

Scaling Study of Pure Gauge Lattice QCD by Monte Carlo Renormalization Group Method

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The scaling behavior of pure gauge SU(3) in the region $\beta=5.85-7.60$ is examined by a Monte Carlo renormalization group analysis. The coupling shifts induced by factor 2 blocking are measured on both 32^4 and 16^4 lattices with high statistics. A systematic deviation from naive 2-loop scaling is clearly seen. The mean field and effective coupling constant schemes explain part, but not all, of the deviation. It can be accounted for by a suitable change of coupling constant, including a correction term $O(g^7)$ in the 2-loop lattice β function. Based on this improvement, $\sqrt{\sigma}/\Lambda_{\overline{\text{MS}}}^{n_f=0}$ is estimated to be 2.2 ± 0.1 from the analysis of the string tension σ .

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Since a confirmation of the approach to the continuum limit has basic importance in lattice quantum chromodynamics (lattice QCD), systematic scaling analyses at larger β are inevitably required. Recent analyses have shown that scaling violations persist in physical quantities such as the string tension and hadron masses up to $\beta=6.8$ [1]. On this problem, it has been argued that a suitably chosen coupling constant reveals perturbative scaling [2-6].

Monte Carlo renormalization group (MCRG) analysis tells us the coupling shift $\Delta\beta$ induced by scale transformation of the lattice spacing, $a \rightarrow sa$ [7]:

$$s = \exp \left[\int_{g(\beta)}^{g(\beta-\Delta\beta)} \frac{dg}{\beta_f(g)} \right] = \frac{f(1/g(\beta-\Delta\beta)^2)}{f(1/g(\beta)^2)}, \quad (1)$$

where $f(1/g^2) = a\Lambda$ and $\beta_f(g)$ is the lattice β function. (For bare lattice coupling constant, $1/g^2 = \beta/6$). This gives us another way to examine the scaling behavior of lattice QCD. In SU(3) lattice gauge systems, MCRG has been performed by several groups on 16^4 lattices in the large β region up to 7.2 [8-12]. However, these results were inconclusive and even controversial. Although Gupta *et al.* [11] have claimed consistency with asymptotic scaling from their $\sqrt{3}$ blocking result, Bowler *et al.* [9,10] have found sizable deviations from 2-loop scaling. Recently Hoek has reanalyzed the same data and claimed

a very slow approach to scaling [12]. These varied conclusions are mainly due to the difficulty of obtaining a precise value for the coupling shift in the high β region, because the deconfining transition prevents an accurate matching between Wilson loops on a small lattice. Studies on larger lattices with better statistics are required to clarify the scaling behavior in the high β region.

In this work, we report results of an MCRG study and scaling analysis in the high β region on both 32^4 and 16^4 lattices with high statistics.

Using a 32^4 lattice gives us the following advantages.

(i) One more blocking level than previous works. It is not *a priori* clear that blocking from 16^4 to 2^4 is sufficient for loop matching. A deeper blocking is preferred to confirm good matching and for closer matching conditions.

(ii) We remain in the confinement phase up to $\beta \sim 6.9$ whereas a 16^4 lattice is above the deconfining transition point for $\beta > 6.35$.

The numerical simulation has been performed on the 512-cell parallel processor Fujitsu AP1000 [13]. Three lattices of size 32^4 , 16^4 , and 8^4 are generated by the overrelaxed pseudo heatbath algorithm [14] (the mixing ratio of overrelaxation to pseudo heat bath is 9:1 in average). Blocking is performed every ten updates. 2-3 K configurations are blocked at each β (more near the deconfining transition point). For error estimation, the

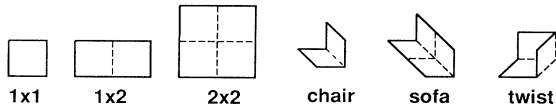


FIG. 1. Wilson loops used in lattice matching.

jackknife method is applied. We monitor autocorrelations of blocked Wilson loops in those measurements to know the statistical validity of sampling (see Fig. 3). Near the deconfining transition point, two more configurations are blocked. The effect of long autocorrelation there is taken into account in the error estimation. Details of the prescription will be presented elsewhere.

In this work, the Swendsen blocking transformation is used to double the lattice spacing ($s=2$) [15]. Blocking is repeatedly performed down to 2^4 for two lattices, one of size L (at β) and the other of size $L/2$ (at $\beta - \Delta\beta$). To match long range physical contents on both lattices, a set of Wilson loops on one blocked lattice is compared with the corresponding one on the other blocked lattice. For early matching between blocking trajectories, the blocking transformation is controlled by a parameter q which governs the size of Gaussian fluctuations around the maximal SU(3) projection of the block link variable [16]. The coupling shift $\Delta\beta$ is determined at the value of q where the mismatch of the two sets of Wilson loops is minimum. Planar 1×1 and 1×2 Wilson loops and non-planar 6-link ("twist" and "chair") and 8-link ("sofa") loops are measured on blocked lattices at $q=0.0, 0.02, 0.04$ for this purpose (see Fig. 1). A brief description is found in our previous report [13].

