Cancellation of Mass Singularities in Thermal Reaction Rates

A. Niégawa
Department of Physics, Osaka City University, Sumiyoshi-ku, Osaka 558, Japan
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Thermal amplitudes exhibit mass singularities when particle(s) with vanishing mass is (are) involved. We show that such singularities do not appear in the perturbation expansions of the thermal reaction rates, provided the theory in consideration does not bring about such mass singularities in zero-temperature reaction rates. The latter is supposed to be the case for physically sensible quantities in a wide class of theories, including QED and QCD (Kinoshita-Lee-Nauenberg theorem).

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Much interest has been taken in gauge theories at high temperature (hot gauge theories), because of their relevance to the early Universe and to the quark-gluon plasma to be produced in heavy-ion collisions. One of the most important findings in recent years in this field is the resummation program developed by Pisarski and Braaten [1,2]. The essential observation is to see that, to one-loop order in the sense of "hard thermal loop" [1,2], there appears two natural scales, i.e., a hard scale $\sim T$ and a soft scale $\sim gT$ with g the gauge coupling constant. To obtain a consistent perturbative expansion, for thermal propagators and (generalized) vertices whose external momenta are soft, $p_\mu \leq O(gT)$, one has to use partially or hard-thermal-loop resummed ones.

In renormalizable quantum field theories with massless particles, we encounter the problem of infrared (IR) and collinear or mass singularities. At zero temperature, it is believed that, in most of the physically relevant theories such as QED and QCD, as far as physically sensible quantities are concerned, IR singularities of Bloch-Nordsiek type and mass singularities disappear when one sums over the set of all degenerate initial and final states [Kinoshita-Lee-Nauenberg (KLN) theorem [3–7]]. At finite temperature, several works have been devoted to this issue [8,9], but the subject is still under study.

In thermal field theories, the resummed soft propagators mentioned above soften or screen the IR singularities appearing in loop integrals. For example, in hot QCD, this softening renders some otherwise divergent physical quantities finite [10]. Some of the other physical quantities (like the damping rate of a moving quark or a gluon) are still IR divergent, but are expected to be rendered finite through resummations of still higher-order loops [1,11]. In this Letter, we shall be concerned with mass singularities, and show that thermal reaction rates are free from them.

We start with an expression of thermal reaction rates [12]. We consider a heat-bath system of temperature T, composed of the fields $\phi^{(\alpha)}$, with α labeling collectively a field type and its internal degrees of freedom. For convenience, we enclose the system inside a large cube with volume $V = L^3$. Employing the periodic boundary conditions, we label the single-particle basis by its momentum $\mathbf{p} = 2\pi\mathbf{i}/L$, $i_j = 0, \pm 1, \pm 2, \ldots, \pm\infty$ $(j = x, y, z)$. At the final stage, we take the limit $V \to \infty$.

Physically interesting thermal reactions are of the following generic type,

$$\{A\} + \text{heat bath} \to \{B\} + \text{anything} . \tag{1}$$

Here $\{A\}$ and $\{B\}$ designate groups of particles, which are not thermalized, such as virtual photons and leptons. (Generalization to more general process, where among $\{A\}$ and/or $\{B\}$ are $\phi^{(\alpha)}$'s, is straightforward [13].) The reaction rate $\mathcal{R}$ of the thermal process (1) is expressed [12] as a statistical average of the transition probability, $W = S^* S$ (with S the S-matrix element), of the *zero-temperature* $(T = 0)$ *process*,

$$\{A\} + \{n_{\mathbf{i}}^{(\alpha)}\} \to \{B\} + \{n_{\mathbf{i}}^{(\alpha)\prime}\} , \tag{2}$$

where $\{n_{\mathbf{i}}^{(\alpha)}\}$ denotes the group of $\phi^{(\alpha)}$'s, which consists of the number $n_{\mathbf{i}}^{(\alpha)}$ of $\phi_{\mathbf{i}}^{(\alpha)}$ ($\phi^{(\alpha)}$ in a mode $\mathbf{i}$):

