The Ratio $(f_{B_s}/f_B)/(f_{D_s}/f_D)$ and Its Implications for B-$\bar{B}$ Mixing

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We observe that quantities like $(f_{B_s}/f_B)/(f_{D_s}/f_D)$ are predicted to be unity *both* by heavy quark *and* by light quark flavor symmetries. Hence, the deviation from the symmetry prediction must be simultaneously small in both symmetry breaking parameters, i.e., order of the ratio of light to heavy quark masses. We estimate the size of the correction. We observe that the ratio of $(\Delta M/\Gamma)$ for B_s-$\bar{B}_s$ to B-$\bar{B}$ mixing can be expressed in terms of the measurable ratio f_{D_s}/f_D with good precision. We comment on applications of these ideas to other processes.

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Heavy quark symmetries [1] have become an important tool in the computation of decay widths and cross sections involving heavy mesons and baryons (for reviews, see Ref. [2]). The limitation of the method stems from the charm and bottom quark masses, m_c and m_b, being only slightly heavier than the typical hadronic scale. Depending on the particular quantity under investigation, the corrections to heavy quark symmetry predictions can be more or less sizable. For example, there are many indications that corrections to the scaling law for the pseudoscalar meson decay constant, $f_D \sim 1/\sqrt{M_D}$, are large, of order of 50% [3]. And deviations from the predicted value of the form factors for semileptonic $B \to D$ and $B \to D^*$, and $\Lambda_b \to \Lambda_c$ transitions at zero recoil, are expected to be small [4]. It is of utmost importance to determine which quantities are expected to be afflicted by large deviations from the symmetry limit, and which are not.

In this Letter we introduce a class of relations between observables that arise as a result of heavy quark symmetry and separately as a result of chiral symmetry of the light quarks. Therefore, deviations from these relations must be simultaneously small in the light quark masses and in the inverse of the heavy quark masses, $1/m_Q$. That is, corrections are order m_s/m_Q. This is to be contrasted with the size of corrections to predictions from chiral symmetry alone, m_s/Λ, or from heavy quark symmetries alone, Λ/m_Q, with Λ a typical hadronic scale.

To be specific consider the double ratio

$$R_1 = \frac{f_{B_s}/f_B}{f_{D_s}/f_D} \; . \tag{1}$$

Light quark flavor symmetries predict the ratio in the numerator and the one in the denominator to deviate from unity by a quantity of order m_s/Λ. Heavy quark flavor symmetries relate the correction itself. It can be expanded in powers of Λ/m_Q, thus: $\frac{m_s}{\Lambda}(a_0+a_1\frac{\Lambda}{m_Q}+\cdots)$, for some constants a_i. The ratio (1) is therefore

$$R_1 = 1 + a_1\left(\frac{m_s}{m_b} - \frac{m_s}{m_c}\right) + \cdots \; . \tag{2}$$

If we set, conservatively, $a_1 \approx 1$, and $m_s = 150$ MeV, $m_c = 1.5$ GeV, and $m_b = 4.5$ GeV, then the correction is only $R_1 - 1 \approx 7\%$.

Before we attempt a better estimate of the correction to $R_1 - 1$, let us see how this knowledge can be of use. Of considerable interest is the ratio

$$R_2 = \frac{(\Delta M/\Gamma)_{B_s}}{(\Delta M/\Gamma)_B} \; , \tag{3}$$

which is a measurement of the relative strengths of B_s-$\bar{B}_s$ and B-$\bar{B}$ mixing. In the standard model many uncertain factors, such as top quark mass dependence, drop out in the ratio:

$$R_2 = \frac{\tau_{B_s} M_{B_s}}{\tau_B M_B} \left|\frac{V_{ts}}{V_{td}}\right|^2 \left(\frac{f_{B_s}}{f_B}\right)^2 \frac{B_{B_s}}{B_B} \; . \tag{4}$$

To extract the fundamental ratio $|V_{ts}/V_{td}|$ from an experimental determination of R_2, one needs knowledge of f_{B_s}/f_B and the ratio of the mixing parameters B_{B_s}/B_B. Both quantities have been studied recently [5,6] using a phenomenological Lagrangian incorporating heavy quark and chiral symmetries [7]. The ratio B_{B_s}/B_B is 1 with small corrections, but the corrections to $(f_{B_s}/f_B)^2-1$ are sizable and cast a doubt on the reliability of this computation. Clearly, a better approach is to use for f_{B_s}/f_B the (soon to be) measured ratio f_{D_s}/f_D. Therefore, to good approximation

$$R_2 = \frac{\tau_{B_s} M_{B_s}}{\tau_B M_B} \left|\frac{V_{ts}}{V_{td}}\right|^2 \left(\frac{f_{D_s}}{f_D}\right)^2 \; . \tag{5}$$

As an attempt to better estimate the corrections to R_1-1 we use the phenomenological Lagrangian for heavy mesons and light pseudoscalars that incorporates heavy and chiral symmetries. We use the Lagrangian and notation of Ref. [5]. There are terms of order m_q/m_Q in the Lagrangian and in the operator that represents the axial current $\bar{q}\gamma^\mu\gamma_5 Q$ in terms of effective fields. Generally, these involve unknown coefficients arising from strong interactions dynamics. But the coefficients of terms in the Lagrangian responsible for chiral SU(3) symmetry breaking in the mass difference of heavy vector and pseudoscalar mesons can be inferred from experiment. Let

