Quantized Persistent Currents in Quantum Dot at Strong Magnetic Field

Y. Avishai[1] and M. Kohmoto[2]
[1]Department of Physics, Ben Gurion University, Beer Sheva, Israel
[2]Institute for Solid State Physics, University of Tokyo, 7-22-1 Roppongi, Minato-ku Tokyo 106, Japan
(Received 17 February 1993)

We investigate equilibrium electron currents and magnetization in an ideal two-dimensional disk of radius R placed in a strong magnetic field H. The most striking results emerge when the conditions for the existence of edge and bulk states are met, namely $R \gg a_H = (\hbar c/eH)^{1/2}$. When the Fermi energy is locked on a Landau level, the current as a function of electron density is quantized in units of $(e/h)\hbar\omega_c/2$, where ω_c is the cyclotron frequency. We argue that this effect survives against weak disorder. It is also shown that the persistent current has an approximately periodic dependence on $1/H$.

PACS numbers: 73.50.Jt, 72.10.Bg, 73.20.Dx

Persistent current [1] is an equilibrium property of an electronic system having the topology of a closed loop, expressing its response to an applied magnetic flux passing through the hole. The simplest example of such a system is an electron gas confined within a one-dimensional ring threaded by a flux. In the present Letter, we raise the issue of persistent currents in simply connected two-dimensional systems subject to a strong magnetic field. The motivation is related to the question of what happens if there are edge states traveling along one direction only, as the sample has only one boundary. We would especially like to find out whether the current is a monotonic function of electron density. As we shall see, the answer to this question is positive but only locally. In particular, novel quantization takes place. It is worth pointing out here that for systems with many transverse channels, we cannot expect the magnetization to be simply related to the persistent current. In addition, at strong magnetic field, we shall see that bulk states do not contribute to the current, but do so for the magnetization.

Unlike the concept of local current density, the definition of total current in a simply connected system requires some clarification. As long as a system has a hole, there is no ambiguity as to what is the relevant persistent current. One way to define the current in a simply connected system is then to start from a system with a hole of radius ρ threaded by a flux Φ, and then let $\rho \to 0$. This procedure introduces the somewhat unrealistic concept of flux line, but so long as $\Phi \neq 0$, it forces the wave function to vanish at the point where the flux threads the sample, and that makes the concept of flux line well defined. Experimentally, however, we have a two-dimensional system in a perpendicular magnetic field without an additional central flux. We can arrive at this situation by letting $\Phi \to 0$ (after taking the limit $\rho \to 0$). Note that, due to the presence of the (constant) magnetic field, the ground state is not Kramer degenerate [2]. Yet, some subtle point remains open with this limiting procedure. It is best exemplified in the clean disk geometry upon which we concentrate henceforth. The formalism developed below is based on the independent particle approximation in which the effect of the electron-electron interaction is neglected. The pertinent quantity is then the total azimuthal current along a radial cross section between $r=0$ and $r=R$ (which is independent of angle due to current conservation). Since there is a rotational symmetry, the angular momentum m is a good quantum number. The s state wave function does not vanish at the origin if $\Phi=0$, and therefore, in that particular case, the limiting procedure described above does not carry the characteristics of a system with a hole. We shall return to this point in connection with Eq. (4) below.

The total current is an integral of the current density along a straight line stretched from the flux point to the edge of the sample. It is determined by the energy spectrum, or, more precisely, through the dependence of the ground state energy on the central flux Φ. (We are interested in the particular value $\phi=0$.) Before studying the energy spectrum in a disk in a strong magnetic field H, it is useful to familiarize oneself with that of an infinite system. We use the symmetric gauge and adopt units such that $\hbar = 2m^* = 1$ where m^* is the mass of an electron. The scaled magnetic field is given by $b = 1/a_H^2 = eH/\hbar c$ where a_H is the magnetic length. The eigenvalues $E_n(m)$ depend on the radial quantum number n ($n=0,1,2,\ldots$) which denotes a Landau level, and the angular momentum m ($m=0,\pm1,\pm2,\ldots$). They are given by [3]

$$E_n(m) = 2[n + \tfrac{1}{2}(|m| - m + 1)]b \tag{1}$$

(note that in physical energy units b is equivalent to $\hbar\omega_c/2$). The corresponding wave functions are

$$\Psi_{nm}(r,\theta) = C r^{|m|} e^{im\theta} e^{-br^2/2} L_n^{|m|}(br^2), \tag{2}$$

