Local Time-Dependent Perturbation in Luttinger Liquid

Alexander O. Gogolin*

Landau Institute for Theoretical Physics, Russian Academy of Sciences, Kosygina str.2, Moscow, 117940, Russia

(Received 2 July 1993)

The response of the Luttinger liquid to a local potential is studied at zero temperature. It exhibits a crossover behavior. On the largest time scale and in the case of repulsive electron-electron interaction, the backscattering from the potential contributes to the core-hole Green's function $G(t)$ the factor $(t_0/t)^{1/8}$—a universal (potential independent) decay law. The orthogonality catastrophe remains of the same analytical type as for the simple metals.

PACS numbers: 72.10.Fk, 05.30.Fk

The theory of the local time-dependent perturbation in metals deals with infrared divergences, whose importance is explicitly demonstrated by Anderson's orthogonality catastrophe [1]. Knowledge of the response of a metal to the local potential provides one with the general tool for studying a variety of problems (x-ray response [2], two-level systems [3], etc.). The behavior of a simple metal under a local perturbation is well understood due to the asymptotically exact solution by Nozières and De Dominicis [4]. However, in one-dimensional metals the situation turns out to be qualitatively different. Although one arrives at results similar to those for a simple metal, provided that either the correlation effects [5] or the backscattering from external potential [6] is neglected, it is the interference of both which leads to an unusual behavior of the response functions [7]. Because of one-dimensional correlation effects the perturbation theory in an external potential breaks down for large time (low energy). Therefore the natural question [7] of the local response behavior on the largest time scale becomes highly nontrivial. The present paper is answering this question [8].

Furthermore, the interest in physical properties of one-dimensional metals has recently been renewed in connection with attempts to understand the physics of high-T_c superconductors, using an intuition developed for one-dimensional systems [9] and also in response to the progress in technology of one-dimensional semiconductor structures (quantum wires).

The spin-charge separation in low-energy properties of pure one-dimensional metals is well known [10]. The external potential mixes spin and charge degrees of freedom [11]. Although the effects due to this mixing are interesting by themselves, they seem to be not very important for the response functions studied below. Anyway, for the qualitative understanding in this Letter the spinless model is considered.

The Hamiltonian of the problem is

$$H = H_L + V(t)\,,$$
$$H_L = \int dx \{\psi_+^\dagger(-i\partial_x)\psi_+ + \psi_-^\dagger(i\partial_x)\psi_-\} \tag{1}$$
$$+ \tfrac{1}{2}\int\int dx\,dy \{2g_2(x-y)\rho_+(x)\rho_-(y) + g_4(x-y)[\rho_+(x)\rho_+(y) + (+\to -)]\}\,,$$

where H_L is the Luttinger's model Hamiltonian [12], $\psi_\pm$ are the left and right moving electron fields, $\rho_\pm$ are the density operators, and $g_{2,4}$ are the interaction potentials in accordance with the standard "g-ology" notation [10]. The units such that $v_F = 1$ and x is dimensionless are implied. The term $V(t)$ describes the interaction of the electron system with the external field and may be written in the form

$$V(t) = \int dx \{V_f(x,t)[\psi_+^\dagger\psi_+ + (+\to -)] + V_b(x,t)[\psi_+^\dagger\psi_- + \text{H.c.}]\}\,. \tag{2}$$

This expression decomposes the original potential V [which is assumed to be even: $V(x) = V(-x)$] into two parts: $V(x) \to V_f(x) + V_b(x)\cos(2k_F x)$, corresponding to the forward and backward scattering. Studying the x-ray response one should consider V time independent but multiply it by $d^\dagger d$ ($d^\dagger$ being the core-hole creation operator).

In this Letter the overlap integral $\langle 0|V\rangle$ of the ground states of the Luttinger liquid with and without the external potential is studied, as well as the core-hole Green's function $G(t) = i\langle 0|T\{d(0)d^\dagger(t)\}|0\rangle$, which determines the x-ray photoemission spectrum and serves also as a starting point for an investigation of two-level systems.

In the absence of backscattering ($V_b = 0$) the interaction with the potential [Eq. (2)] is reduced to the expression linear in the density operators and therefore the problem can be completely solved by a canonical transformation. This fact has recently been used by a number of authors [6]. In the absence of correlations ($g_{2,4} = 0$ but $V_b \neq 0$), the problem admits the usual asymptotical solution [5].