$$\mathcal{R} = \frac{\overline{\sum_{\{n_{\mathbf{i}}^{(\alpha)}\}}} \rho \overline{\sum_{\{n_{\mathbf{i}}^{(\alpha)\prime}\}}} W(\text{process (2)})/2\pi\delta(0)}{\overline{\sum_{\{n_{\mathbf{i}}^{(\alpha)}\}}} \rho \overline{\sum_{\{n_{\mathbf{i}}^{(\alpha)\prime}\}}} W_0(\{n_{\mathbf{i}}^{(\alpha)}\} \to \{n_{\mathbf{i}}^{(\alpha)\prime}\})} , \tag{3}$$

$$\rho = N^{-1} \exp\left(-\beta \sum_{\alpha} \sum_{\mathbf{i}} n_{\mathbf{i}}^{(\alpha)} E_{\mathbf{i}}^{(\alpha)}\right) . \tag{4}$$

Here $2\pi\delta(0) = t_f - t_i$ $(= \infty)$ is the time interval during which a "measurement" is made. $W_0 = S_0^* S_0$ is the $T = 0$ transition probability of the process indicated, $E_{\mathbf{i}}^{(\alpha)}$ is the energy of $\phi_{\mathbf{i}}^{(\alpha)}$, and N is the normalization factor. In (3) $\overline{\sum}$ stands for the summation with symmetry factors being respected, and, for a bosonic (fermionic) $\phi^{(\alpha)}$, $n_{\mathbf{i}}^{(\alpha)}$ runs over $0, 1, 2, \ldots, \infty$ (0 and 1).

The $T = 0$ S-matrix element is obtained through an application of the reduction formula. As an illustration, we take a heat bath of thermal neutral scalars ϕ, and we take $\{A\}$ to be $\{\Phi(\mathbf{p}_j)\,;\ j = 1, \ldots, m\}$ and $\{B\}$ to be $\{\Phi(\mathbf{q}_k)\,;\ k = 1, \ldots, n\}$ with Φ a nonthermalized heavy neutral scalar. Assuming $\Phi - \phi$ coupling to be of the form $\Phi\phi^l$, we have [12]

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$$S=\prod_{\mathbf{i}}\Bigg[\sum_{j_\mathbf{i}(j_\mathbf{i}')=0}^{n_\mathbf{i}(n_\mathbf{i}')}\delta(n_\mathbf{i}-j_\mathbf{i}\,;\,n_\mathbf{i}'-j_\mathbf{i}')\left\{\binom{n_\mathbf{i}'}{j_\mathbf{i}'}\binom{n_\mathbf{i}}{j_\mathbf{i}}\frac{1}{j_\mathbf{i}'!\,j_\mathbf{i}!}\right\}^{1/2}\prod_{n'=1}^{j_\mathbf{i}'}\left(iK^*_{\mathbf{i},n'}\right)\prod_{n=1}^{j_\mathbf{i}}\left(iK_{\mathbf{i},n}\right)\Bigg]$$
$$\times\prod_{j=1}^{m}\left(iK_{p_j,\Phi_j}\right)\prod_{k=1}^{n}\left(iK^*_{q_k,\Phi_k}\right)\langle 0\mid T\Bigg[\prod_{n'=1}^{j_\mathbf{i}'}\phi_{n'}\prod_{n=1}^{j_\mathbf{i}}\phi_n\prod_{j=1}^{m}\Phi_j\prod_{k=1}^{n}\Phi_k\Bigg]\mid 0\rangle\,, \tag{5}$$

where $\delta(\cdots;\cdots)$ denotes the Kronecker's δ symbol,

$$K_{\mathbf{i},n}\cdots\phi_n\equiv\frac{1}{\sqrt{2E_\mathbf{i}VZ_\phi}}\int d^4x\,e^{-ip_\mathbf{i}\cdot x}(\Box+m^2)\cdots\phi(x)\,,$$