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$\Delta^{(b)} \equiv M_{B^*} - M_B$ and $\Delta_s^{(b)} \equiv M_{B_s^*} - M_{B_s}$ and define $\Delta^{(c)}$ similarly with B replaced by D. A class of computable corrections to $R_1 - 1$, involving nonanalytic dependence on the light quark masses, e.g., $\Delta \ln m_s$, can be obtained by including the effects of the mass shifts Δ in the renormalization of the heavy meson fields. This is identical to the computation of f_{D_s}/f_D of Ref. [5], except that one must now keep track of the dependence on Δ, and neglect terms that cancel in the ratio R_1. It is also similar in spirit to the computation of $(1/m_c)^n$ effects of Ref. [8]. The result of the computation is expressed in terms of the function

$$J(m,x) = (m^2 - 2x^2)\ln(m^2/\mu^2) - 4x^2 F(m/x) \ , \tag{6}$$

where

$$F(y) \equiv \begin{cases} \sqrt{1-y^2}\,\mathrm{arctanh}\sqrt{1-y^2}, & y \le 1 \ , \\ -\sqrt{y^2-1}\arctan\sqrt{y^2-1}, & y \ge 1 \ . \end{cases} \tag{7}$$

The factor of $m^2 \ln m^2$, which cancels in R_1, is retained for comparison with the results of Ref. [5]. It is also useful to introduce the chiral symmetry breaking parameters $\delta^{(b)} = m_{B_s} - m_B$ and $\delta^{(c)} = m_{D_s} - m_D$.

Neglecting isospin breaking one obtains

$$R_1 - 1 \approx \frac{3g^2}{32\pi^2 f^2}\left[\frac{2}{3}J(M_\eta,\Delta_s^{(b)}) + 2J(M_K,\Delta^{(b)}-\delta^{(b)}) - \frac{3}{2}J(M_\pi,\Delta^{(b)}) - \frac{1}{6}J(M_\eta,\Delta^{(b)}) - J(M_K,\Delta_s^{(b)}+\delta^{(b)})\right] - (b \to c) \ . \tag{8}$$

The dependence on the renormalization point μ is cancelled by a counterterm. By choosing $\mu \sim 1$ GeV one avoids formally large logarithms in the counterterm.

To estimate this numerically we take $g^2 = 0.4$, its present experimental upper bound [9], $f = f_K = 170$ MeV, $\Delta^{(b)} = \Delta_s^{(b)} = \frac{1}{3}\Delta^{(c)} = \frac{1}{3}\Delta_s^{(c)} = 50$ MeV, $\delta^{(b)} = \delta^{(c)} = 150$ MeV and the physical pseudoscalar masses. With these, the correction is -3.3%. The deviation of each ratio, f_{D_s}/f_D and f_{B_s}/f_B, from unity is itself a factor of 4 larger.

We conclude by pointing out that there are many other symmetry predictions of interest which are corrected at order m_q/m_Q only. An important example is the ratio of ratios of form factors for semileptonic (or rare) B and D decays into light pseudoscalars (π and K) or vector (ρ and K^*) mesons. To be more specific, let us parametrize the hadronic matrix elements in terms of the Lorentz invariant form factors

$$\langle K(p_k)|\bar b\gamma^\mu s|B(p_B)\rangle = f_+^{(B\to K)}(p_B+p_K)^\mu + f_-^{(B\to K)}(p_B-p_K)^\mu \ , \tag{9}$$

$$\langle K(p_K)|\bar b\sigma^{\mu\nu} s|B(p_B)\rangle = -2ih^{(B\to K)}[p_B^\mu p_K^\nu - p_B^\nu p_K^\mu] \ ,$$

with analogous definitions for $f_\pm^{(B\to\pi)}$ and $h^{(B\to\pi)}$ and for $D \to K$ and $D \to \pi$ matrix elements. Writing the form factors as functions of $v\cdot p_K = p_B\cdot p_K/m_B = p_D\cdot p_K/m_D$, one has the heavy quark flavor symmetry relations

$$f_+^{(B\to K)}/f_+^{(D\to K)} = \sqrt{m_b/m_c} \ ,$$
$$f_+^{(B\to \pi)}/f_+^{(D\to \pi)} = \sqrt{m_b/m_c} \ . \tag{10}$$

This gives

$$\frac{f_+^{(B\to K)}/f_+^{(B\to\pi)}}{f_+^{(D\to K)}/f_+^{(D\to\pi)}} = 1 \ . \tag{11}$$

The same relation is obtained by an application of chiral symmetry to the numerator and denominator separately. Hence, the double ratio deviates from unity in order m_s/m_c. Analogous relations can be written for f_- and h.

Both form factors $f_+^{(D\to K)}$ and $f_+^{(D\to\pi)}$ are experimentally accessible. Computation of the rate for the rare decay $B \to K\mu^+\mu^-$ requires knowledge of the form factors $f_+^{(B\to K)}$ and $h^{(B\to K)}$, while the rate for $B \to \pi e\nu$ is expressed in terms of $f_+^{(B\to\pi)}$. A precise determination of the rate for $B \to K\mu^+\mu^-$ therefore requires measurement of $B \to \pi e\nu$ and an application of heavy quark spin symmetries, which imply

$$2m_Q h = -f_- = f_+ \ . \tag{12}$$

Note that one needs to invoke the use of heavy quark spin symmetry only for B-meson form factor relations, and not for the lighter D meson.

The chiral symmetry prediction of the ratio of form factors [10] for $B \to K$ and $B \to \pi$ is afflicted by rather large corrections [11]. As we have seen, the predictions of the ratio of ratios is expected to hold to much better accuracy. Therefore, a good measurement of $D \to Ke\nu$ and $D \to \pi e\nu$ form factors could greatly aid in the interpretation of a measurement of $B \to K\mu^+\mu^-$ and $B \to \pi e\nu$.

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