For the case when both $g_{2,4}$ and V_b are nonzero [7,13], the lowest order of the linked cluster expansion contributes the term,

$$-\frac{1}{2}\left[\frac{V_b(0)}{\pi}\right]^2 \gamma \frac{(Wt)^{\nu_0} - 1}{\nu_0}\,, \tag{3}$$

0031-9007/93/71(18)/2995(4)$06.00

to $\ln G(t)$, where W is the high-energy cutoff (a bandwidth), V_b is the zero Fourier component of $V_b(x)$ [or the $2k_F$ Fourier component of $V(x)$], and γ is a numerical coefficient (which tends to unity for vanishing interaction). The exponent

$$\nu_0 = 2\left(1-\frac{1}{A}\right), \quad A=\left(\frac{1+(g_4+g_2)/2\pi}{1+(g_4-g_2)/2\pi}\right)^{1/2}, \tag{4}$$

$g_{2,4}$ being the zero Fourier components of $g_{2,4}(x)$. For clarity $g_2=g_4=g$ is assumed below. The scaling factor $A>1$ for the repulsive interaction ($g>0$) and $A<1$ for the attractive one ($g<0$).

Equation (3) demonstrates the crucial role of the backscattering for the local response: The singularity is enhanced (for repulsion) and its analytical form is changed from a logarithmic to a power-law one. The physical reason for that is the enhancement of the electron-hole excitations density of states for momenta $k \approx 2k_F$: $\rho_{\rm eh}(\omega) \propto \omega^{1+\nu_0}$. That is closely connected with the well-known enhancement of the Peierls susceptibility [14]. Actually the fact that in the simple metals $\rho_{\rm eh} \propto \omega$ makes the perturbation theory applicable for an arbitrary large time (all closed loops have the same logarithmic singularities [15]). Therefore the local response of the simple metals can be qualitatively described in the framework of the Tomonaga-boson approach [16]. The exception is the model case when Van Hove points lie in the vicinity of the Fermi surface [here an enhanced $\rho_{\rm eh}(\omega)$ is built in simply due to the band structure]. This problem is exactly solvable; in the large time limit the usual logarithmic behavior is restored: $\ln|G(t)| = -\frac{1}{4}\ln t$, but the exponent is universal (potential independent) which corresponds to the unitary limit for the scattering phase $\delta = \pm\pi/2$ [17]. It is shown below that a very similar scenario holds also for the Luttinger liquid.

The second nonvanishing order of the linked cluster expansion contributes to $\ln G(t)$ a term $\propto V_b^4(0)(Wt)^{\nu_1}$, where $\nu_1 = 2\nu_0$. Thus, the result Eq. (3) makes sense only in the case of a weak potential ($V_b \ll W$) and serves as the intermediate-t asymptotics ($1/W \ll t \ll t_0$). Here $t_0 \sim 1/\omega_0$ and ω_0 is a crossover energy, which can be estimated as

$$\omega_0 = W\left(\frac{V_b(0)}{W}\right)^{2/\nu_0}. \tag{5}$$

It is convenient to reformulate the problem, making use of the identification [10]:

$$\psi_\pm(x) \to (2\pi\alpha)^{-1/2}\exp[\pm i\sqrt{4\pi}\phi_\pm(x)],$$

α being a cutoff ($\alpha \sim 1/W$) and $\phi_\pm(x)$ the right and left movers of the scalar phase field $\phi(x)$. A standard algebra leads to the equivalent Hamiltonian:

$$H_{\rm str} = \int dx\{\tfrac{1}{2}[\Pi^2+(\partial_x\phi)^2] + v_f(x,t)\partial_x\phi + v_b(x,t)\cos(\sqrt{4\pi/A}\,\phi)\}, \tag{6}$$

where $\Pi(x)$ is the canonical momenta and the renormalized potentials are $v_f = V_f/\sqrt{\pi}(1+g/\pi)^{3/4}$, $v_b = V_b/\pi\alpha(1+g/\pi)^{1/2}$.