and likewise for $K_{p_j,\Phi_j}\cdots\Phi_j$. S_0 is given by a similar expression to (5), where factors related to the Φ fields are deleted. In (5), among $n_\mathbf{i}$ ($n_\mathbf{i}'$) of $\phi_\mathbf{i}$'s in the initial (final) state, $j_\mathbf{i}$ ($j_\mathbf{i}'$) of $\phi_\mathbf{i}$'s participate *directly* in the reaction(s). Remaining $n_\mathbf{i}-j_\mathbf{i}$ of $\phi_\mathbf{i}$'s are merely spectators, which reflects only on the statistical factor in $\mathcal{R}$ in (6) below.

Let us turn back to the general formula (3) and (4). The thermal reaction rate $\mathcal{R}$ can be reduced to a "forward" thermal amplitude evaluated in the framework of the real-time thermal field theory based on the time path, $-\infty\to+\infty\to-\infty\to-\infty-i\beta$, in a complex time plane (thermal optical theorem). It is shown [12] that, when the number density of each incident particle in $\{A\}$ [Eq. (1)] is normalized to $2E$ (E being its energy), (3) goes to

$$\frac{d\mathcal{R}}{dV}\Big/\prod_{k=1}^{n}\left(\frac{d\mathbf{q}_k}{(2\pi)^3 2E_k}\right)=\mathcal{A}\,, \tag{6}$$

in the limit $V\to\infty$. Here $\mathbf{q}_k$ (E_k), $k=1,2,\ldots,n$, is the momentum (energy) of the kth particle in $\{B\}$ [Eq. (1)]. $\mathcal{A}$ stands for the connected real-time thermal amplitude, summed over spins and internal degrees of freedom of particles involved, for the "forward process," $\{A_1\}+\{B_2\}\to\{A_2\}+\{B_1\}$, where $\{A_1\}$ ($\{A_2\}$) signifies that all the fields in $\{A\}$ are the physical (thermal-ghost) fields, and likewise for $\{B\}$. [The Dirac spinors are normalized such that the spin sum of $u(p)\bar{u}(p)$ and $v(p)\bar{v}(p)$ give $\not{p}+m$ and $\not{p}-m$, respectively.] Inclusion of the contributions from disconnected $\mathcal{A}$'s is straightforward.

In reducing (3) to (6), one expands the numerator, $\mathcal{N}$, and the denominator, $\mathcal{D}$, of (3) in powers of coupling constant g and then gathers the same-order terms. The following type of contribution coming from $\mathcal{N}$, say $g^N\mathcal{N}_N$, involves infinity; a heat-bath particle $\phi_\mathbf{i}^{(\alpha)}$ participates in the reaction with $\{A\}$ and $\{B\}$, and "another" $\phi_\mathbf{i}^{(\alpha)}$ in the same mode $\mathbf{i}$ undergoes graphically disconnected reaction with other $\phi^{(\beta)}$'s. This type of infinity corresponds to an infinite lifetime of $\phi_\mathbf{i}^{(\alpha)}$ that is on the mass shell—thus, as it were, mass (-shell) singularity. In such case, one can always find the corresponding diverging term in $\mathcal{D}$, say $g^M\mathcal{D}_M$, which originates from the fact that the thermal reaction probability among $\phi^{(\alpha)}$'s are proportional to the time interval t_f-t_i ($=\infty$) of "measurement." When this term, $g^M\mathcal{D}_M$, is combined with the relevant lower-order contribution in $\mathcal{N}$, $g^{N-M}\mathcal{N}_{N-M}$, and added to $g^N\mathcal{N}_N$ above, there emerges [12] the "correct" form of thermal propagator with thermal self-energy correction, which takes a seat in the thermal amplitude $\mathcal{A}$ in (6). That $\mathcal{A}$ is free [14] from ill-defined singularities like $\{\delta(p^2-m^2)\}^n$ ($n\geq 2$) guarantees the absence of mass (-shell) singularities of this type. Thus, in the sequel, we will not be concerned about them. It is to be noted that these types of singularities are absent not only in reaction rates but also in thermal amplitudes. On the contrary, the *genuine* mass singularities discussed below are absent only in reaction rates.

