New Noise Exponents in Random Conductor-Superconductor and Conductor-Insulator Mixtures

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Time dependent fluctuations of the fraction of normal-conducting part in random resistor-superconductor (RS) and resistor-insulator (RI) networks lead to a novel effect close to the percolation threshold. The normalized noise scales as a function of the resistance with a characteristic exponent λ. The value of λ is different from the value found in classical percolation models but can be related to the resistivity exponent s (t) of the RS (RI) transition by a simple scaling relation: $\lambda=2/s$ $(2/t)$. Results of recent experiments on high-T_c superconducting thin films are interpreted in terms of this new effect and a crossover from three to two dimensional percolation behavior is found.

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The electrical conductance and conductance noise in random resistor-superconductor (RS) and resistor-insulator (RI) composites have been extensively studied during the last decade [1–10]. Such studies are interesting not only for fundamental reasons but also for technological applications. An important example of such materials is high-T_c superconductors, which show a random RS composite nature in the superconducting transition region (e.g., [11–14]). Another example is thick film resistors [15–19], which are widely used in electronics. Such resistors consist of metal particles embedded in a glass medium and behave as random RI composites [19].

Random resistor networks are applied in modeling the physical behavior of granular superconductors and metal-insulator composites [1–10]. One can start from a spatially homogeneous lattice of resistors representing a homogeneous material. Introducing short circuits parallel to some resistor elements, selected at random, yields a network representing a random RS mixture. Cutting out some resistors, selected at random, yields a network representing a random RI mixture.

The physics of random resistor networks is governed by the characteristic cluster size ξ of the superconducting phase in RS composites and of the conducting phase in RI composites. In the percolation region, these length scales can be expressed as a function of the volume fraction p_r of resistors $(0 \le p_r \le 1)$,

$$\xi_{rs} \propto (p_r - p'_{cr})^{-\nu} \quad (p_r > p'_{cr}) \tag{1a}$$

and

$$\xi_{ri} \propto (p_r - p_{cr})^{-\nu} \quad (p_r > p_{cr}), \tag{1b}$$

where $p'_{cr} = 1 - p_{cs}$, p_{cs} is the percolation threshold of the superconductor component, p_{cr} is the percolation threshold of the conducting component, and ν is a critical exponent ($\nu=\frac{4}{3}$ in 2D; $\nu=0.89\pm0.01$ in 3D). This critical behavior leads to the following scaling behavior of the macroscopic resistance R:

$$R_{rs} \propto (p_r - p'_{cr})^{s} \quad (p_r > p'_{cr}) \tag{2a}$$

and

$$R_{ri} \propto (p_r - p_{cr})^{-t} \quad (p_r > p_{cr}), \tag{2b}$$

where R_{rs} is the resistance of the RS composite and R_{ri} is the resistance of the RI composite. Values of the universal exponents s and t [1–10] are shown in Table I.

Studies of noise in classical percolation models are based on the assumptions that the resistor elements in the network fluctuate independently of each other and that the fluctuation is small in comparison to the mean value of the resistance,

TABLE I. Scaling exponents of the resistance and the normalized noise in random resistor networks. The resistance of the normal-conducting elements fluctuate independently; i.e., the number of noise sources is given by the number of resistors.

	RS composite $(p_r > p_c)$		RI composite $(p_r > p_c)$	
	$R_{rs} \propto (p_r - p_c)^s$; $\frac{S_{rs}(f)}{R_{rs}^2} \propto R_{rs}^{-l_{rs}}$		$R_{ri} \propto (p_r - p_c)^{-t}$; $\frac{S_{ri}(f)}{R_{ri}^2} \propto R_{ri}^{-l_{ri}}$	
	s	l_{rs}	t	l_{ri}
1D	1	1	...	...
2D	1.297 ± 0.07	0.86 ± 0.02	1.297 ± 0.07	0.86 ± 0.02
3D	0.73 ± 0.011	0.9 ± 0.32	1.96 ± 0.1	0.80 ± 0.1

