# Fusion Cross Section from Chaotic Scattering

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Dynamics of scattering in the classical model of $\alpha$-cluster nuclei is studied in terms of transport theory. Behavior typical for hyperbolic chaotic scattering is found, which results in an exponential decay of the survival probability. This allows a determination of the unitarity deficit of the $S$ matrix and thus the probability for compound-nucleus formation.

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Recent progress in understanding the chaotic aspects of nonlinear dynamical systems has revitalized interest in the properties of compound nuclei. Indeed, because of the richness of experimental data the compound nucleus provides a very useful laboratory for studying the correspondence between classical and quantum dynamical systems. In fact, the first empirical evidence supporting the connection between classical chaos and the fluctuations characteristic of the Gaussian orthogonal ensemble comes from the study of compound nucleus resonances [1]. What makes the problem even more interesting is that, as an open phase-space phenomenon, it brings up the concepts of transient chaos associated with chaotic scattering [2]. Semiclassical considerations for the energy autocorrelation function $C_{ij}$ of an $S$-matrix element, $C_{ij}(\epsilon) = \langle S^*_{ij}(E)S_{ij}(E+\epsilon)\rangle_{\Delta E}$, then make a link between the quantum and the classical pictures [3]. We have

$$C_{ij}(\epsilon) \sim \int dt\, \langle P_{ij}(E,t)\rangle_{\Delta E} \exp(i\epsilon t/\hbar), \tag{1}$$

where $P_{ij}(E,t)$ is the classical survival probability for the system to remain in the interaction region with respect to a $j \to i$ transition and $\langle\ \rangle_{\Delta E}$ denotes averaging over a quantum mechanically large but classically small energy interval $\Delta E$.

Chaotic scattering connected with the existence of only unstable periodic trajectories (hyperbolic chaotic scattering) results in an exponential decay: $P(E,t) \propto \exp(-\gamma t)$. For $\Delta E$ sufficiently small on the classical energy scale, so that the energy variation of $\gamma$ can be neglected, the corresponding autocorrelation function has a Lorentzian form: $C(\epsilon) \sim \hbar/(\epsilon + i\hbar\gamma)$. On the quantum level this is the region of Ericson fluctuations [4]. Thus, $\gamma$ can be identified with the correlation width $\Gamma_{\rm corr}$. In certain situations, for instance, in the region close to the ground state, the classical phase space may reveal more solid structures: the Kolmogorov-Arnold-Moser (KAM) surfaces. Then a power law, $P(E,t) \propto t^{-z}$, is expected. Accordingly [5], $C(\epsilon) \approx C(0) + C_1(\epsilon/\hbar)^{z-1}$ which, for instance, for $z < 2$ leads to the cusp shape typical for isolated resonances. The survival probability $P(E,t)$ reflecting the classical transport properties of the system (degree of mixing, fractal structures, and stability) thus establishes a fundamental link between the structure of the underlying classical phase space and the experimental, quantum mechanical observables. $P(E,t)$ also defines that part of the incident flux which spends a long enough time in the interaction region so that the stochastic fluctuations characteristic of the compound nucleus appear. In other words, $P(E,t)$ provides an estimate for the unitarity deficit of the average $S$ matrix and thus determines the transmission coefficients [6].

Such a new kind of approach is of basic importance for a global understanding of a nucleus as a complex dynamical system. This points to the need of more quantitative study in realistic nuclear models. However, when talking about the classical limit of the nuclear system, which is at the heart of the related considerations, one has to keep in mind that the nucleus is a collection of fermions. It is not fully clear what is a corresponding classical counterpart. More manageable in this sense are the models based on the $\alpha$-cluster effects. There exists evidence that such clustering effects define the most relevant degrees of freedom of light nuclei in the low energy region. For all these reasons, the model in the present study is specified as a classical limit of the time dependent cluster model [7]. The elementary constituents are thus the pointlike alpha particles, and the dynamics is treated classically. The two-body interaction between the alpha particles is extracted from an adiabatic time dependent Hartree-Fock calculation [8]. The corresponding effective potential has a van der Waals–type form; i.e., it includes long-range weak repulsion, intermediate-range attraction, and short-range strong repulsion [9]. The model of this kind has been successful, for instance, in describing the nuclear fragmentation effects [10]. Since many effective interactions in physics have a qualitatively similar form the present analysis can be assigned a more general meaning.

