Evidence for Color Fluctuations in Hadrons from Coherent Nuclear Diffraction

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A QCD-based treatment of projectile size fluctuations is used to compute inelastic diffractive cross sections $\sigma_{\rm diff}$ for coherent hadron-nuclear processes. We find that fluctuations near the average size give the major contribution to the cross section with < few % contribution from small size configurations. The computed values of $\sigma_{\rm diff}$ are consistent with the limited available data. The importance of coherent diffraction studies for a wide range of projectiles for high energy Fermilab fixed target experiments is emphasized. The implications of these significant color fluctuations for relativistic heavy ion collisions are discussed.

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We demonstrate that color fluctuations in the projectile wave function play an important role in high energy (E_h) hadron (h), nuclear reactions. This is done by studying the cross section $\sigma_{\rm diff}$ for inelastic coherent nuclear diffraction which would vanish without such fluctuations. Not many data exist. But making the necessary measurements seems much more feasible now due to the development of microvertex detectors [1]. Using these detectors can ensure that the nucleus is left in its ground state in the diffractive process. Furthermore, such measurements will ultimately allow the computation of fluctuations of various observables available in heavy ion collisions [2,3]. This means that new experiments, which can be done during the 1995 Fermilab fixed target run, have gained a new urgency.

We begin our analysis with a general discussion. It has long been known that the time scale given by the uncertainty principle for quantum fluctuations from a hadronic state h into a state X is $2E_h/(m_X^2 - m_h^2)$. Such fluctuations are inhibited at large enough energies, so that one may treat h as frozen in its initial configuration [4]. The natural approach to describe such collisions is the scattering-eigenstates formalism [5,6]. It accounts for the high energy coherence effects: the projectile can be treated as a coherent superposition of scattering eigenstates, each with an eigenvalue σ. The probability that a given configuration interacts with a nucleon with a total cross section σ is $P(\sigma)$. It is possible to reinterpret $P(\sigma)$ by relating the size of a given configuration with its cross section (forward scattering amplitude) in a monotonic fashion; see, e.g., [7–10] and the references therein. For example: (1) In color transparency physics a configuration with a small size has small interactions. (2) Nucleonic configurations with and without a pionic cloud have different interactions with a target. (3) In string models of hadrons the transverse size is approximately proportional to the sine of the angle between the momentum and string directions.

In quantum mechanics a system fluctuates among its components. Such fluctuations lead to different kinds of interactions. Since color dynamics determines all the interactions, we use the term color fluctuations.

What is known about the $P(\sigma)$? The convenient procedure is to consider moments of this quantity: $\langle\sigma^n\rangle = \int d\sigma \sigma^n P(\sigma)$. The zeroth moment is unity, by conservation of probability, and the first is the total cross section. The analysis of diffractive dissociation data from the nucleon [11] as well as inelastic corrections to the total hadron-deuteron cross section [12] determined the second moment of $P(\sigma)$ (see the summary in [13]) while diffractive dissociation data from the deuteron targets determined the third moment of $P(\sigma)$ for protons [13]. The functional form of $P(\sigma)$ was then determined by taking the behavior for small values of σ from QCD [14,15] and also including the rapid decrease of $P(\sigma)$ for large values of σ. Thus it is possible to obtain a realistic form of $P(\sigma)$ for a wide range of σ [13,15].

The ideas behind the formulas for $\sigma_{\rm diff}(A)$ were suggested a long time ago; see, e.g., Refs. [11,16–18] and Ref. [19] and references therein. Reference [20] reviews the attempts to describe data in terms of a few scattering eigenstates. However, a realistic model based on QCD for the cross section fluctuations was missing. For example, pre-QCD models contained terms corresponding to a $\delta(\sigma)$ piece of $P(\sigma)$ [11,19] which QCD does not allow. Here we provide an expression for $\sigma_{\rm diff}(A)$ in terms of the independent information given by $P(\sigma)$. We start with the standard formula for the diffractive cross section in