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$$\langle \Delta r_k(t) \Delta r_j(t) \rangle = \delta_{k,j} \varepsilon_k^2; \quad \varepsilon_k^2 \ll r_k^2, \tag{3}$$

where r_k is the mean resistance, $\Delta r_k(t)$ the instantaneous noise amplitude, and ε_k^2 the mean square noise of the kth resistor element. One most important result is that in the percolation region, the normalized mean-square fluctuation, and consequently the normalized power density spectrum of the noise $\Delta R(t)$ of the macroscopic resistance R scale with the resistance as

$$\frac{S_{rs}(f)}{R_{rs}^2} \propto \frac{\langle \Delta R_{rs}^2(t) \rangle}{R_{rs}^2} \propto R_{rs}^{-l_{rs}} \tag{4a}$$

and

$$\frac{S_{ri}(f)}{R_{ri}^2} \propto \frac{\langle \Delta R_{ri}^2(t) \rangle}{R_{ri}^2} \propto R_{ri}^{-l_{ri}}, \tag{4b}$$

where $\Delta R_{rs}(t)$ and $\Delta R_{ri}(t)$ are the noises of R_{rs} and R_{ri}, respectively, and $S_{rs}(f)$ and $S_{ri}(f)$ are the corresponding power density spectra at a fixed frequency f. Values for the noise exponents l_{rs} and l_{ri} [1] are given in Table I.

It is important to note that in order to obtain the scaling behavior of the normalized noise as given by Eq. (4), the p_r dependence of the noise $\Delta r_k(t)$ has to be negligible,

$$S_k[f,p_r(1)] \approx S_k[f,p_r(2)], \tag{5}$$

where $S_k(f,p_r)$ is the power density spectrum of $\Delta r_k(t)$, and $p_r(1)$ and $p_r(2)$ are two arbitrary values of p_r in the percolation region.

In high-T_c superconductors, the quantity p_r is controlled by the sample temperature. In the low temperature part of the superconducting transition, the material can be described as a network of Josephson junctions formed by the connections between neighboring superconducting grains. At a given temperature T, the critical current $I_c^{(i)}(T)$ of the ith junction can be written as [12]

$$I_c^{(i)}(T) = \frac{2e}{h} E^{(i)}(T), \tag{6}$$

where $E^{(i)}(T)$ is the Josephson coupling energy of this particular junction. If the sample is fed by a current I, two situations might occur: either the local current $I^{(i)}$ which flows through the ith junction is less than $I_c^{(i)}(T)$ and the junction is superconducting, or $I^{(i)}$ is larger than $I_c^{(i)}(T)$ and the junction is resistive. Since all high-T_c materials will contain some defects (at least on the scale of the superconducting coherence length), it seems reasonable to assume that a more or less wide distribution of junction critical currents will exist in these materials. As a consequence, for a given applied current, the transition temperatures of the junctions will be distributed according to a function $g(T_c;I)$. The relative number $p_r(T)$ of normal-conducting junctions at the temperature T is determined by $g(T_c;I)$ as follows:

$$p_r(T) = \int_0^T g(T';I)\,dT'. \tag{7}$$

In most cases, the actual distribution $g(T;I)$ is unknown, which implies that Eq. (2a) cannot be directly tested by conductance measurements. This fact emphasizes the importance of Eq. (4a) since in this case all quantities can be measured, which makes it possible to verify the existence of percolation as well as to study in more detail the behavior of the network close to percolation. This requires measurements of the temperature dependencies of the resistance R and its noise $S(f)$.

The quality of high-T_c superconducting thin films has improved significantly during the last couple of years. This development of fabrication techniques has led to a decrease of the strength of the microscopic noise, i.e., a reduction of ε_i^2 in Eq. (3). As a consequence, the normalized noise of these materials has decreased by 5–8 orders in magnitude [20] which in turn has made it possible to distinguish other noise sources than the source considered in classical percolation noise models.