The appearance of chaos in classical nuclear scattering is expected to be a generic effect. Already a global deformation of nuclei introduces strong irregularities in the deflection angle as a function of the impact parameter [11]. The $\alpha$-$^{12}$C scattering in a simplified configuration (the target $^{12}$C composed of three alpha particles frozen in the plane at the corners of an equilateral triangle) shows [12] irregularities developing self-similar structures with the well defined fractal dimension and the dynamics is governed by positive Lyapunov exponents. Analyzed in the language of the transport theory [13] already such scattering in most cases leads to an exponential decay of the corresponding survival probability. This feature is well understood in terms of the fractal dimension and the Lyapunov exponent [12].

Here, for more realistic nuclear physics estimates, we consider the target consisting of three interacting alpha particles with all the degrees of freedom released. Furthermore, the target to be interpreted as a real $^{12}$C nucleus is defined as a statistical ensemble. Each configuration is constructed so as to ensure the proper binding energy of the $^{12}$C nucleus and its total linear and angular momentum zero. Within these constraints all phase space coordinates are chosen uniformly. This means that any quantity $f$ is evaluated by integrals

$$\int d\mathbf{r}_1\, d\mathbf{r}_2\, d\mathbf{r}_3 \int d\mathbf{p}_1\, d\mathbf{p}_2\, d\mathbf{p}_3\, \delta\Big(\sum_{i=1}^{3}\mathbf{r}_i\Big)\delta\Big(\sum_{i=1}^{3}\mathbf{p}_i\Big)\delta\Big(E_B-\sum_{i=1}^{3}\mathbf{p}_i^2/2m - V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\Big)\delta\Big(\sum_{i=1}^{3}\boldsymbol{l}_i\Big) f(\mathbf{r}_1,\ldots,\mathbf{p}_3), \quad (2)$$

where $\mathbf{r}_i$, $\mathbf{p}_i$, and $\boldsymbol{l}_i$ denote the position, momentum, and angular momentum of the $i$th alpha particle, with respect to the $^{12}$C center of mass. The delta functions over the linear momenta and energy are evaluated by the method described in Ref. [10] and samples with angular momentum greater than $0.05\,\hbar$ rejected.

Numerical scattering experiments of the projectile alpha particle on the so-constructed $^{12}$C nucleus are performed for given values of the bombarding energies and impact parameters. In this way one obtains an ensemble of trajectories connected with various internal configurations of the target. This determines the survival probability $P(t)$ measured as a ratio of events $N(t)$ such that all the particles still remain in the interaction region up to time $t$, to the total number of collisions $N_0$. As an example, Fig. 1(a) presents the resulting $P(t)$ for $E = 5$ MeV and for four values of $l$. Typically, $N_0$ is of the order of $10^4$ in each case. Asymptotically, in all the cases shown, the decay is exponential, which reflects the existence of hyperbolic instabilities. However, for central collisions essentially all the trajectories are governed by this kind of dynamics, while in more peripheral cases only a small fraction of them are. With increasing $l$ the dynamics becomes dominated by fast, nontransient processes. What is also interesting is that the slope (and thus the correlation width $\Gamma_{\rm corr}$) of the exponential component systematically decreases with increasing $l$.

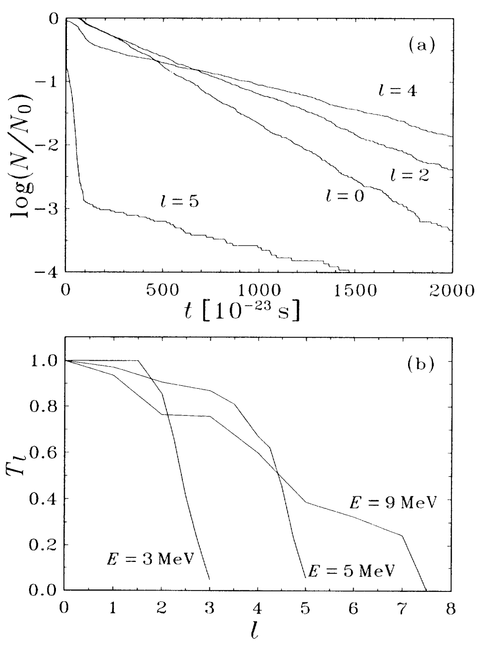


FIG. 1. (a) The ratio of trajectories surviving up to the time $t$ [$P(t)$] for the system $\alpha$-$^{12}$C for various initial relative angular momenta $l$ at the projectile energy of 5 MeV. (b) Transmission coefficients for the same system calculated by extrapolating $P(t)$ backwards in time. The angular momentum and energy dependencies are indicated.

