Comment on "From Isotropic to Anisotropic Superconductors: A Scaling Approach"

In a recent Letter [1], Blatter, Geshkenbein, and Larkin (BGL) proposed a scaling approach to obtain physical results for anisotropic superconductors from the isotropic counterparts. Based on the rule for the isotropic-to-anisotropic mapping [Eq. (4) of Ref. [1]], BGL concluded that the effect of anisotropy is (1) to reduce the field component in the superconducting planes, (2) to enhance the effective strength of pinning, and (3) to increase the temperature T of thermal fluctuations. In this Comment, we point out the errors in the BGL scaling approach and show that the point (1) of the BGL conclusion should be stated differently, and the points (2) and (3) are incorrect.

The starting point of the scaling argument of BGL is the idea of Klemm and Clem (KC) [2] that in the framework of the Ginzburg-Landau (GL) theory the free energy density F of an anisotropic superconductor can be expressed in isotropiclike form by a transformation of coordinates (which involves a length rescaling and a rotation); i.e., the expression for F is the same as its isotropic counterpart except that the GL parameter κ is replaced by $\tilde{\kappa}$ which depends upon the orientation of the applied field $\mathbf{H}$ (note that κ is the only parameter in the GL theory). Although this conclusion is not exact when $\mathbf{H}$ is tilted with respect to the principal axes [3], as shown by later works [4, 5], it is a good approximation provided $\kappa^2 b \gg 1$ ($b = B/B_{c2}$), which can be easily satisfied by high-κ materials over a wide field region including intermediate and high fields [5]. Other thermodynamic quantities can be derived from F and the expressions are the same as their isotropic counterparts except that κ is replaced by $\tilde{\kappa}(\hat{\mathbf{H}})$ (note the exception that the transverse component of the magnetization has no isotropic counterpart) [4–6]. The rule for the isotropic-to-anisotropic mapping is simply $\kappa \to \tilde{\kappa}(\hat{\mathbf{H}})$ [or $H_{c2} \to H_{c2}(\hat{\mathbf{H}})$], which *is different* from that of BGL [Eq. (4) of Ref. [1]]. An example showing that the BGL scaling rule is incorrect has been given in Ref. [6]. Also note that for the properties associated with the flux-line dynamics the situation is more complicated, and the general applicability of the simple mapping rule $\kappa \to \tilde{\kappa}(\hat{\mathbf{H}})$ (which is valid for equilibrium thermodynamic properties) has not been proved [6].

Concerning the point (1) of the BGL conclusion, we point out that $\mathbf{H}$ itself, an independent variable, has nothing to do with anisotropy. The effect of anisotropy is not to reduce the component of $\mathbf{H}$ in the superconducting planes, but is that the physical properties depend upon both H and $\hat{\mathbf{H}}$. For fixed H, the response of a sample to $\mathbf{H}$ is different for different $\hat{\mathbf{H}}$, because of the difference in the values of $\tilde{\kappa}(\hat{\mathbf{H}})$ [or $H_{c2}(\hat{\mathbf{H}})$] [3]. Concerning the points (2) and (3) of the BGL conclusion, similarly, we point out that both the pinning strength and the temperature are independent variables and should not have any direct connection with the anisotropy. We notice that these conclusions are based on the BGL isotropic-to-anisotropic mapping rules for temperature T and the disorder strength γ: $T \to T/\epsilon$ and $\gamma \to \gamma/\epsilon$, where the anisotropy ratio $\epsilon < 1$ [1]. These mapping rules are derived as follows [1]. In the transformation, since energy scales as $\mathcal{G} = \epsilon\tilde{\mathcal{G}}$, one obtains $T = \epsilon\tilde{T}$. Since the disorder correlator $\langle\delta\alpha(\mathbf{r})\delta\alpha(\mathbf{r}')\rangle = \gamma\delta(\mathbf{r}-\mathbf{r}')$ is transformed into $\langle\delta\tilde{\alpha}(\tilde{\mathbf{r}})\delta\tilde{\alpha}(\tilde{\mathbf{r}}')\rangle = (\gamma/\epsilon)\delta(\tilde{\mathbf{r}}-\tilde{\mathbf{r}}')$, one obtains $\gamma = \epsilon\tilde{\gamma}$. We now show that these scaling rules do not imply the points (2) and (3). To understand physics at the scale μ, we should keep scale differently. Therefore, no physics can be extracted from observing the relation $\mathcal{G} = \epsilon\tilde{\mathcal{G}}$ (or $T = \epsilon\tilde{T}$). In particular, the GL free energy functional does not contain explicit T dependence when written in dimensionless form [7]; therefore T has nothing to do with the transformation, which is simply a *mathematical* technique and has no physical content. Similarly, the δ function is not invariant; the related invariant quantity is the form of $\int d^3r\delta(\mathbf{r}-\mathbf{r}')$. The factor $1/\epsilon$ in the front of $\delta(\tilde{\mathbf{r}}-\tilde{\mathbf{r}}')$ is canceled by another factor ϵ associated with the transformation of the volume element $d^3r = \epsilon d^3\tilde{r}$. Therefore, γ has nothing to do with the transformation. Thus, the results and conclusions of [1], which are based upon incorrect isotropic-to-anisotropic mapping rules, are not valid. For example, that the results of Ref. [1] for the thermal depinning temperature T_{dp} and the melting temperature T_m are smaller by a factor of ϵ than their isotropic counterparts, respectively, is an artifact of the incorrect mapping rule.

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