$\Delta\beta$ at $\beta=5.85-7.60$, measured by matching 16^4 and 8^4 lattices after three blocking steps, is shown by open circles in Fig. 2(a). Improved statistics give a clear systematic behavior for the new data in comparison with those of previous works [Fig. 2(b)] although they are consistent. The data are significantly below the 2-loop scaling result (solid curve). Therefore naive 2-loop scaling does not hold in this region. The deviation rapidly decreases as β increases. But, even at $\beta=7.60$, 10% deviation remains.

A notable feature of the present data is the following. For $\beta < 6.3$, the quality of the data is sufficient and shows the approach to the 2-loop scaling result. On the other hand, the data above $\beta=6.3$ suffer relatively large errors. The matching of blocked Wilson loops becomes difficult in this region. This difficulty is caused by the deconfining transition on the 16^4 lattice [17]. As shown in Fig. 3, the autocorrelation time of blocked Wilson loops sharply peaks at $\beta=6.35 \pm 0.05$ (on an 8^4 lattice, $\beta=5.90$). An increase of fluctuations of the blocked Wilson loops is also evident at this point. Thus, the 16^4 lattice turns into the deconfining region at this point. It is noted that the value $\beta=6.35$ is smaller than the previously expected value 6.45 ± 0.05 from an analysis of the finite temperature phase transition [18]. Above this β , Wilson loops are dominated by perturbative contributions and the match-

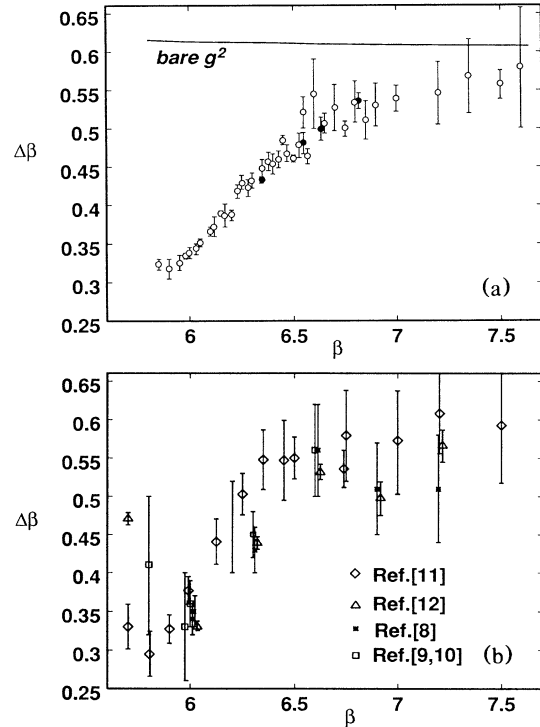


FIG. 2. Coupling shift $\Delta\beta$ measured in this work (a) and that in previous works (b). Solid curve shows 2-loop asymptotic scaling with bare coupling constant.

ing suffers large errors.

Matching between 32^4 and 16^4 has been tried at $\beta=6.35-7.00$. The deconfining transition point on a 32^4 lattice is pushed up to around $\beta=6.9$ since the coupling shift is ≈ 0.54 in this region and the deconfining transition point is $\beta=6.35$ on a 16^4 lattice. At $\beta=6.35, 6.55, 6.65,$ and 6.80 , where the lattice is in the confinement phase, the matching can be performed successfully and the resultant coupling shifts after three and four blockings agree within error as shown in Fig. 4. Thus, the data become stable for blockings greater than three and this fact gives reliability to the 16^4 lattice (three blockings) measurements.

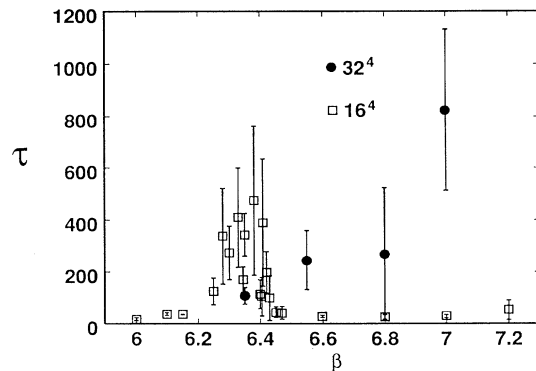


FIG. 3. Autocorrelation time τ on 16^4 and 32^4 lattices.