Recent experimental results [20,21] of conductance noise in high-quality high-T_c superconducting thin films have shown that the noise exponent can be much larger than the scaling exponents predicted by classical percolation models [l_{rs} in Eq. (4a)]. The new noise exponent, which in the following will be called λ_{rs}, takes the following experimental values (see Fig. 1): In 3D $\lambda_{rs} \approx 2.7$ (instead of the classical $l_{rs} = 0.9 \pm 0.3$), while in 2D $\lambda_{rs} \approx 1.5$ (instead of the classical $l_{rs} = 0.86 \pm 0.02$).

In this Letter, we give a possible explanation for this new effect. It will be shown that time-dependent fluctuations of the intergrain Josephson coupling energy in RS mixtures can lead to a new class of universal noise exponents (this model was first described in [20]). The corresponding effect in RI mixtures is also predicted and the relevant noise exponents are derived.

We will start by considering the Josephson junctions in the network. As the system is in the percolation region, Eq. (2a) holds in the following form:

$$R_{rs}[p_r(T)] \propto [p_r(T) - p'_{cr}]^s, \tag{8}$$

where $R_{rs}[p_r(T)]$ is the mean value of the macroscopic

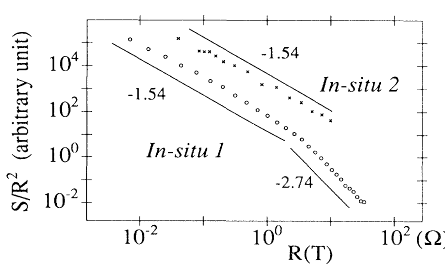

FIG. 1. Normalized noise versus resistance in high-quality high-T_c superconducting thin films in the percolation region. *In situ 1*: sample fabricated by coevaporation. *In situ 2*: sample fabricated by dc magnetron sputtering. The solid lines correspond to slopes predicted by scaling theory.

resistance at the temperature T. It will be assumed that a time-dependent perturbation of the Josephson coupling energy E_j exists, sufficiently strong to induce a random switching (on-off) of some of the junctions. The random switching of junctions represents a spontaneous fluctuation (noise) $\Delta p_r(t)$ of $p_r(T)$, which will induce a noise $\Delta R_{rs}(T;t)$ in the macroscopic resistance,

$$\Delta R_{rs}(T,t) = \frac{dR_{rs}[p_r(T)]}{dp_r}\Delta p_r(t) \propto [p_r(T) - p'_{cr}]^{s-1}\Delta p_r(t) = R_{rs}^{1-1/s}\Delta p_r(t)\,. \quad (9)$$

The normalization noise power can thus be expressed as

$$\frac{\langle \Delta R_{rs}^2(T,t)\rangle}{R_{rs}^2(T)} \propto R_{rs}^{-2/s}\langle \Delta p_r^2(t)\rangle\,. \quad (10)$$

Correspondingly, the normalized noise spectrum is

$$\frac{S_{rs}(f,T)}{R_{rs}^2(T)} \propto R_{rs}^{-2/s}(T)S_p(f,T)\,, \quad (11)$$

where $S_p(f,T)$ is the noise spectrum of $\Delta p_r(t)$.

In the case when the temperature dependence of the p noise is negligible (or, is at least weak in comparison to the temperature dependence of $R^{-2/s}$) we have

$$\frac{S_{rs}(f,T)}{R_{rs}^2(T)} \propto R_{rs}^{-\lambda_{rs}}(T);\ \ \lambda_{rs} = 2/s\,. \quad (12)$$

In this case, the normalized noise can still be expressed as a power function of the resistance. However, the value of the exponent differs significantly from the value of the exponent l_{rs} of the classical percolation problem (see Table II). To obtain the simple scaling law given in Eq. (12), it is essential that the temperature dependence of $S_p(f,T)$ is weak. This condition is similar to the condition given by Eq. (5) for the classical percolation noise problem and has important implications for the possible microscopic mechanisms of this new noise effect.

