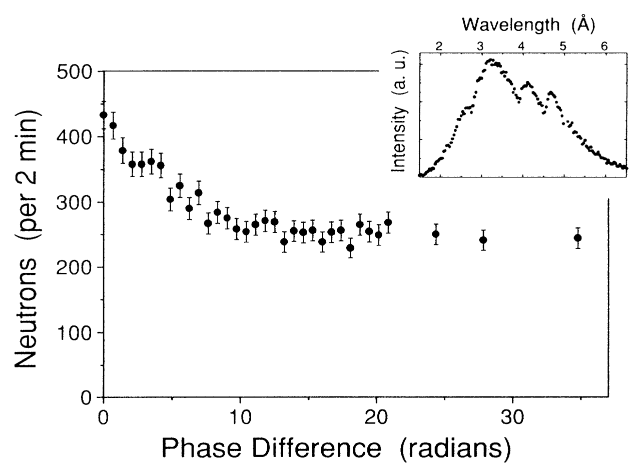

FIG. 3. Neutrons counted after the second supermirror while a static magnetic field is on in the region between polarizer and analyzer. The horizontal axis displays the phase difference between the spin eigenstates as introduced by the magnetic field. The intensity decreases rapidly to a median value because the phase difference is accompanied by a relative displacement of the two eigenstates which exceeds the coherence length. The inset shows the neutron spectrum measured after transmission through the whole apparatus; the modulation is caused by the neutron guide tube leading to the experiment.

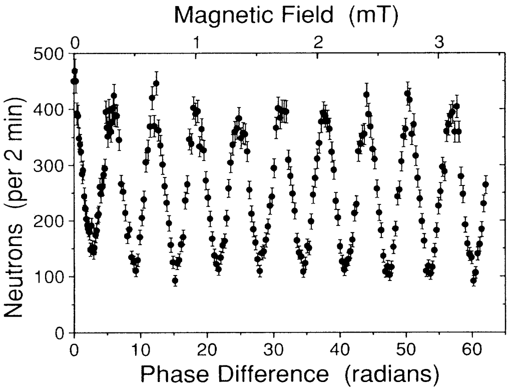

FIG. 4. As in Fig. 3 but now with the magnetic field switched on and off while the neutrons are inside the magnetic field region. The oscillations observed signify the fact that the relative phase shift between the two spin eigenstates in this case is not accompanied by a displacement of the wave packets. Note that the horizontal axis covers phase differences about twice as large as those of Fig. 3.

tron detector a second supermirror analyzes the $|+\hat{z}\rangle$ spin state of the transmitted beam. The intensity is recorded by a PC-based multiscalar synchronously triggered with the chopped neutron bursts by a proper reference signal derived from an optical pickup mounted at the chopper rotor.

At first, a static magnetic field was applied. Figure 3 shows the result of such a measurement. No intensity oscillations behind the analyzer can be observed as the magnetic field is increased due to the immediate loss of polarization. This result follows from the fact that even for a field of only 0.06 mT (which would be required in our setup to achieve a phase difference of 2π between the two spin eigenstates, i.e., one full spin precision for the mean wavelength of 3.7 Å of the incident beam) the relative longitudinal displacement s of the wave packets of the two spin states is about 3.7 Å, and already larger than the coherence length of 3.3 Å.

However, the situation changes dramatically if the switching times are chosen appropriately according to the requirements of the force-free SAB situation as described above. For the given values of the length of the flight path and the spectral distribution of the beam it follows that the optimum turn-on time interval of the field is $\Delta t_F = 100\ \mu$s. At the maximum field of 3.5 mT this corresponds to a phase difference between the two spin eigenstates of about 60 rad. If the phase shift would be due to a spatial shift of the wave packets, the relative displacement of the two spin eigenstates would be about ten mean wavelengths, i.e., 40 Å, which exceeds the coherence length by more than an order of magnitude. Figure 4 shows quite impressively that in fact no damping of the observed intensity oscillations occurs with this experimental arrangement. The slight distortion of the oscillations at low fields can definitely be attributed to the influence of the earth's magnetic field which causes a deviation of the net magnetic field from the $\hat{x}$ direction [18]. The experiment demonstrates unambiguously that there the phase shift caused by the magnetic field is not accompanied by a reduction of contrast and hence there is no limit on the number of observable intensity oscillations whatever the spectral width of the incident beam may be, provided one has unlimited power resources to produce sufficiently large and adequately pulsed magnetic fields.

The neutron polarization interference experiments reported here demonstrate for the first time the nondispersivity of the Aharonov-Bohm effect. Whereas the coherence length would have implied an immediate decrease of visibility, it was this nondispersivity which made the observation of phase shifts of up to 60 rad possible without loss of contrast.

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applicable magnetic fields) did not allow to show that the observed effect is really topological. In fact, here, too, the effect is so small that it is doubtful if it will easily be possible to achieve phase shifts exceeding the coherent length.

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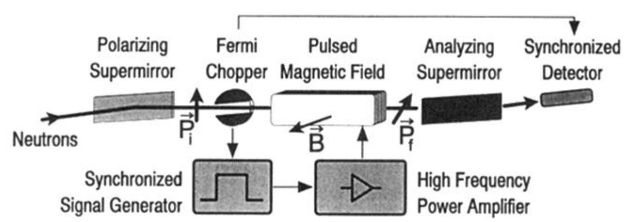

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