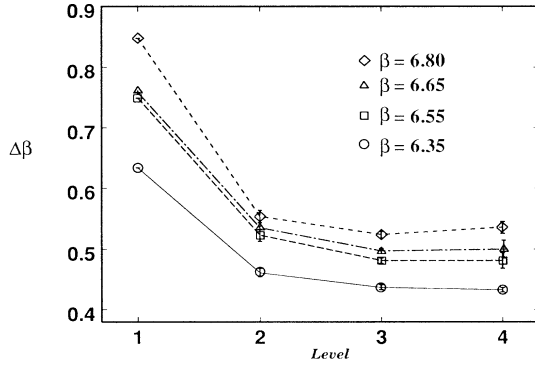


FIG. 4. Coupling shift $\Delta\beta$ on 32^4 lattice in different blocking level.

The measured coupling shifts [black circles in Fig. 2(a)] are naturally connected with those of $\beta < 6.35$ on the 16^4 lattice. Thus, a systematic deviation from naive 2-loop scaling in this region is further confirmed by these measurements.

At $\beta = 7.00$ where the 32^4 lattice is above the deconfining transition point, definite data could not be extracted from our 3 K blocked configurations (30 K sweeps) due to very long range fluctuations.

Although a trend to approach the 2-loop scaling value is seen, the coupling shift shows significant deviation. This deviation can be partly absorbed by a mean field scheme [2,3] or an effective coupling constant scheme [4-6]. In the mean field scheme, the modified-minimal-subtraction (MS) coupling constant is given by $1/g_{\overline{MS}}(\pi/a)^2 = \langle U_{\text{plaq}} \rangle / g^2 + 0.025$ where $\langle U_{\text{plaq}} \rangle$ is the average plaquette. Similarly the effective coupling constant is defined as $g_e^2 = 3(1 - \langle U_{\text{plaq}} \rangle)$. Both coupling constants are obtained here based on our measurement of $\langle U_{\text{plaq}} \rangle$ at $\beta = 5.70-7.60$. The coupling shift is calculated assuming the 2-loop scaling form for $g_{\overline{MS}}$ [19],

$$f_2(x) = \pi \left(\frac{x + (b_1/b_0)}{b_0} \right)^{b_1/2b_0^2} \exp \left(-\frac{x}{2b_0} \right), \quad (2)$$

$$x = 1/g_{\overline{MS}}^2,$$

where $b_0 = 33/48\pi^2$ and $b_1 = (102/121)b_0^2$. As shown by curve *M* in Fig. 5, this scheme partly explains the present MCRG data. Similarly, the effective coupling constant g_e improves the agreement somewhat, as shown by curve *E* in Fig. 5.

However, a sizable deviation still persists between these schemes and the present data. Therefore we need a different prescription to approach the continuum limit from the presently accessible region of β . In Refs. [2,3], to get the continuum limit, a linear or bilinear extrapolation in terms of the lattice constant a was assumed for physical quantities expressed in units of $\Lambda_{\overline{MS}}$ while the authors of Ref. [6] used a linear extrapolation in $1/\ln a$. Here instead, the continuum limit is extracted by taking into account the lattice β function and expressing our

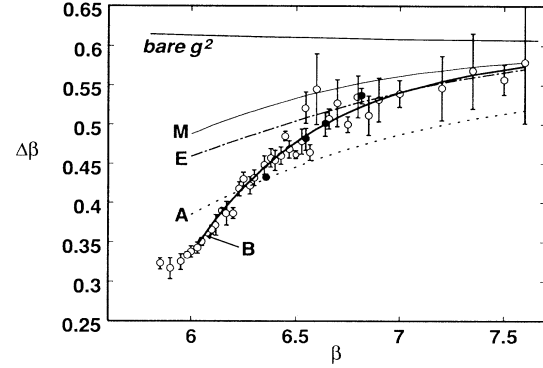


FIG. 5. Coupling shift reproduced by different coupling constant schemes. Notations are described in the text.

MCRG data in terms of an effective coupling constant. We fit the data by the lattice β function including a next-order correction, and define our effective coupling $1/g_u^2$ by a free shift of $1/g_{\overline{MS}}^2$ as follows:

$$-\frac{dx_u}{2d \ln a} = b_0 + \frac{b_1}{x_u} + \frac{b'}{x_u^2}, \quad (3a)$$

with

$$x_u \equiv \frac{1}{g_u^2} = \frac{1}{g_{\overline{MS}}^2} - x_0. \quad (4)$$

Assuming the correction term b' is small, we actually use the following equation instead of Eq. (3a):

$$-\frac{dx_0}{2b_0} \left[\frac{1}{(1 + b_1/b_0 x_u)} - \frac{b'}{b_0 x_u^2} \right] = d \ln a. \quad (3b)$$