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terms of the transition matrix $\hat{T}$:

$$d\sigma_{\rm diff}(A) \propto \sum_{\alpha, M_X^2} \delta^4(P_f - P_i) \cdots |\langle A; \alpha, M_X^2 \,|\hat{T}|\, A, h\rangle\,|^2, \tag{1}$$

where the ellipsis represents the standard phase space and flux factors. The frozen approximation allows us to use completeness to sum over the diffractive excitations α, M_X^2. Then the only hadronic information resides in the square of the hadronic wave function. The key step introduced in Ref. [11] and revived in Refs. [2,13,15] is to reexpress the integral over that squared wave function in terms of an integral over σ that involves the independently determined probability $P(\sigma)$. For coherent nuclear processes the scattering wave function can be obtained using the optical potential, now also a function of the integration variable σ. We consider values of A greater than about 10, so that the t dependence of the nuclear form factor is much more important than that of the hN diffractive amplitude. Then the coherent nuclear diffractive cross section $\sigma_{\rm diff}(A)$ can be expressed as

$$\sigma_{\rm diff}(A) = \int d^2B \left[\int d\sigma P(\sigma) \sum_n \left[\langle h \mid F(\sigma, B) \mid n\rangle^2 \right] - \left(\int d\sigma P(\sigma) \langle h \mid F(\sigma, B) \mid h\rangle \right)^2 \right], \tag{2}$$

where $F(\sigma, B) = 1 - e^{-\frac{1}{2}\sigma T(B)}$ and $T(B) = \int_{-\infty}^{\infty} \rho_A(B, Z) dz$. Here the direction of the beam is $\hat{\mathbf{Z}}$ and the distance between the projectile and the nuclear center is $\mathbf{R} = \mathbf{B} + Z\hat{\mathbf{Z}}$. Equation (2) is similar to the one used in [19] which did not introduce the notion of $P(\sigma)$. The advantage of Eq. (2) as compared to the related equation of Ref. [21] (for a review see [22]) within the two gluon exchange model is that we do not need to assume the validity of PQCD at average interquark distances in hadrons where α_S is large (with an extra prescription for dealing with gauge noninvariant effects due to introduction of nonzero gluon mass), nor use constituent quark model wave functions instead of parton wave functions.

It is instructive to consider the extreme black disk (bd) positions inside the nucleus and zero otherwise, so that $\sigma_{\rm diff}(A)$ vanishes. In particular, the black disk model gives $\sigma_{\rm tot}^{\rm bd} = 2\pi R_A^2$, $\sigma_{\rm el}^{\rm bd} = \pi R_A^2$, and $\sigma_{\rm diff}^{\rm bd}(A) = 0$. But we shall see that including the effects of color fluctuations leads to observable diffractive cross sections rapidly increasing with A, which are consistent with existing measurements of semi-inclusive diffraction [23,24]. Another way to show that color fluctuations cause $\sigma_{\rm diff}(A)$ is to observe that taking $P(\sigma)$ to be a delta function, e.g., $P(\sigma) = \delta(\sigma - \bar{\sigma})$, gives $\sigma_{\rm diff}(A) = 0$.

We first display results for the pion projectile and use three parametrizations of $P(\sigma)$ of Blättel *et al.* [15] of the form $P(\sigma) = N(a,n) e^{-(\sigma - \sigma_0)^n/(\Omega\sigma_0)^n}$. All of these distributions have approximately the same value of $\omega_\sigma \equiv (\langle\sigma^2\rangle - \langle\sigma\rangle^2)/\langle\sigma\rangle^2 = 0.4$–$0.5$. The resulting $\sigma_{\rm diff}(A)$ are shown in Fig. 1(a). Note that for each $P(\sigma)$, $\sigma_{\rm diff}(A)$ varies as $A^{1.05}$ for $A \approx 16$ and as $A^{0.65}$ for $A \approx 200$.

Next we examine the $\sigma_{\rm diff}(A)$ that can be observed in proton scattering. The current data indicate [13] that $\langle\sigma^2\rangle \approx 1.25\langle\sigma\rangle^2$ (for proton energies of about 400 GeV) so we may expect interesting effects. The results are shown in Fig. 1(b) for three versions of $P(\sigma)$ of Ref. [13]. We see that the shape of $P(\sigma)$ plays an important role in obtaining the magnitude of $\sigma_{\rm diff}(A)$. The A dependence is of the approximate form $A^{0.80}$ for $A \sim 16$ and $A^{0.4}$ for $A \sim 200$, which is smaller than for the pion case because here $P(\sigma = 0) = 0$ and the average value of σ is larger.