where C is a normalization constant, r and θ are plane polar coordinates, and $L_n^{|m|}(x)$ is the Laguerre polynomial. For a fixed n, all the energies with $m \geq 0$ are degenerate, forming the bulk states of the Landau level. In particular, for $n=0$, the wave functions (2) for $m \geq 0$ can be written in terms of the complex variable $z = re^{i\theta}$ as $Cz^m e^{-b|z|^2/2}$. On the other hand, for $m<0$, the energies are not degenerate. In particular, for $n=0$, the pertinent states correspond to the wave functions $Cz^{*m} e^{-b|z|^2/2}$ One may regard the energies for negative values of m as belonging to higher Landau levels. Thus, for example, in this construction $E_0(-1)=E_1(0)=3b$ belong to the second Landau level. Besides the fact that such a newly defined Landau level contains functions with different numbers of radial nodes, there is one other point to be noticed. If we add a central flux of strength Φ the corresponding energy eigenvalues are obtained from Eq. (1) through the substitution $m \rightarrow m+\phi$ where $\phi=\Phi/\Phi_0$ with $\Phi_0=hc/e$ being the magnetic flux quantum. We may then speak of continuous energy curves, since, as ϕ is varied continuously between 0 and 1, the energies $E_n(m;\phi)$ slide along the curve $2[n+\frac{1}{2}(|x|-x+1)]b$. This adiabatic change is subject to the condition $E_n(m;\phi+1)=E_n(m+1;\phi)$, as is required by gauge invariance. Turning to our example above, the energy $E_1(0)$ will slide rightward to become $E_1(1)=3b$, namely, it is not changed. On the other hand, the eigenvalue $E_0(-1)$ will slide right and downward to become $E_0(0)=b$. We then conclude that the dependence of the energies for $m>0$ on a central flux which is allowed to vary adiabatically is different from those with $m<0$. Once we turn to a finite system, this has an important consequence with respect to the persistent currents.

We now concentrate on the qualitative features of the energy spectrum for a disk of finite radius R in a strong magnetic field such that the condition $R \gg a_H$ holds. Following Halperin [4] we define, for $m>0$, an m dependent radial parameter r_m through

$$\frac{1}{2} b r_m^2 = m-\phi \,. \tag{3}$$

Then, if $R-r_m > a_H$, the energies $E_n(m;\phi)$ are almost identical to those for the infinite system [given by Eq. (1) for $\phi=0$], and the corresponding wave functions $\Psi_{nm}(r,\theta)$ are localized in the radial coordinate near r_m. If $R-r_m < a_H$ (or negative), the energies $E_n(m;\phi)$ form a sequence which increases monotonically with m, and the corresponding eigenfunctions are edge states. The energies for $m<0$ are practically unaffected by the finite size, and hence, they are also given to an excellent approximation by Eq. (1). In Fig. 1 we display the single-particle spectrum for the disk in a strong magnetic field, which is computed by integrations of the relevant Schrödinger equation. The consequence of this structure of the spectrum to the behavior of the persistent currents is rather peculiar, and consists of the central point of the present work.

The technical details for obtaining the solutions are explained elsewhere [5]. The fact that the system has a finite radius R introduces a new length unit into the problem in addition to the magnetic length $a_H=(\hbar c/eH)^{1/2}$. Since we are going to study the current as a function of electron density and as a function of the magnetic field, it is useful to express energies $E_n(m;\phi)$ and magnetic fields b in terms of this unit (recall that $\hbar=2m^*=1$ so that both E and b have dimension of inverse length square). Besides the eigenvalues $E_n(m;\phi)$, the numerical procedure produces also the wave functions $\Psi_{nm}(r,\theta) =(2\pi)^{-1/2}\psi_{nm}(r)e^{im\theta}$, which vanish at $r=R$ (hard wall boundary conditions) and are normalized to unity on the disk area with volume element $r\,dr\,d\theta$. The contribution of an occupied state at energy $E_n(m;\phi)$ to the azimuthal current between $r=0$ and $r=R$ can be evaluated directly by integrating the corresponding current density. The result is

$$I_{nm}=\int_0^R |\psi_{nm}(r)|^2 \left(2\frac{m-\phi}{r}-br\right) dr$$
$$=2\int_0^R |\psi_{nm}(r)|^2 \frac{m-\phi}{r} dr - b \tag{4}$$

[the current in physical units is $(e/h)(\hbar^2/2m^*)I_{nm}$]. Under certain conditions (see below) the current can also be computed in terms of the Byers-Yang [6] relation,

$$I_{nm}=-\frac{\partial E_n(m;\phi)}{\partial \phi}=\frac{\partial E_n(m;\phi)}{\partial m} \,. \tag{5}$$