According to recent experimental studies [20,21] on $YBa_2Cu_3O_7$ high-T_c superconducting thin films obtained by various fabrication methods, high quality films display the scaling behavior predicted by Eq. (12). As can be seen in Fig. 1, the extracted values for noise exponents in films obtained by the so-called *in situ* fabrication method are in remarkable agreement with the predicted λ_{rs} values. In sample "*in situ 1*" (fabricated by coevaporation) the scaling behavior can be followed over seven decades in normalized noise spectral density which gives strong evidence for the existence of percolation in this sample as well as strong support for the proposed noise model. The dimensional crossover [from $\lambda_{rs} \approx 2.8$ to $\lambda_{rs} \approx 1.5$ around $R_{rs}(T) \approx 2\,\Omega$] indicates that the percolation length ξ reaches the smallest linear size (the sample thickness $\sim 0.08\ \mu$m) of the sample. Sample "*in situ 2*" (fabricated by dc magnetron sputtering) shows the 2D value of λ_{rs} (≈ 1.5) in the whole of the percolation region which is experimentally accessible. This 2D behavior is in accordance with scanning electron microscopy characterization of the film, which shows that the grain size of this film is of the same order of magnitude as the sample thickness ($\sim 0.1\ \mu$m).

The temperature dependence of the normalized noise has been compared with the square of the normalized temperature derivative of the resistance in order to check the possibility of temperature fluctuations [22] being the origin of the $\Delta p_r(t)$ fluctuations. This experimental check gave a negative result; temperature fluctuations can be excluded as the origin of the $\Delta p_r(t)$ fluctuations [20].

In the last part of this Letter, the corresponding $\Delta p_r(t)$ noise in RI mixtures will be considered. Starting from Eq. (2b) and the assumption of a noise Δp_r in p_r lead to the same sort of equations as for the RS mixtures [Eqs. (8-12)] with the final result

$$\frac{S_{ri}(f,T)}{R_{ri}^2(T)} \propto R_{ri}^{-\lambda_{ri}}(T);\ \ \lambda_{ri} = 2/t\,. \quad (13)$$

The predicted values of the λ_{ri} exponents are shown in Table II. The existence of $\Delta p_r(t)$ noise in RI composites has not been verified by experiments. This might be due to the fact that in 3D systems the classical and new exponents are rather similar. In 2D RI systems, however, the existence of noise in $\Delta p_r(t)$ should be clearly reflected in experiments.

In conclusion, a noise of the quantity p_r in percolating RS (RI) composites can lead to a new class of universal noise exponents. The value of the new noise exponent λ is different from the value found in classical percolation

TABLE II. Comparison of classical (l) and new (λ) noise exponents.

	RS composite ($p_r > p_c$)		RI composite ($p_r > p_c$)	
	$\frac{S_{rs}(f)}{R_{rs}^2} \propto R^{-l_{rs}}$;	$\frac{S_{rs}(f)}{R_{rs}^2} \propto R_{rs}^{-\lambda_{rs}}$	$\frac{S_{ri}(f)}{R_{ri}^2} \propto R_{ri}^{-l_{ri}}$;	$\frac{S_{ri}(f)}{R_{ri}^2} \propto R_{ri}^{-\lambda_{ri}}$
	l_{rs}	$\lambda_{rs} = 2/s$	l_{ri}	$\lambda_{ri} = 2/t$
1D	1	2	...	...
2D	0.86 ± 0.02	1.54 ± 0.09	0.86 ± 0.02	1.54 ± 0.09
3D	0.9 ± 0.32	2.74 ± 0.04	0.80 ± 0.1	1.02 ± 0.05

models but can be related to the resistivity exponent s (t) of the RS (RI) transition by a simple scaling relation: $\lambda = 2/s$ ($2/t$). Recent experimental results on (high-quality) high-T_c superconducting thin films originating from different fabrication methods are interpreted in terms of this new effect. In Table II, the theoretical values of the "old" and "new" noise exponents in RS as well as RI mixtures are shown for comparison.

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