Note that for the case of a point potential the forward and backscattering processes are completely decoupled (actually they are always decoupled on the large time scale: see below). Indeed, the forward-scattering term in Eq. (6) affects only the odd parity (with respect to the origin $x=0$) components of the field $\phi(x)$, whereas the backscattering term affects the even parity ones. In the point potential limit the Hamiltonian may be written as a sum of two commuting parts, corresponding to the forward and backscattering. Thus,

$$G(t) = G_f(t)G_b(t) \tag{7}$$

and the same for the overlap integral, provided that $v_b(x) = v_b\delta(x)$. The forward-scattering contribution $G_f(t)$ has been found in the papers [6] and in what follows only $G_b(t)$ is considered (v_f can be omitted).

Clearly, in order to evaluate the large-t response one has to understand the structure of the ground state (and low-energy excitations) of the system in the presence of a static external potential (impurity) first. The important step is to recognize that the dynamics of the phase field $\phi(x=0,\tau)$ at the impurity position is just the same as the dynamics of a quantum dissipative particle in a periodic potential (Ohmic case in the classification of Ref. [3]), which has been intensively studied in the eighties [18]. This mapping can be achieved by integrating out all variables $\phi(x,\tau)$ with $x \neq 0$; that requires the following identification of the parameters: the high-energy cutoff $\leftrightarrow$ the inverse mass of the particle and $A \leftrightarrow 2\pi\eta$, η being the friction coefficient. This analogy is known [19]. It has been used for the solution of the quantum dissipation problem along the critical line [20]: $\eta_c = 1/2\pi \leftrightarrow A_c = 1$ (free fermions). A useful visualization for the problem is an elastic string in a periodic potential (an elastic string as a dissipative object has also been considered in Ref. [21]). The attraction ($g<0$) corresponds to $\eta < \eta_c$ (delocalization), the repulsion, to $\eta > \eta_c$ (localization).

The quantum dissipation problem has been studied by different methods, mainly by mapping it to a logarithmically interacting gas (of kinks or instantons [18]). Fortunately, there is a simple variational approach (originally used in the theory of roughening transitions [22]), which provides one with a good idea of how the ground state looks. The variational wave function is

$$\Psi_{\rm var}[\phi] = [\mathrm{Det}(2\pi\hat{K})]^{-1/4}\exp\{-\tfrac{1}{4}\phi\hat{K}^{-1}\phi\}, \tag{8}$$

where $K(x,y) = \langle\phi(x)\phi(y)\rangle$ is the static correlation function, subject to the self-consistency equation (which is to be obtained by minimizing the energy).

In the case $v_b(x) = v_b\delta(x)$ the algebra is straightforward and leads to the expression (similar to that discussed in Ref. [21])

$$K(x,y) = \int_0^\infty \frac{dp}{2\pi}\frac{e^{-ap}}{p}\{\cos(p|x|+\delta_p^{\rm ev}) \times\cos(p|y|+\delta_p^{\rm ev}) + \sin px\sin py\}, \tag{9}$$

where $\delta_p^{\rm ev} = -\tan^{-1}(v_{\rm eff}/p)$ is the effective scattering

phase appearing in the Schrödinger-type equation for normal coordinates and $v_{\rm eff}=(v_b/\eta)\exp[-K(0,0)/\eta]$. For $\eta>\eta_c$ one finds a convergent $\langle\phi^2(0)\rangle=(1/2\pi)\times\ln(1/\alpha\omega_0)$ and $v_{\rm eff}=\alpha^{-1}(\alpha v_b)^{2/\nu_0}$. This value is in agreement with the scaling results [18] and with the perturbation theory, Eq. (5). So there is the single crossover in the problem and one can identify $v_{\rm eff}$ as $\sim\omega_0$.

Formally, for a potential which is not strictly δ function (but still localized within a radius a), $\phi(x=0)$ has no meaning of the dissipative particle coordinate any more. Nevertheless, the situation remains qualitatively the same: $\langle\phi^2(0)\rangle$ is still convergent for $\eta>\eta_c$ due to the general fact of the vanishing of the penetration coefficient for a one-dimensional barrier in the low-energy limit [23]. The variational approach can be easily generalized for this case (the forward scattering also has to be included) although the explicit calculation of $v_{\rm eff}(x)$ seems formidable. The effective scattering phase $\delta_p^{\rm odd}$, corresponding to the odd parity components of the field ϕ, vanishes as $\sim pa$ at low energies. Thus, it should be emphasized that, on the relevant energy scale ($\omega\ll\omega_0$), any potential affects the system like a point one [particularly, Eq. (7) holds for any potential, provided that $t\gg t_0$].