Notice, however, that this relation is derived only for systems with a hole. Following our discussion of the limiting procedure it can be applied in simply connected systems even at $\phi=0$ provided $m\neq 0$, since the corresponding wave functions vanish at $r=0$. On the other hand, the energy derivative is discontinuous at $\phi=0$ for $m=0$, as can easily be verified from a glance at Eq. (1) pertaining to an infinite system. Similar behavior is anticipated for our finite system as well. Thus, $I_{n0}(\phi)$ does not have a definite limit as $\phi \rightarrow 0$. This is not surprising, since we have already stressed that the case $m=0$ and $\phi \rightarrow 0$ can-

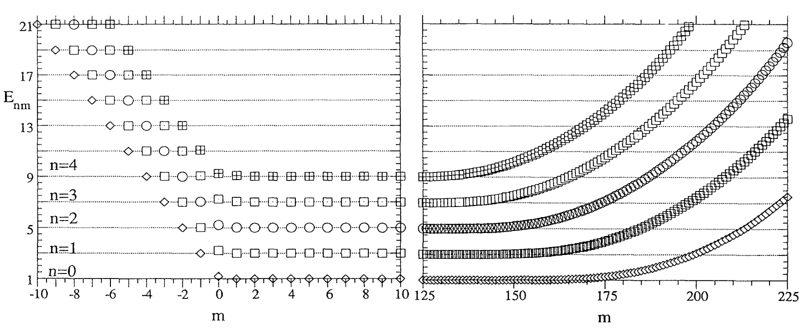

FIG. 1. Single particle spectrum of a clean disk of radius $R=20$ x, placed in a magnetic field $b=1$ (in units of x^{-2}). Energies $E_n(m;\phi=0)$ (in units of x^{-2}) are displayed as a function of the angular momentum quantum number m. In the left part the spectrum is shown for m around 0, while in the right part it is shown for m in the edge region. The flat parts are given by odd multiples of b (in physical units, b is equivalent to $\hbar\omega_c/2$) corresponding to the Landau levels for the infinite system $(2n+1)b=1$, 3, 5, 7, and 9.

not be regarded as having the topology of a system with a hole. The procedure which we adopt in this particular case $m=0$ is to set $\phi=0$ at the onset, evaluate the current density from the wave function, and then compute the current by integration. Evidently, the angular momentum part of the current density, $2m|\psi_{nm}(r)|^2/r$ will vanish for $m=0$ and hence only the second term on the right-hand side of Eq. (4) will survive. The result is then $I_{nm=0}(\phi=0)=-b$. Curiously, this is just the average of the two values obtained from the right and left derivatives. Note that this procedure does not imply that we are, in fact, considering a disk with a hole. Indeed, for a disk with a pointlike hole, the wave function must vanish at $r=0$ even for $\phi=0$. Thus, the $m=0$ state is excluded in this case and does not contribute at all to the current. On the other hand, in the procedure described above, we do get a nonzero contribution from the $m=0$ state through the second term in Eq. (4). The vanishing of the first term and the nonvanishing of the second term on the right-hand side of Eq. (4) for $m=0$ and $\phi=0$ are the hallmark of a current in a simply connected disk in the presence of a magnetic field.

We now construct and examine the behavior of the persistent currents at zero temperature $I=\sum_{nm}I_{nm}\theta[E_F-E_n(m)]$, where θ is the step function. The Fermi energy E_F depends on the magnetic field H and on the electron number N. We first consider the dependence of I on N (for a fixed value of the magnetic field) as we add electrons to the system, starting from $N=0$. Experimentally, variation of electron density is achieved in terms of an applied gate voltage. From the structure of the spectrum as shown in Fig. 1, we conjecture the following three points: (1) If a state belongs to the flat part (bulk states) of the Landau level, then $I_{nm}=0$. Thus, if the Fermi energy is locked on the bulk states of a given Landau level, the current I, as a function of N, is flat. This structure of plateaus has been noticed by us previously when we have discussed the annulus geometry [5]. In the present case, however, we will see that the value of the current on these plateaus is quantized in an integral multiple of $(e/h)\times\hbar\omega_c/2$. (2) If m belongs to one of the edge states then $I_{nm}>0$. Thus, when electrons are added on the edge states, the current I increases monotonically with N. (3) When the Fermi energy crosses an $m=0$ state, the total current is reduced by a half unit (that is, $-b$), while crossing an $m<0$ state reduces the current by exactly one unit (that is, $-2b$). As a consequence of the above three points we expect a combination of plateaus, monotonically increasing parts, and abrupt reductions of a half or one unit. This is indeed the case as we can see from Fig. 2 where we display the Fermi energy and the persistent current as a function of electron density $n_e=N/\pi R^2$.