At present there are no data available on the A dependence of $\sigma_{\rm diff}(A)$. However, the A dependence of the reaction $\pi^+ + A \to \pi^+ + \pi^+ + \pi^- + A$ was studied in [23] for $p_{\pi^+} = 200$ GeV. These data integrated over the mass interval $0.8 \le M_{3\pi} \le 1.5$ GeV (and corrected for the small contribution of Coulomb excitations)

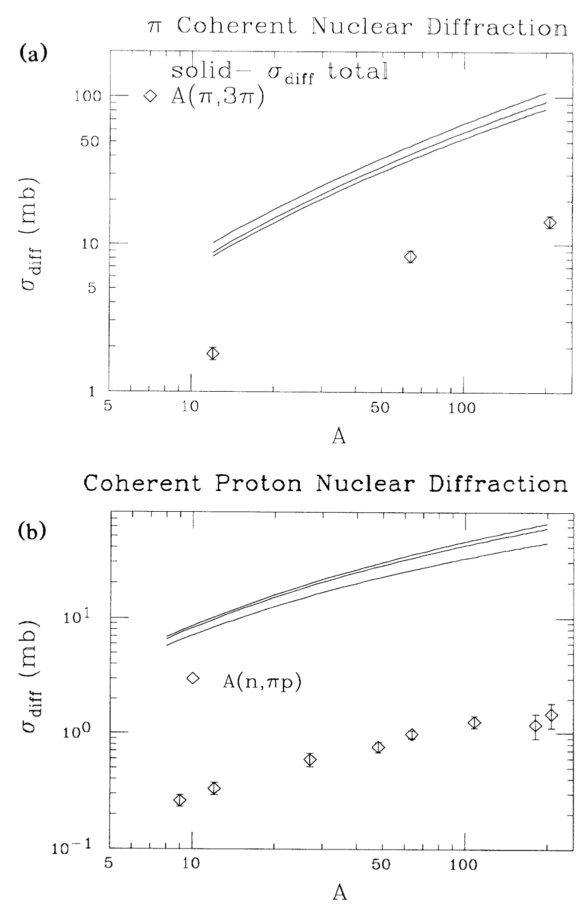

FIG. 1. (a) $\sigma_{\rm diff}(A)$, pion beam, for the different $P(\sigma)$ of [15]. The data are from Ref. [23]. (b) $\sigma_{\rm diff}(A)$ for a proton beam. The data are from Ref. [24].

are shown in Fig. 1(a). In Fig. 1(b) we present the data of [24] on $n + A \to p\pi^- + A$ for the the mass interval $1.35 \le M \le 1.45$ GeV with the Coulomb contribution subtracted using the analysis of [24]. The A dependence of both pion and nucleon semi-inclusive diffraction is reproduced well by our calculation. *A priori*, the A dependence for a given semiexclusive channel could be different from that of $\sigma_{\rm diff}$. However, if fluctuations near the average value of σ dominate then, as we discuss below, the A dependence given by Eq. (2) is sensitive mainly to the value of $\sigma_{\rm tot}$; see Eq. (3). Our numerical results [using $P(\sigma)$] for $\sigma_{\rm diff}(A)$ are reasonably close to the calculation of the pre-QCD model of [19] because similar values of ω_σ are used. But Ref. [19] showed that their calculation agrees with the only experimental data taken for emulsion targets. Hence we also agree with these data.