Representing the survival probability asymptotically for given $l$ as

$$P_l(t) = T_l \exp(-\Gamma^l_{\rm corr} t) \quad (3)$$

allows us to evaluate what fraction of the incident flux decays exponentially and, consequently, to determine the transmission coefficients $T_l$ for a compound nucleus formation. Such a prescription is intuitively natural and consistent with the more formal considerations [6]. Examples of the so-determined transmission coefficients $T_l$ are presented in Fig. 1(b). As expected, they decrease with angular momentum and are stretched to larger $l$ for higher energy.

The fusion cross section expressed by $T_l$ reads

$$\sigma_f = \frac{\pi}{mE}\int T_l l\, dl. \quad (4)$$

Figure 2 presents $\sigma_f$ as a function of energy (solid line) for the considered system of $\alpha$-$^{12}$C. It initially rises and then begins to fall at about 7 MeV. For energies higher than 9 MeV the exponential form of the decay is less obvious and the above procedure becomes inapplicable.

As a further generalization of our system we consider the collision $^{12}$C-$^{12}$C. The projectile has been constructed in the same way as the target: the phase space has been sampled uniformly for initial configurations, preserving the binding energy and total linear momentum and ensuring angular momentum vanishing. As previously, the decay rates and transmission coefficients have been calcu-

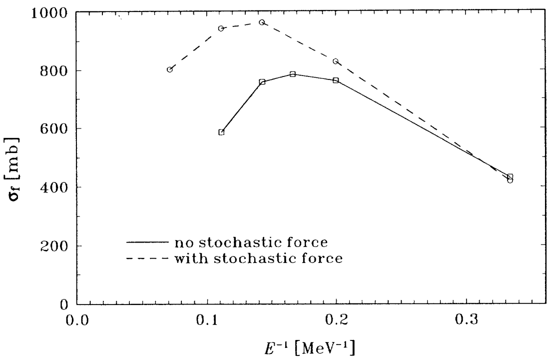


FIG. 2. Calculated fusion cross section (solid line) as a function of energy for the system $\alpha$-$^{12}$C using Eq. (3). The dashed line includes the stochastic force.

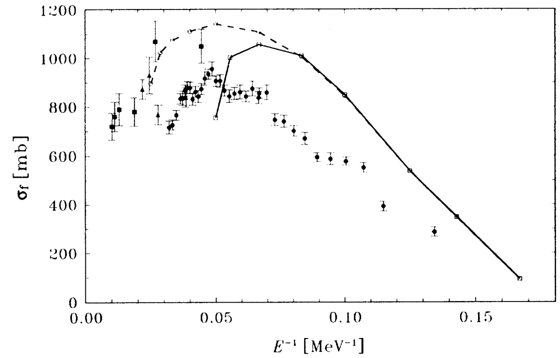


FIG. 3. Theoretical fusion cross sections for the system $^{12}$C-$^{12}$C without (solid line) and with (dashed line) the stochastic force. The data are from Refs. [14] (full circles), [15] (full squares), and [16] (triangles).

lated for various impact parameters as well as bombarding energies. The resulting fusion cross section corresponds to the solid line in Fig. 3. Our exploratory model calculation does not pretend to reproduce the experimental data already at this stage. Nevertheless, as comparison with the existing experimental data (collected from Refs. [14–16]) shows, it properly reproduces the general tendency especially at low energies where the model is expected to account for the most important degrees of freedom. The fact that the calculated fusion cross section systematically overestimates the data in this energy region seems to partly originate from a too large radius of our model $^{12}$C nucleus. With the interaction of Ref. [8] it exceeds the empirical radius by 4.6%, which should result in about 10% enhancement of the cross section. Still, the difference between our calculated result and the data is larger than 10%. Taking this correction into account our result appears consistent with the calculations based on realistic phenomenological models (e.g., Ref. [17]). It is also interesting to note that those calculations [17], overestimating the measured fusion cross section for the $^{12}$C-$^{12}$C system, provide a very good description of other systems.