Note also that for the case of attraction the wave function of the string in the potential is qualitatively the same (at large distances from the origin) as the one of a free string (i.e., delocalization). Thus, there is no reason to expect orthogonality and one may argue that $\langle 0|V\rangle_b$ [as well as $G_b(t)$ for large t] is nonvanishing for $g<0$ (of course, the orthogonality due to the forward scattering remains).

The low-energy excitations above the ground state Eqs. (8) and (9) are described by the effective Hamiltonian

$$H_{\rm eff}=\int dx\ \tfrac{1}{2}[\Pi^2+(\partial_x\phi)^2]+\tfrac{1}{2}\omega_0\phi^2(0)\,. \tag{10}$$

One has to stress that the above simple approach reproduces all the essential results of more sophisticated methods (such as scaling) applied to the quantum dissipation problem. It correctly describes not only the main feature of the problem, the localization-delocalization transition, but also results in a correct behavior of the $\langle\phi(0)\phi(\tau)\rangle$ correlation function (and gives a correct energy scale ω_0 for the low-lying excitations). The physical reason is that the further x is from the origin, the better is the approximation of the problem by the wave function Eqs. (8) and (9). It correctly describes just the tails of the string, those responsible for the low-energy properties. Therefore the effective quadratic approximation may be used for the calculation of the response functions [treating the potential energy term in Eq. (10) as time dependent: $\omega_0\to\omega(\tau)$] and, moreover, I believe that the corresponding results are asymptotically exact (i.e., for $t\gg t_0$).

Following Ref. [4] I introduce the local transient bosonic Green's function $D(\tau,\tau')$, satisfying the equation

$$\partial_\tau D(\tau,\tau')=\partial_\tau D_0(\tau-\tau')+\int d\tau''\partial_\tau D_0(\tau-\tau'')\omega(\tau'')D(\tau'',\tau')\,, \tag{11}$$

where $D_0(\tau)=-i\langle T\{\phi(0)\phi(\tau)\}\rangle$ is the Green's function in the absence of the potential. Since $\partial_\tau D_0(\tau)=i/[\tau-i\alpha\,\mathrm{sign}(\tau)]$ coincides asymptotically with the kernel of the Hilbert transformation, Eq. (11) is of the form appearing in the theory of aircraft wings [24]. Generally speaking it is not exactly solvable. However, one should keep in mind that the reduction of the problem to the quadratic Hamiltonian makes sense only asymptotically, for time intervals $\gg t_0$. So, one is forced to retain only the leading asymptotical terms for all responses. Fortunately it is possible to find them even despite the absence of the exact solution of Eq. (11) for arbitrary $\omega(\tau)$.

First calculate the overlap integral. It may be written as [25]

$$\langle 0|V\rangle_b=\exp\left\{\frac{\omega_0}{2}\int_0^1 d\lambda\int_0^\infty d\tau D_\lambda(\tau,\tau)\right\}, \tag{12}$$

where $D_\lambda(\tau,\tau)$ is the solution of Eq. (11) with $\omega(\tau)=\lambda\omega_0 e^{-\delta\tau}\theta(\tau)$, $\delta=0+$.

Note that the bosonic Green's functions have the same analytical properties as the fermionic ones for zero chemical potential [26]:

$$D_0(\omega)=\int_0^\infty dE P(E)[(\omega-E+i\delta)^{-1}-(\omega+E-i\delta)^{-1}]\,,$$

where $P(E)=1/2\pi E$. Therefore Hamann's technique [25] can be straightforwardly applied to the problem [27]. That leads to the following expressions for the transient Green's function ($\tau,\tau'>0$):

$$D_\lambda(\tau,\tau')=i\int\int\frac{d\omega d\omega'}{(2\pi i)^2}\frac{e^{-i\omega\tau+i\omega'\tau'}\tilde D_0(\omega)}{\omega'-\omega-i\delta}\frac{X^+(\omega)}{X^+(\omega')} \tag{13}$$