Remarkably, the value of the current at each plateau is quantized and equal to an integer multiple of b. The reason for this quantization is as follows. The sum of the currents I_{nm} in Eq. (5) can be divided into a sum over

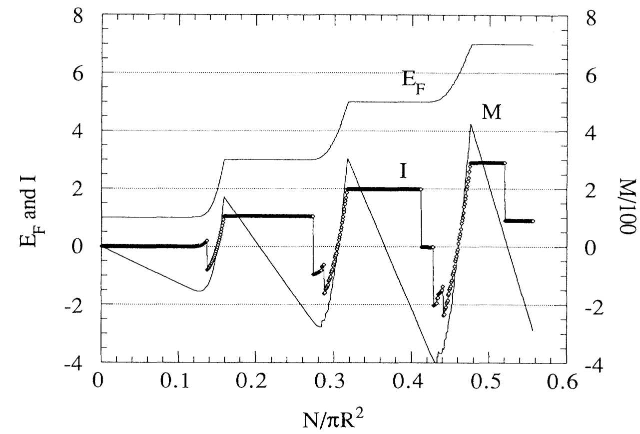

FIG. 2. Fermi energy and current (left ordinate) and magnetization (right ordinate) as functions of electron density for the system with radius R and magnetic field b as in Fig. 1. The current is also given in units of x^{-2} [equivalent in physical units to $(e/h)\hbar\omega_c/2$]. The magnetization is given in units of μ_B.

$m\le 0$ and a sum over $m>0$. We have just shown that the first sum is composed of quantized and negative contributions, $-b$ for $m=0$ and $-2b$ for $m<0$. As for the second sum, let us fix n, and assume that it runs over $m>0$ up to m', such that $E_n(m')=E_{n'}(m=1)=(2n'+1)b$ for some $n'>n$. We notice that the energy curve is very smooth in that region of $m>0$, so that a replacement of the derivative $\partial E_n(m)/\partial m$ by a difference $E_n(m+1)-E_n(m)$ should be an excellent approximation. The total sum over $m>0$ should then be equal to $E_n(m')-E_n(m=1)=2b(n'-n)$. To wit, when the Fermi energy is locked on a Landau level we get negative contributions $-2b$ from states with $m<0$, negative contributions $-b$ from states with $m=0$, and positive even multiples of b from states with $m>0$. In physical units, the current is then an integer multiple of $(e/h)\hbar\omega_c/2$. This is somewhat surprising, since the energy quantization is in steps of $\hbar\omega_c$. But remember that the current is related to the derivative of the energy. Although we have adopted the hard wall boundary condition, it is apparent that the quantization takes place in other boundary conditions as well (say, soft wall boundary conditions).

It is also of interest to study the persistent current as a function of the magnetic field (or rather, its inverse) for a fixed number of electrons. If we start with a strong field and fix the number of particles so that initially the filling factor is less than 1, then the current will be zero since only bulk states are occupied. Now we start to decrease the strength of the field and the Fermi energy decreases as well since the energy curve is lowered. At the same time, the degeneracy of the Landau level is decreased, so that eventually, the Fermi energy is pushed up, as edge states from the first level are filled. This leads to a sawtooth structure of the Fermi energy. As before, the current will get positive contributions from the edge states, and abrupt, negative contributions from states

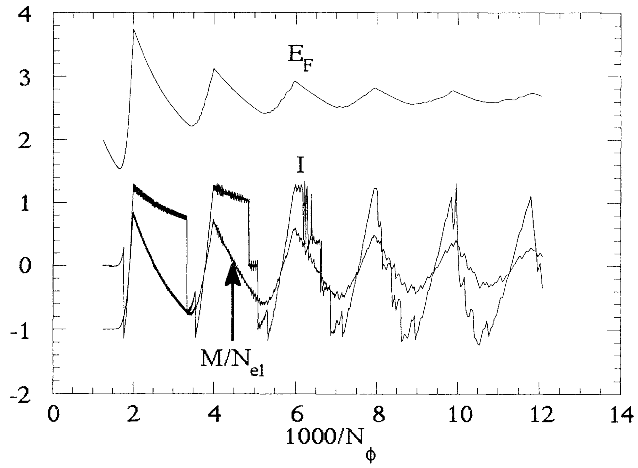

FIG. 3. Fermi energy and persistent current (in units of x^{-2}), and magnetization per particle (in μ_B) as a function of $1000/N_\phi$ where N_ϕ is the number of flux quanta in the sample, for the disk with radius R as in Figs. 1 and 2.

with $m < 0$. Typical behavior is displayed in Fig. 3. The conclusion is that the current is approximately periodic in $1/H$. Although this is expected from the behavior of the Fermi energy, it is the first time that de Haas–van Alphen type oscillations are predicted for the persistent current and not only for the magnetization.

