Directed Polymer Localization in a Disordered Medium

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(Received 11 May 1993)

The localization of a directed polymer onto an extended defect (such as a line or a plane) in the presence of competing bulk disorder is examined. Based on scaling ideas and exact analysis on a hierarchical lattice, we develop a new renormalization scheme to study the directed polymer localization problem. We establish absence of delocalization transition for attractive columnar defect in the marginal dimension $d_c = 2$, and for attractive planar defect in $d = 3$. For columnar defect in three dimensions, our simulations yield a localization length exponent $\nu_\perp = 1.8 \pm 0.6$.

PACS numbers: 64.60.Fr, 02.70.–c, 05.40.+j, 74.60.Ge

Through various mathematically equivalent formulations, a directed polymer (DP) sampling quenched random bond or site energies unifies a diverse set of problems in the theory of disordered and nonequilibrium systems [1]. The DP model has been extended by including in the Hamiltonian an additional line or plane potential along the principal direction to describe, e.g., wetting in the presence of bulk disorder [2], pinning of a flux line by a columnar defect or twin boundary in type-II superconductors close to the lower critical field H_{c1} [3–5], inhomogeneous surface growth [6], and shock wave generation in driven lattice gas with a blockage [7]. In an attractive potential, a DP may either be localized, making only excursions of finite distance away from the potential well, or be delocalized by energetically favorable configurations in the whole d-dimensional disordered medium. Thermodynamically, the two states are separated by a phase transition in that the (free) energy per unit length exhibits a singularity at a critical depth Δ_c of the potential well [8]. The case of a repulsive potential is also interesting as it approximates that of two directed lines, which is of concern in a many-line problem [9,10].

Despite previous theoretical and extensive numerical studies [2,6,11–14], a coherent picture on the DP localization transition is still lacking. Since temperature is irrelevant for the transition, perturbative renormalization group (RG) analysis based on a high temperature expansion [10] tends to obscure real physics. On the numerical side, direct determination of localization length close to criticality in a transfer matrix simulation [2,11,14] is severely limited by finite-size effects, making it difficult to draw definitive conclusions on the transition.

In this Letter we propose a RG scheme which combines phenomenological scaling ideas with transfer matrix data in constructing a flow equation for the analysis of the localization transition at zero or finite values of Δ_c. By working *within* the critical regime, our method enables one to determine the critical exponents using relatively small systems in a numerical simulation. The method is applied to a defect hierarchical lattice and to a line potential at $d = 2$ and 3 and a plane potential at $d = 3$. Our numerical results confirm a zero-temperature perturbation theory whose predictions have hitherto been challenged by several other numerical studies [2,11,12,14].

Using a continuum description, a DP is specified by its transverse displacements $\mathbf{x}(t) \in R^{d-1}$ along the principal direction t. The energy of the DP is given by

$$H = \int dt \left[\frac{\epsilon}{2} \left(\frac{d\mathbf{x}}{dt} \right)^2 + \eta(\mathbf{x}, t) + V(\mathbf{x}_\perp) \right]. \quad (1)$$

Here ϵ is the line tension, $\eta(\mathbf{x}, t)$ is a quenched random potential with a short-range correlation, and $V(\mathbf{x}_\perp)$ is a potential due to an n-dimensional ($n < d$) extended defect (e.g., a line or a plane) located at $\mathbf{x}_\perp = 0$, with $\mathbf{x}_\perp$ being the component of $\mathbf{x}$ perpendicular to the defect. In this paper we limit ourselves to the case where V has a simple symmetric shape and extends over only a finite range a, e.g., $V(\mathbf{x}_\perp) = -\Delta$ for $|\mathbf{x}_\perp| < a$ and $V(\mathbf{x}_\perp) = 0$ otherwise.

In the absence of the potential V, the configuration which minimizes (1) has a typical transverse extension of linear size ξ which grows with the length t of the DP as $\xi(t) \sim t^\zeta$, where ζ is known as the roughness exponent [1]. If (1) is minimized with one end of the DP fixed at some position $\mathbf{x}$, the resulting ground state energy has a fluctuation of order $\delta E(t) \sim t^\omega$ with respect to either different realizations of η or the end position $\mathbf{x}$. An attractive potential V of finite depth Δ lowers the energy of sections of the DP which fall in the region $|\mathbf{x}_\perp| < a$ by an amount Δ per unit length. This energy gain is not sufficient to localize the DP completely within $|\mathbf{x}_\perp| < a$, unless Δ is significantly larger than the typical variation of η. Thus even if the DP is in a localized state, it may still wander out of the potential well to take advantage of low values of η within a transverse localization length $\xi_c > a$. The typical span of such excursions along the t axis is of the order of t_c, usually known as the correlation length [8]. For $\xi_c \gg a$, which is the case close to a continuous delocalization transition, the ground state configuration inside the localization volume $|\mathbf{x}_\perp| < \xi_c$ is dominated by the disorder fluctuation η instead of the potential V.