The solution of Eq. (3b) is

$$f(x_u) = f_2(x_u) \exp \left(\frac{-b'}{2b_0^2 x_u} \right). \quad (5)$$

Using Eqs. (4) and (5), we have attempted two fits for the data above $\beta = 6.00$: (A) Restrict $x_u = 1/g_{\overline{MS}}^2$ (i.e., $x_0 = 0$) with b' as a free parameter; (B) allow a shift of the effective coupling constant; i.e., both x_0 and b' are free parameters. The results are shown in Fig. 5. In case (A), only a poor fit is obtained. The coefficient b' is also relatively large as $b'/b_0 = -0.0850(0.0013)$ while $b_1/b_0 = 0.0587$. Thus, it is difficult to explain the present data by an additional $O(g_{\overline{MS}}^7)$ term only. On the other hand, in case (B), we can fit the data quite well by values of parameters as (curve *B*)

$$x_0 = 0.442(0.004), \quad b'/b_0 = -0.0119(0.0008). \quad (6)$$

In this case, the coefficient of the $O(g_u^7)$ term is small but the shift parameter x_0 is non-negligible. It is noted that we have also tried fitting in the effective coupling scheme and have gotten similar results. Since the sign of b' is negative, we say that our lattice is coarser than predicted by 2-loop scaling.

Based on this scheme, we can discuss the continuum

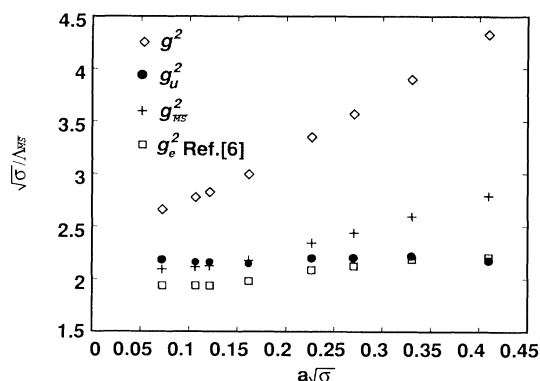


FIG. 6. Scaling of string tension in different coupling constant schemes.

limit of physical quantities in units of $\Lambda_{\overline{\text{MS}}}^{n_f=0}$. In Fig. 6, scaling of the string tension of Refs. [6,20,21] is examined. As expected, good scaling is obtained for $\sqrt{\sigma}/\Lambda_{\overline{\text{MS}}}^{n_f=0}$ for $\beta=5.7-6.8$, as shown by black circles in the figure, and the value is

$$\sqrt{\sigma}/\Lambda_{\overline{\text{MS}}}^{n_f=0} = 2.2(0.1). \quad (7)$$

This value is also interpreted as the continuum limit value in the present scheme. Our result (7) is larger than that of the static quark potential in Ref. [6] (1.80 ± 0.06) and Ref. [22] (1.72 ± 0.13). We note also that $\Lambda_{\overline{\text{MS}}}^{n_f=0} = 200$ MeV for $\sqrt{\sigma} = 440$ MeV. It is slightly smaller than that extracted from the $1p-1s$ splitting of charmonium in Ref. [2] (234 MeV).

In this work, the scaling behavior of SU(3) lattice theory in the interval $\beta=5.85-7.60$ is studied by MCRG analysis. A significant deviation from naive 2-loop scaling is seen. This is clearly observed in the data by measurements kept in the confinement phase up to $\beta=6.80$. Thus naive 2-loop scaling does not hold in this region. It is shown that a large part of the deviation is accounted for by the mean field and/or the effective coupling constant schemes. We show further that the deviation can be absorbed by a next-order correction to the 2-loop β function, together with a shifted mean field coupling constant. The coefficient of the next-order term, b' , is consistently small. Based on the scaling behavior of this lattice β function, the string tension remains constant in the region $\beta=5.70-6.80$ where the lattice spacing changes by a factor of 6. The estimated $\Lambda_{\overline{\text{MS}}}^{n_f=0}$ in the continuum limit is $0.46(0.03)\sqrt{\sigma}$. Though there is still a 20% discrepancy between different methods [2,6,22], this seems an encouraging confirmation of the approach to the continuum limit. It is also noted that the suggested coupling constant g_u diverges at some value of the bare coupling constant ($\beta \sim 4.5$). An interesting possibility is that this divergence is related with the transition point from the strong to the weak coupling region studied by Bhanot and Creutz in a space of couplings of Wilson action and that of adjoint representation [23].

Finally the significance of MCRG measurements in the 3066

confinement phase is stressed. Above the deconfining transition point, the matching procedure inevitably suffers large errors. Although most of the data measured above the transition are consistent, those in the confinement phase have higher quality and reliability. Study on a larger lattice is apparently preferred. In order to cover the region up to $\beta=7.5$, we need a 64^4 lattice.

The present calculations have been carried out on the parallel computer AP1000 at the Fujitsu Parallel Computing Research Facilities. We would like to acknowledge Dr. M. Ishii and members of the facilities for warm hospitality and valuable advice.

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