and the overlap integral

$$\ln|\langle 0|V\rangle_b|=-\frac{1}{2\pi^2}\int_0^1 d\lambda\int_0^\infty\int_0^\infty dE\,dE'\times\frac{\delta_\lambda(E)\partial_\lambda\delta_\lambda(E')}{(E+E')^2}\,, \tag{14}$$

where

$$\ln X^+(\omega)=(2\pi i)^{-1}\int d\omega'\ln[\tilde D_0(\omega')/D_0(\omega')]\times(\omega'-\omega-i\delta)^{-1}$$

and $\tilde D_0(\omega)$ is the Green's function in a static potential. The scattering phase $\delta_\lambda(E)=\tan^{-1}(\lambda\omega_0/2E)$ has the same origin as in Eq. (9). Note that the only difference in the final expression for the overlap integral [Eq. (14)] between the bosonic problem and the fermionic one is the additional factor $\frac{1}{2}$ [cf. Eq. (15) of Ref. [25]]. Performing the integrations in Eq. (14) one obtains the remarkably simple result,

$$\ln|\langle 0|V\rangle_b|=-\tfrac{1}{16}\ln(\omega_0/\Delta)\,, \tag{15}$$

where $\Delta=2\pi/L$, L being the length of the string. For the core-hole Green's function a similar algebra leads to

$$\ln|G_b(t)| = -\tfrac{1}{8}\ln(t/t_0), \quad \text{for } t \gg t_0. \tag{16}$$

In conclusion, I have proposed the following scenario for the Luttinger liquid response. First, for $1/W \ll t \ll t_0$, the asymptotics Eq. (3) holds. Note, that this asymptotics is meaningful; it determines, for example, the energy scale for a two-level system weakly coupled to the Luttinger chain. For a larger time more and more singular contributions come into play, which finally result in the standard logarithmic behavior, Eqs. (15) and (16), but with universal exponents (i.e., independent of the strength and shape of the potential). The zero potential ($v_b \to 0$) limit corresponds to the vanishing of the region of applicability of Eqs. (15) and (16): $t_0 \to \infty$. In other words the local response of the Luttinger liquid on the largest time scale may be described by the effective scattering shift for the bosonic field ϕ: $\delta = \pm \pi/2$.

The results can also be mapped for those for noninteracting electrons. Indeed, in the latter case the overlap integral may be written as [5]

$$\ln|\langle 0|V\rangle|/\ln L = -\left(\frac{\varphi}{2\pi}\right)^2 - \left[\frac{1}{2\pi}\tan^{-1}\sqrt{R/(1-R)}\right]^2, \tag{17}$$

where φ is the forward-scattering phase and R the reflection coefficient at the Fermi energy [28]. The first term in Eq. (17) is the forward-scattering contribution (i.e., zero interaction limit of the exponents obtained in Ref. [6]) and the second one is due to the backscattering and coincides with the result Eq. (15) in the limit of $R \to 1$.

I am thankful to S. Brazovskii, H. Capellmann, G. M. Eliashberg, D. Khmelnitskii, P. Nozières, and K. Schönhammer for comments on various ideas presented here and to A. Ioselevich, V. Janiš, and V. Meden for many useful discussions. This work was supported by A. v. Humboldt Stiftung.

*Present address: Institut Laue Langevin, Boîte postale 156, 38042 Grenoble, France.