The similarity of the results for different models of $P(\sigma)$ suggests that fluctuations of σ near the average give the major contributions. See Fig. 2, where we plot $\sigma_{\rm diff}(A)/\omega_\sigma$. Thus we compute an approximate diffractive cross section $\sigma_{\rm diff}^{\rm appr}$ by using a Taylor series about $\sigma = \langle\sigma\rangle$ in the integrals $\int d\sigma P(\sigma) f(\sigma)$:

$$\sigma_{\rm diff}^{\rm appr} = \frac{\omega_\sigma \langle\sigma\rangle^2}{4} \int d^2B T^2(B) e^{-\langle\sigma\rangle T(B)}. \qquad (3)$$

We see that the approximation is quantitatively accurate for $A < 50$ and qualitatively good for all values of A. This is because for realistic $P(\sigma)$ the dominant contribution to the inelastic diffraction cross section arises from impact parameters B near the nuclear surface where $\langle\sigma\rangle T(B)$ is small. As a result, the second cumulant (dispersion of the cross section) dominates the diffractive cross section. This shows that the A dependence is mainly determined by the value of $\langle\sigma\rangle$. The deviation at large A must be due to configurations further from those of average cross section, in particular the ones of relatively small σ (and therefore small size). At the same time our numerical analysis shows that the series given by the sum of cumulants of the cross sections is badly convergent. This is similar to the poor convergence of the standard Glauber series for $\sigma_{\rm tot}(hA)$ which has an A dependence of $A^{2/3}$ while the first term $\sim A$.

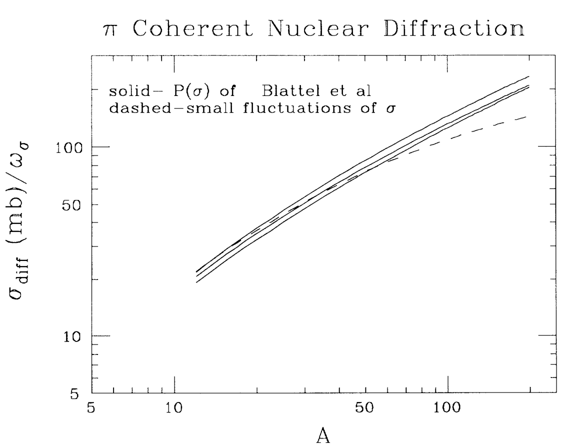

FIG. 2. $\sigma_{\rm diff}(A)/\omega_\sigma$ (pion beam). Solid curves are from Eq. (2). Dashed curve is from Eq. (3).

In the limit that A becomes infinite, configurations of small size can be expected to dominate since the nucleus acts as a black disk for all other configurations. Indeed Refs. [21,25] suggested that the effects of such small-sized configurations would dominate the pion-nucleus inelastic diffractive cross section. This early result is inherent in our Eq. (2). The integration over small values of σ in Eq. (2) gives a result $\propto 1/T(B)$ for values of B within the nucleus. The integration over d^2B leads to $\sigma_{\rm diff} \propto R_A \propto A^{1/3}$. Our realistic functions $P(\sigma)$ do not vanish at $\sigma = 0$, so we search for the dominance of small-sized configurations simply by increasing A. Numerical evaluations of Eq. (2) show that the behavior is close to $A^{0.33}$ for fantastic values of A greater than about 10 000. An additional result indicating that small-sized configurations play a small role is obtained by simply cutting off the integrals over σ at a maximum value of $\sigma_{\rm max} = 5$ mb. This contribution varies from 2% to 5% as A increases from about 12 to 200. This can be considered as an upper limit on the PQCD contribution suggested in [21,25].

But it is possible to find small-sized configurations. Reactions in which the pion diffractively dissociates into two high P_t jets select those configurations [26] and leads to an A^2 variation of the forward diffractive cross section. One could also look for the transition to this A^2 regime by considering production of states where the value of $M_{\rm diff}$ is mainly determined by the transverse momenta of produced hadrons.