At higher energies, on the other hand, the "chaos determined" fusion cross section drops down too fast. This, actually, is expected because at higher energies our model certainly does not account for all physical degrees of freedom and this must suppress the amount of instabilities as compared to the real physical process. That this is really a reason is also suggested by a calculation incorporating the stochastic force of Ref. [10]. Such a force is assumed to globally account for an internal structure of the alpha particles. It is activated when any two alpha particles during the collision process come to a distance smaller than the position of minimum in the $\alpha$-$\alpha$ potential and results in a random change of direction of their relative momentum. The procedure does not affect the total energy and momentum of the entire system. A similar stochastic force recently incorporated [18] in the molecular dynamics approach has been a crucial ingredient in a successful description of nuclear fragmentation effects. The results of our calculation including the stochastic force are indicated in Figs. 2 and 3 by the dashed lines. The exponential decay still holds and the resulting enhancement of the cross section at higher energies originates essentially from an increase of the transmission coefficients for larger angular momenta.

The analysis presented in this Letter is the first quantitative attempt to explore the role and the consequences of classical chaos on physical observables in the context of the compound nucleus formation. It provides further arguments that the semiclassical approach can be considered [2] an interesting alternative to quantum stochastic approaches based on the random matrix model [19]. A particularly nice feature of the treatment discussed here is that it allows a direct study of the time scales involved and the magnitude and mechanism of competition between the fast and the slow processes for different initial conditions.

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[1] T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Panday, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981); R.V. Haq, A. Panday, and O. Bohigas, Phys. Rev. Lett. **48**, 1086 (1982); O. Bohigas, M.J. Giannoni, and C. Schmit, Phys. Rev. Lett. **52**, 1 (1984).
[2] C.H. Lewenkopf and H.A. Weidenmüller, Ann. Phys. (N.Y.) **212**, 53 (1991); Phys. Rev. Lett. **68**, 3511 (1992).
[3] R. Blümel and U. Smilansky, Phys. Rev. Lett. **60**, 477 (1988).
[4] T. Ericson, Phys. Rev. Lett. **5**, 430 (1960).
[5] Y.-C. Lai, R. Blümel, E. Ott, and C. Grebogi, Phys. Rev. Lett. **68**, 3491 (1992).

[6] H.L. Harney, F.-M. Dittes, and A. Müller, Ann. Phys. (N.Y.) **220**, 159 (1992).

[7] S. Drożdż, J. Okołowicz, and M. Ploszajczak, Phys. Lett. **109B**, 145 (1982); W. Bauhoff, E. Caurier, B. Grammaticos, and M. Ploszajczak, Phys. Rev. C **32**, 1915 (1985).

[8] D. Provoost, F. Grümmer, K. Goeke, and P.G. Reinhardt, Nucl. Phys. **A431**, 139 (1984).

[9] T. Srokowski, A. Szczurek, and S. Drożdż, Phys. Rev. C **41**, 2159 (1990).

[10] K. Möhring, T. Srokowski, and D.H.E. Gross, Nucl. Phys. **A533**, 333 (1991).

[11] A. Rapisarda and M. Baldo, Phys. Rev. Lett. **66**, 2581 (1991).

[12] S. Drożdż, J. Okołowicz, and T. Srokowski, Phys. Rev. E (to be published).

[13] J.D. Meiss, Rev. Mod. Phys. **64**, 795 (1992).

[14] D.G. Kovar *et al.*, Phys. Rev. C **20**, 1305 (1979).

[15] M.N. Namboodiri, E.T. Chulick, and J.B. Natowitz, Nucl. Phys. **A263**, 491 (1976).

[16] R.L. Parks, S.T. Thornton, L.C. Dennis, and K.R. Cordell, Nucl. Phys. **A348**, 350 (1980).

[17] P. Fröbrich, Phys. Rep. **116**, 337 (1984).

[18] A. Ono, H. Horiuchi, T. Maruyama, and A. Onishi, Phys. Rev. Lett. **68**, 2898 (1992).

[19] J.J. Varbaarschot, H.A. Weidenmüller, and M.R. Zirnbauer, Phys. Rep. **129**, 367 (1985).