[1] P. W. Anderson, Phys. Rev. Lett. **18**, 1049 (1967).
[2] For a recent review, see K. Ohtaka and Y. Tanabe, Rev. Mod. Phys. **62**, 929 (1990); G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1990).
[3] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
[4] P. Nozières and C. T. De Dominicis, Phys. Rev. **178**, 1097 (1969).
[5] K. Schönhammer, Prog. Theor. Phys. **106**, 147 (1991).
[6] T. Ogawa, A. Furusaki, and N. Nagaosa, Phys. Rev. Lett. **68**, 3638 (1992); D. K. K. Lee and Y. Chen, Phys. Rev. Lett. **69**, 1399 (1992). See also D.-H. Lee and J. Toner, Phys. Rev. Lett. **69**, 3378 (1992), whose consideration of the orthogonality catastrophe implies in fact a neglection of the backscattering contribution.
[7] A. O. Gogolin, JETP Lett. **57**, 50 (1993).
[8] Recently N. Prokof'ev [The University of British Columbia report (to be published)] has independently approached the problem by a different method; his conclusions are qualitatively similar.
[9] See M. Ogata and P. W. Anderson, Phys. Rev. Lett. **70**, 3087 (1993), and references therein.
[10] J. Solyom, Adv. Phys. **28**, 201 (1979).
[11] A. M. Tsvelik (to be published).
[12] J. M. Luttinger, J. Math. Phys. **4**, 1154 (1963); D. C. Mattis and E. H. Lieb, J. Math. Phys. **6**, 304 (1965).
[13] V. Meden, Diploma thesis, The Göttingen University, 1993.
[14] A. Luther and I. Peschel, Phys. Rev. B **9**, 2911 (1974).
[15] See P. Nozières, J. Gavoret, and B. Roulet, Phys. Rev. **178**, 1084 (1969). For the metals with usual Fermi liquid properties that is also correct in the presence of correlations: K. Yamada and K. Yosida, Prog. Theor. Phys. **60**, 353 (1978); **62**, 363 (1979).
[16] K. D. Schotte and U. Schotte, Phys. Rev. **182**, 479 (1969); K. Schönhammer, Z. Phys. B **45**, 23 (1981).
[17] A. O. Gogolin, JETP Lett. **57**, 311 (1993).
[18] A. Schmid, Phys. Rev. Lett. **51**, 1506 (1983); S. A. Bulgadaev, Zh. Eksp. Teor. Fiz. **90**, 634 (1986) [Sov. Phys. JETP **63**, 369 (1986)]; M. P. A. Fisher and W. Zwerger, Phys. Rev. B **32**, 6190 (1985).
[19] For a recent discussion, see C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **68**, 1220 (1992); Phys. Rev. B **46**, 15233 (1992); L. I. Glazman, I. M. Rusin, and B. I. Shklovskii, Phys. Rev. B **45**, 8454 (1992); A. Furusaki and N. Nagaosa, Phys. Rev. B **47**, 3827 (1993); **47**, 4631 (1993).
[20] F. Guinea, V. Hakim, and A. Muramatsu, Phys. Rev. Lett. **54**, 263 (1985).
[21] A. Schmid, Ann. Phys. (N.Y.) **170**, 333 (1986).
[22] Y. Saito, Z. Phys. B **32**, 75 (1978).
[23] L. D. Landau and I. M. Lifshitz, *Quantum Mechanics* (Pergamon, Oxford, 1977).
[24] N. I. Muskhelishvili, *Singular Integral Equations* (Nordhoff, Groningen, 1953).
[25] D. R. Hamann, Phys. Rev. Lett. **26**, 1030 (1971).
[26] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, in *Methods of Quantum Field Theory in Statistical Physics*, edited by A. R. Silvermann (Prentice-Hall, Englewood Cliffs, NJ, 1963).
[27] Mathematical aspects of the problem of a scalar field with a source were intensively discussed in connection with the meson pair theory [see G. Wentzel, Z. Phys. **118**, 227 (1941); A. Klein and B. H. McCormick, Phys. Rev. **98**, 1428 (1955)]; the important physical difference here is that the field ϕ of the present problem [Eqs. (7) and (11)] is massless, which leads to the infrared divergences.
[28] The quantities $R = |A|^2$ and φ ($B = |B|e^{i\varphi}$) are defined by the single electron scattering solution, $\psi_+(x \to -\infty) \to e^{ik_F x} + Ae^{-ik_F x}$ and $\psi_+(x \to +\infty) \to Be^{ik_F x}$. The right-hand side of Eq. (17) may also be written as $(\delta_+^2 + \delta_-^2)/2\pi^2$, where the phase shifts for the even (odd) parity scattering solutions are $\delta_\pm = \{\varphi \pm \tan^{-1}[R/(1-R)]^{1/2}\}$ (cf. Ref. [8]). For the case of a strong δ potential [$V(x) = V\delta(x)$, $V \gg W$] both the forward and backscattering contributions to the orthogonality exponent approach the same value $\frac{1}{16}$.