Armed with the above machinery, it is rather straightforward to show that the reaction rate $\mathcal{R}$ is free from mass singularities. Let us rewrite (3) as

$$\mathcal{R}=\frac{\sum_E\left(N^{-1}e^{-\beta E}\right)\Gamma(E)/2\pi\delta(0)}{\sum_E\left(N^{-1}e^{-\beta E}\right)\Gamma_0(E)}\,, \tag{7}$$

$$\Gamma(E)=\overline{\sum_{\{n_\mathbf{i}^{(\alpha)}\}}}\delta(E\,;\,\sum_\alpha\sum_\mathbf{i}n_\mathbf{i}^{(\alpha)}E_\mathbf{i}^{(\alpha)})\overline{\sum_{\{n_\mathbf{i}^{(\alpha)\prime}\}}}W\,. \tag{8}$$

$\Gamma_0(E)$ in (7) is given by an expression similar to (8). Γ and Γ_0 are the transition probabilities of the reactions in consideration, with the initial energy of $\phi^{(\alpha)}$'s being E. An important observation here is that $\Gamma(E)$ and $\Gamma_0(E)$ are to be evaluated *in vacuum field theory* [cf. (5)]. Since all degenerate sets in the *initial* and *final* states are summed up in (8), we can apply the KLN theorem [3,4]; $\Gamma(E)$ and $\Gamma_0(E)$ are free from mass singularities in each order of perturbation expansion in a wide class of theories.

Now we will see in some detail how the argument by KLN [3–7] goes, referring to $\Gamma(E)$ in (8). Because all the degenerate sets are summed up in (8) allows us to introduce a set of double-cut diagrams [3,7], through which $\Gamma(E)$ is evaluated. A double-cut diagram is a no-leg diagram and has two kinds of cut lines, the initial-state cut lines and the final-state cut lines, which divide the diagram into two parts; the one corresponding to S and the other to S^*. We divide the set of double-cut diagrams into subsets: All diagrams belonging to a subset have the same topology if two kinds of cut lines are removed. They differ in that the way of cuttings are different, corresponding to different processes. (An example is given below.) IR and mass singularities in $\Gamma(E)$ can come about when subsets of (internal) lines in a double-cut diagram go on-shell. We name any point in momentum space where this is the case a "singular point." At this stage, we restrict ourselves to examining mass singularities, and then ignore throughout the soft modes, $E_{\mathbf{i}}^{(\alpha)} < \lambda \sim O(gT)$. Mass singularity may appear [6,7] only if momentum contour integral (in the $V \to \infty$ limit form) is trapped at a singular point. Each double-cut diagram belonging to a subset develops, in general, mass singularities in the vicinities of some singular points. The KLN theorem states that if we add all contributions coming from all the diagrams in the subset, the cancellation of mass singularities takes place in the vicinity of the singular point from which they come about. It should be emphasized that the cancellation occurs in the vicinity of the singular point, and is not affected by cutting off the IR region. The situation is exactly the same for $\Gamma_0(E)$.