With this picture in mind, we propose the following RG scheme *within* the localization volume. On a given length scale t along the DP, we compute the mean ground state energy $\gamma(x_\perp, t)$ for a DP of length t with one end

0031-9007/93/71(17)/2745(4)$06.00

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fixed at a distance $x_\perp$ from the center of the potential. At $x_\perp = 0$, this energy differs from its value $\gamma_0(t)$ at $V = 0$ by an amount $\Delta(t) \equiv \gamma_0(t) - \gamma(0,t)$. Since the transverse fluctuation of the DP is limited to a distance $\xi(t) \sim t^\zeta$, we expect $\gamma_0(t) - \gamma(x_\perp, t)$ to fall off rapidly to zero for $x_\perp > \xi$. Thus if we choose t as the coarse-grain scale along the DP and $\xi(t)$ ($> a$) the corresponding scale perpendicular to it, the effective potential then has a depth $\Delta(t)$ and extends over a distance $\xi(t)$ in the $\mathbf{x}_\perp$ direction. This potential now competes with the ground state energy fluctuation $\delta E(t) \sim t^\omega$ of DP sections contained in the coarse-grained cells outside the potential well (or barrier). The ratio of the two energies $u(t) \equiv \Delta(t)/\delta E(t)$ then sets the strength for the renormalized potential. At the transition, we expect $u(t)$ to approach a constant u_c^* as $t \to \infty$. On the localized side, $u(t) - u_c^*$ reaches a value of order 1 at $t = t_c$, at which point the renormalization process is to be terminated. In terms of t_c, the transverse localization length ξ_c and the condensation energy $e_c \equiv \lim_{t\to\infty} \Delta(t)/t$ are given by

$$\xi_c \sim t_c^\zeta, \quad e_c \sim t_c^{\omega-1}. \tag{2}$$

To demonstrate how the scheme works in practice, let us first consider the case of a DP on a hierarchical lattice [15] with a defect line, where the block transformation described above can be implemented exactly. The hierarchical lattice is constructed by an iterative rule. Upon each iteration, a bond is split into b branches, each containing two connected bonds. The case $b = 2$ is illustrated in Fig. 1. A defect line is introduced following the iteration rule for a defect bond as illustrated in Fig. 1(b)

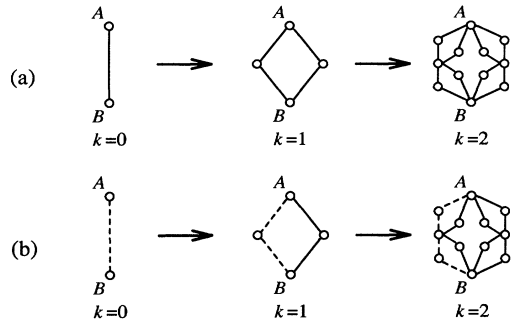

FIG. 1. First three generations of a hierarchical lattice at branch number $b = 2$: (a) normal lattice; (b) lattice containing a line of defect bonds (dashed lines).

[16]. The energy of a defect bond has a distribution $\tilde{P}_0(x)$ distinct from that of a normal bond, $P_0(x)$. The DP configurations are the shortest paths that join the two ends A and B; all have the same length $t = 2^k$ on a given lattice. The energy of a DP is simply the sum of energies of all the bonds it traverses.

The hierarchical construction of the lattice enables one to find the ground state of the DP recursively. In particular, the distribution $P_k(x)$ of the ground state energy on successive generations of the normal lattice satisfies the recursion relation [15]

$$\int_x^\infty P_{k+1}(y)dy = \Big(\int_x^\infty dy \int dz\, P_k(z)P_k(y-z)\Big)^b. \tag{3a}$$

A similar relation holds for the ground state energy distribution $\tilde{P}_k(x)$ on the defect lattice,

$$\int_x^\infty \tilde{P}_{k+1}(y)dy = \Big(\int_x^\infty dy \int dz\, \tilde{P}_k(z)\tilde{P}_k(y-z)\Big)\Big(\int_x^\infty dy \int dz\, P_k(z)P_k(y-z)\Big)^{b-1}. \tag{3b}$$

These equations serve as the basis of our numerical investigation of localization on the hierarchical lattice, which is defined by the probability for the ground state to be realized on the defect branch in the limit $k \to \infty$.