The same formalism and $P(\sigma)$ used to obtain $\sigma_{\rm diff}(A)$ for pion and proton projectiles also allow us to compute the A dependence of the zero angle differential cross section for coherent nuclear diffractive dissociation as well as the total cross section. The diffractive angular distribution is related to the square of the scattering amplitude $\mathcal{M} \sim \int d^2B e^{i\mathbf{q}_t\cdot\mathbf{B}} \langle h \mid F(\sigma, B) \mid X\rangle$. Squaring $\mathcal{M}$ and summing over the diffractively produced states X yields the angular distribution. The result is

$$\frac{d\sigma_{\rm diff}}{dt}(0^\circ) = \pi \int d\sigma P(\sigma) f^2(\sigma) - \pi \left[\int d\sigma P(\sigma) f(\sigma)\right]^2, \qquad (4)$$

where $f(\sigma) \equiv \int BdB \left(1 - e^{-\frac{\sigma}{2}T(B)}\right)$. Numerical evaluation yields the result that $d\sigma_{\rm diff}/dt(0^\circ)$ varies approximately as $A^{1.24}$ for the pion (see Fig. 3) and A for the proton. The total cross section $\sigma_{\rm tot}(A)$ is given by the expression

$$\sigma_{\rm tot}(A) = 2 \int d\sigma P(\sigma) \int d^2B \langle h \mid F(\sigma, B) \mid h\rangle. \qquad (5)$$

Color fluctuations (also known in this case as inelastic shadowing [16]) have fairly small effects on total cross sections. Numerical evaluation shows that the results of using the above equation are similar to those of the more

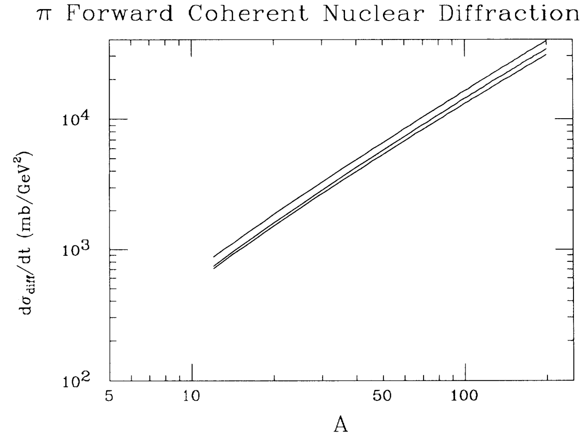

FIG. 3. π forward coherent nuclear diffraction obtained using different $P(\sigma)$ of [15].

detailed calculations of Ref. [27].

At what energies are our calculations valid? If E_h is much less than about 100 GeV, $\sigma_{\rm diff}(A)$ may increase significantly with energy due to the effects of the nuclear longitudinal form factor F_A. Such effects are omitted here, but are important in computing the inelastic shadowing correction to the total nuclear cross section [27] if $E_h \leq 100$ GeV. In diffractive dissociation of a projectile h into a state of mass M_X the minimal longitudinal momentum q_L transferred to the target is $(M_X^2 - M_h^2)/2E_h$. For small q_L the nuclear form factor can be described using the parametrization $F_A(\mathbf{q}^{\,2}) = \exp(-\mathbf{q}^{\,2}R_A^2/6)$. Thus if one uses $P(\sigma)$ for energies that vary as $A^{0.33}$ one can effectively use the same form factor for different nuclear targets. This effect is a small correction if E_h is greater than about several hundred GeV. But this is just the energy range where diffractive data and data on inelastic shadowing corrections to $\sigma_{\rm tot}^{hA}$ are available.

Our results have broader significance because of their implications for heavy ion collisions. In particular, the realistic $P(\sigma)$ used here correspond to a significantly larger probability for multiple scattering processes to occur than the usual Glauber approximation. Previous work [2] has shown that color fluctuations lead to significant fluctuations of transverse energy in nucleus-nucleus collisions in agreement with current data. Moreover, the work of Ref. [3] shows that the probability for percolation phase transitions depends strongly on the quantities $p_n(\sigma) \equiv \int_\sigma^\infty dy y^n P(y)/\langle\sigma^n\rangle$. These give the probability that n nucleon-nucleon inelastic collisions occur with a cross section larger than σ. The differences between the different versions of $P(\sigma)$ are large and influence the predictions of whether or not a percolation phase transition could occur in heavy ion collisions. Measuring $\sigma_{\rm diff}(A)$ for proton beams would strongly constrain $P(\sigma)$.

Color fluctuations have an important intrinsic interest through their close relation to QCD. New Fermilab measurements of coherent nuclear diffraction could determine finer details of $P(\sigma)$ and therefore have a wide impact for studies of heavy ion collisions.

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