The KLN theorem is believed to hold for physically sensible quantities in a wide class of theories that include QED and QCD. Thus we come to the conclusion: In the above class of theories, *the thermal reaction rate* $\mathcal{R}$ *in* (7) *is automatically free from mass singularities.*

When some or all of the particles in $\{A\}$ and/or $\{B\}$ in (1) are massless (thermalized) particles, care should be taken. For example, if the particle with $\mathbf{q}_1$ in $\{B\}$ is massless φ—then necessarily thermalized—the inclusive reaction rate (7) or (6) has mass singularity. However, such a massless particle cannot be "measured" experimentally, since the single $\varphi(\mathbf{q}_1)$ state and states containing a massless φ plus a number of parallel-moving massless particles degenerate in energy. According to the KLN theorem, the inclusive reaction rate (7) is free from mass singularity provided that, in any given order of perturbation series, the contributions from all such degenerate states [including the single $\varphi(\mathbf{q}_1)$ state)] are summed up. Employing the double-cut diagrams, one can easily identify the *relevant set of degenerate states.* As an example, consider the Drell-Yan ($\mu^-\mu^+$-pair production) process in massless QCD at zero temperature. Examples of the relevant double-cut diagrams are depicted in Fig. 1, where, solid (dotted) cut lines represent initial (final) state cuts, and directed, dashed, and wavy lines are massless-quarks, gluons, and virtual photons (γ^*), respectively. Figure 1(a) represents $q(p) + \bar{q} \to g(q) + \gamma^*$, while Fig. 1(b) represents $q(p-q) + g(q) + \bar{q} \to g(p) + \gamma^*$. Figure 1(b) includes a spectator gluon in S^* or in S. In the Coulomb gauge, each one of Fig. 1 develops mass singularity at the singular point, $\mathbf{p} \parallel \mathbf{q}$ ($|\mathbf{p}|, |\mathbf{q}| > \lambda$), but

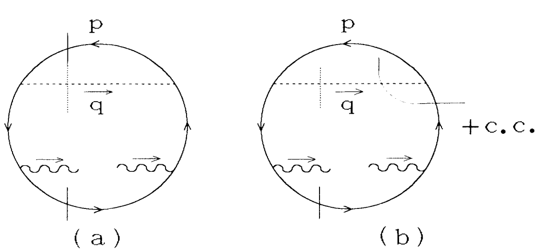

FIG. 1. Double-cut diagrams for the Drell-Yan process.

in the sum of them, the cancellation of these singularities occurs. An important observation here is that the cancellation takes place at every fixed value of $|\mathbf{p}|$ and $|\mathbf{q}|$. Now, suppose, for example, that we insert some projection operator $\mathcal{P}$ into the quark-line segment with momentum p in each diagram in Fig. 1. Here $\mathcal{P}$ picks up, say, the red-quark state. Since all the algebras concerning internal degrees of freedom are common for Figs. 1(a) and 1(b), the above-mentioned cancellation still takes place. Now the initial state of Fig. 1(a) is $q^{\text{red}}(p) + \bar{q}$, while the initial state of Fig. 1(b) is $\{q(p-q) + g(q)\}^{\text{red}} + \bar{q}$. Thus, we have identified the *relevant set of degenerate states* from the double-cut diagrams, Fig. 1.

Note that the reasoning above only requires that the statistical weight ρ in (3) depends on the $\phi_{\mathbf{i}}^{(\alpha)}$'s energy $E_{\mathbf{i}}^{(\alpha)}$, only through $E = \sum_\alpha \sum_{\mathbf{i}} n_{\mathbf{i}}^{(\alpha)} E_{\mathbf{i}}^{(\alpha)}$. Then, our conclusion is valid for, e.g., reaction rates defined on the basis of microcanonical or grand-canonical ensemble.

The renormalization does not ruin the proof of the absence of mass singularities sketched above, provided that suitable renormalization schemes, e.g., an off-shell scheme, are employed [3–7].