Let γ_k and δ_k be the mean and standard deviation of the ground state energy on the normal lattice at generation k, respectively, and $\tilde{\gamma}_k$ the mean ground state energy on the defect lattice. In the limit $\Delta_k \equiv \gamma_k - \tilde{\gamma}_k \ll \delta_k$ or $u_k \equiv \Delta_k/\delta_k \ll 1$, it is possible to show that, if $\tilde{P}_k(x) = P_k(x + \Delta_k)$, then $\tilde{P}_{k+1}(x) = P_{k+1}(x + 2\Delta_k/b) + O(u_k^2)$. Hence we have

$$u_{k+1} = 2^{1-\omega}b^{-1}u_k + cu_k^2 + O(u_k^3) \equiv \phi(u_k), \tag{4}$$

where $\delta_{k+1} = 2^\omega \delta_k$ is used [15]. The first term on the right-hand side of (4) is easily identified as the result of first-order perturbation theory where one ignores possible change of the ground state configuration, and hence it gives an underestimate to u_{k+1} (or equivalently, an overestimate to $\tilde{\gamma}_{k+1}$). The coefficient c of the (postulated) second term should thus be positive. Away from the perturbative regime, knowing u_k alone may not be sufficient to determine u_{k+1}. In such a situation the full distribution $\tilde{P}_k$ should in principle be considered for renormalization. However, close to an unstable fixed point, a unique mapping $u_{k+1} = \phi(u_k)$ is fixed on the fastest growing direction. Near a stable fixed point the direction with slowest convergence plays a similar role. Our numerical investigation of (3) supports Eq. (4) and the existence of a well-defined one-dimensional map after irrelevant parameters of the distribution $\tilde{P}_k$ have been iterated away.

On the basis of (4), one can distinguish three generic situations as illustrated in Fig. 2: (i) $\phi'(0) > 1$ [Fig. 2(a)], the fixed point $u_0^* = 0$ is unstable; (ii) $\phi'(0) = 1$ [Fig. 2(b)], $u_0^* = 0$ is marginally (un)stable; (iii) $0 < \phi'(0) < 1$ [Fig. 2(c)], $u_0^* = 0$ is stable. In case (i) a DP in an attractive potential is always localized. At a vanishing well depth Δ, there is a diverging correlation length $t_c \sim \Delta^{-\nu_\parallel}$ with the exponent $\nu_\parallel = \ln 2/\ln \phi'(0)$. On the repulsive side, one expects a stable fixed point at u_1^*. Inter-

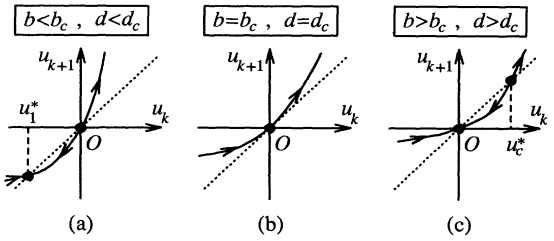

FIG. 2. Three types of generic flow diagrams describing the localization transition from the map $u_{k+1} = \phi(u_k)$ (shown by solid lines). Dashed lines give $u_{k+1} = u_k$.

estingly, the existence of such a fixed point implies that any repulsive potential renormalizes to the same strength $u_1^* \delta E(t)$ on sufficiently large length scales t. In case (iii), there is a localization transition at a critical depth $\Delta_c > 0$. On the localized side, $t_c \sim (\Delta - \Delta_c)^{-\nu_\parallel}$ with $\nu_\parallel = \ln 2 / \ln \phi'(u_c^*)$, where $u_c^* > 0$ is the unstable fixed point of the map as shown in Fig. 2(c). For $u_c^* \ll 1$, a map of the form (4) yields $\nu_\parallel^{-1} = -\ln \phi'(0)/\ln 2 + O(u_c^{*2})$. In the marginal case (ii) with $\phi''(0) > 0$, the fixed point $u_0^* = 0$ is stable on the repulsive side but unstable on the attractive side. Integrating Eq. (4) yields

$$u_k \simeq u_0/(1 - cu_0 k) = u_0/(1 - cu_0 \ln t/\ln 2). \quad (5)$$

Thus on the repulsive side $u_k \to -c^{-1}(\ln 2/\ln t)$ as $t = 2^k \to \infty$ while on the attractive side it reaches a value of order 1 at $t = t_c \sim 2^{1/cu_0}$. On the hierarchical lattice, the marginal situation occurs at $b_c = 2^{1-\omega(b_c)} \simeq 1.57$. For $b < b_c$ one is in case (i) while for $b > b_c$ one is in case (iii).