We have also calculated the magnetization M of the pertinent electronic system. Unlike the current, the magnetization is computed from the response of the energy spectrum to a slight change in the magnetic field, and not to a variation of the flux. In Fig. 2 we show how the magnetization is changed as a function of electron density. Note that the slope dM/dn_e at each Landau level n is proportional to $n+1$, since the energy equals a constant plus a term proportional to $2(n+1)n_e b$. The magnetization M as a function of the inverse of the magnetic field is displayed in Fig. 3. The interpretation of this quantity for finite systems with hard wall boundary conditions has been already given by Sivan and Imry [7]. Comparing the current and the magnetization in Figs. 2 and 3 shows that the persistent current and the magnetization are not proportional. Magnetization of systems confined by harmonic potentials has been analyzed by Papadopulus [8] and by Yoshioka and Fukuyama [9].

Finally, we argue that the main result of the present Letter (the plateau structure of the persistent current) survives the effects of weak disorder. This property is crucial if one intends to observe it experimentally. Consider a bulk state $\psi_{nm}(r,\theta)(m>0)$ in the clean disk. As we have indicated in connection with Eq. (3) $\psi_{nm}(r,\theta)$ is localized in the radial direction around r_m. When θ is varied from 0 to 2π, $\psi_{nm}(r,\theta)$ goes m times around the origin of the complex plane. Let us now add a disordered potential which we write as $\lambda V(\mathbf{r})$ where the parameter λ is changed continuously from 0 to 1. Then $\psi_{nm}(r,\theta)$ is changed continuously into another function $\tilde{\psi}_{nm}(r,\theta)$. Notice, however, that, since the rotational symmetry is lost, we do not have conserved angular momentum anymore. Yet, the subscript m on $\tilde{\psi}_{nm}$ still represents the topological number described above, and that makes sense also in the absence of rotational symmetry. Since m has to be an integer, it is expected not be changed as λ is varied continuously from 0 to 1 as long as $V(\mathbf{r})$ is small enough.

As is commonly accepted, bulk states are localized in the presence of a disordered potential. Hence $\tilde{\psi}_{nm}(r,\theta)$ is localized in the azimuthal direction. Now let us consider $\tilde{\psi}_{nm+\delta m}(r,\theta)$ which satisfies the Schrödinger equation with an additional flux δm, with slightly modified boundary conditions, namely,

$$\tilde{\Psi}_{nm+\delta m}(r,\theta=2\pi)=e^{2\pi\delta m}\tilde{\Psi}_{nm+\delta m}(r,\theta=0)\,.$$

Since the wave function is localized in the azimuthal direction, the phase coherence is lost during the round trip. So, the energy is insensitive to the small change δm. Thus we have $\partial E_n(m)/\partial m=0$ for localized states, and hence a flat part of the energy curve. The Fermi energy is pinned on this energy as the number of electrons is varied and that is all that is required for the plateau structure of the persistent currents.

After completion of this work, we became aware of a work by Schult *et al.* [10] in which magnetization is considered in a similar system. However, our main results, pertaining to the persistent currents, are not reported herein.

The research of one of us (Y.A.) is partially supported by a grant from the Japanese Ministry of Education and partially by a grant from the Israeli Ministry of Science and Development.

[1] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).
[2] V. E. Kravtsov and M. R. Zirnbauer, Phys. Rev. B **46**, 4332 (1992).
[3] L. D. Landau and E. M. Lifshits, *Quantum Mechanics (Non-Relativistic Theory)* (Pergamon, New York, 1965), problem on page 426.
[4] B. I. Halperin, Phys. Rev. B **25**, 2185 (1982).
[5] Y. Avishai, Y. Hatsugai, and M. Kohmoto, Phys. Rev. B **47**, 9501 (1993).
[6] N. Byers and C. N. Yang, Phys. Rev. Lett. **7**, 4 (1961).
[7] U. Sivan and Y. Imry, Phys. Rev. Lett. **61**, 1001 (1988).
[8] G. J. Papadopulus, J. Phys. A **4**, 773 (1971).
[9] D. Yoshioka and H. Fukuyama, J. Phys. Soc. Jpn. **61**, 2368 (1992).
[10] R. L. Schult, M. Stone, H. W. Wyld, and D. G. Ravenhall, Superlattices Microstruct. **11**, 73 (1992).