At first sight, it seems that the above reasoning equally applies to the proof of cancellation of IR singularities of the Bloch-Nordsiek type. This is, however, not the case: In zero-temperature field theory, one deals with the transition-probability formula like (8) with W's as representing *connected* diagrams. (Note that S or S^* is not necessarily connected, cf. the above example.) Then, at some fixed order of perturbation series, as a matter of course, the number of diagrams or terms contributing to this formula is finite. The KLN theorem is that the cancellations of the Bloch-Nordsiek type IR and mass singularities occur among these terms. In the present finite-temperature case, however, in (8) spectator particles are involved, and, for such configurations, the corresponding W's have "spectator part" that is graphically disconnected with the "reaction part." In contrast to the mass-singularity case, since an IR mode, $\mathbf{i} = \mathbf{0}$, has vanishing energy $E_{\mathbf{0}}^{(\alpha)} = 0$, W's with unlimited number of spectator $\phi_{\mathbf{0}}^{(\alpha)}$'s may contribute to (8), which reflects on the fact that the δ symbol in (8) does not constrain the number of terms in (8) to finite. Noting that W contains some positive powers of $n_{\mathbf{0}}^{(\alpha)}$, we see that the summation over $n_{\mathbf{0}}^{(\alpha)}$ in (8) diverges, and likewise for Γ_0. In the limit $V \to \infty$, this reflects on the appearance of diverging integrals like, e.g., $\int_0^\infty d\mathbf{p}/\{(e^{\beta|\mathbf{p}|}-1)|\mathbf{p}|^2\}$ in $\mathcal{A}$ in (6). Thus, our procedure is incapable of analyzing the IR problem.

In passing, one should be reminded of the fact that the consistent expansion in powers of g is obtained only after the hard-thermal-loop resummations are made for the soft lines [1,2] (cf. first three paragraphs). Then, the analysis of the IR singularities should be performed by taking the hard-thermal-loop resummations into account.

We make a comment on gauge theories. Note that the thermal propagator [12,14] from a physical vertex to a thermal-ghost one $D_{21}(k)$, appearing in $\mathcal{A}$ in (6) has its origin [12] in the emission of a particle with momentum k in the reaction (1). Since the final states in (1) are, of course, physical states, for a gauge boson, we have

$$D_{21}^{\mu\nu}(k) = -i\left(\hat{\delta}^{\mu\nu} + \frac{\kappa^\mu \kappa^\nu}{\kappa^2}\right) 2\pi\delta(k^2)\left[1 + \frac{1}{e^{\beta|\mathbf{k}|} - 1}\right],$$

where $\hat{\delta}^{\mu\nu} = \mathrm{diag}(0,1,1,1)$ and $\kappa^\mu = (0, \mathbf{k})$. For a Faddeev-Popov ghost that comes on the stage in non-Abelian gauge theories, depending on the gauge choice, we have $D_{21}^{\mathrm{FP}} = 0$. The situation is similar for other thermal propagators D_{11}, D_{22}, and D_{12}. Then, for gauge theories, the above proof primarily holds provided one calculates (6) in thermal field theory, with the gauge choice of Landshoff-Revhan's [15], in which the thermal parts of thermal propagators only contain physical degrees of freedom. At this point, the fact that the gauge invariant thermal amplitude like $\mathcal{A}$ in (6) is in fact gauge invariant at the level of thermal field theory [14,15] guarantees the validity of the above proof in any gauges.

One more comment we like to add is that, in actual computation of a thermal reaction rate, one should introduce some regularization. If one employs dimensional regularization (see, e.g., [5]), the gauge invariance is manifest. If some other regularization is chosen, one should be careful as to extracting the gauge invariant reaction rate. This situation is exactly the same as in zero-temperature field theory.

Finally, I would like to add that I have analyzed explicitly the thermal Drell-Yan process, quark-gluon plasma$\rightarrow$ $\mu^- + \mu^+$ +anything, with μ pair being at rest. The calculation has been carried out in the Coulomb gauge to the lowest nontrivial order. Within the framework of thermal field theory, many people demonstrated that the reaction rate of this process is free from IR and mass singularities (see, e.g., [8]). On the basis of formulas (3) and (4), I have reconfirmed the absence of mass singularity in the reaction rate.

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