On a d-dimensional lattice, a result analogous to (4) holds. In the absence of the potential V and after averaging over different realizations of the disorder, the probability for the two ends of a DP of length t to be within a distance a in $\mathbf{x}_\perp$ space is of the order of $[a/\xi(t)]^{d-n} = (t_0/t)^{\zeta(d-n)}$, where we have assumed $\xi(t_0) = a < \xi(t)$. Integrating this probability yields the following first-order estimate for the shift of the ground state energy,

$$\Delta(t) \simeq t_0 \Delta[(t/t_0)^{1-\zeta(d-n)} + O(1)]. \quad (6)$$

In terms of $u(t)$, Eq. (6) leads to, for $\zeta(d-n) < 1$,

$$u(2t) = 2^{1-\omega-\zeta(d-n)} u(t) + cu(t)^2 + O[u(t)^3], \quad (7)$$

where as before the coefficient c of the (postulated) quadratic term is taken to be positive on the premise that Eq. (6) is a lower bound for $\Delta(t)$. That $u(2t)$ can be written as a function of $u(t)$ is a statement of the RG. Equation (7) sets the critical dimension d_c for the relevance of a weak potential at $1 - \omega - \zeta(d_c - n) = 0$. Using the values [1,17] $\zeta = \frac{2}{3}$ at $d = 2$, $\zeta \simeq 0.620$ at $d = 3$, and the scaling relation $\omega = 2\zeta - 1$, one obtains $d_c(1) = 2$ for a line potential ($n = 1$) and $3 < d_c(2) < 4$ for a plane potential ($n = 2$). For $d < d_c(n)$, the exponents $\nu_\parallel$ and $\nu_\perp$ characterizing the diverging behavior $t_c \sim \Delta^{-\nu_\parallel}$ and $\xi_c \sim \Delta^{-\nu_\perp}$ are given exactly by (see also Ref. [14])

$$\nu_\parallel = \frac{1}{1 - \omega - \zeta(d-n)}, \quad \nu_\perp = \frac{\zeta}{1 - \omega - \zeta(d-n)}, \quad (8)$$

where we have used the scaling relation (2).

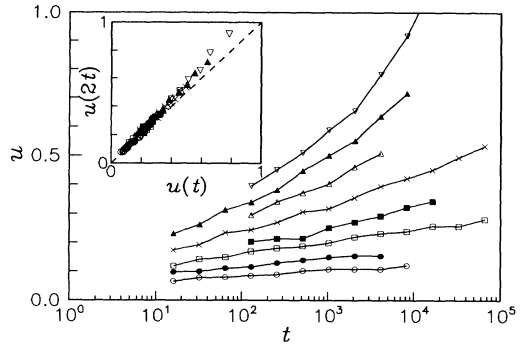

FIG. 3. Scaled height shift $u(t)$ versus t for the inhomogeneous single-step model ($d = 2$, $n = 1$). Data at the same $\tilde{p}$ are connected by a line. Inset: the map $u(2t) = \phi[u(t)]$ constructed using the data shown. Dashed line shows the predicted slope 1.

We have studied the DP localization by a line potential in $d = 2$ and 3 and by a plane potential in $d = 3$ numerically in the context of inhomogeneous surface growth [6]. The reason for adopting such an approach is purely computational. Simulations were performed on the single-step model [18] for $d = 2$ and on the hypercube-stacking model [17] for $d = 3$ using multisite coding techniques. A line potential is introduced by making the growth rate $\tilde{p}$ at a particular column different from the value p for the rest of the system. A plane potential is realized on a row of defect lines. It has been shown that irreversible growth of these surface models is completely equivalent to transfer matrix calculations on suitable lattice versions of the DP model at zero temperature [19].

The length t of the DP, measured in units of updates per site in the growth model, is typically between 10^3 to 10^5 in our simulations. By choosing a system of linear dimension $L \gg t^\zeta$, finite-size effects in the transverse direction can be eliminated. To suppress fluctuations in $\Delta(t)$ (which corresponds to the height shift of the defect column in the growth model), we have used a scheme similar to the idea of damage spreading [20] by performing two simulations in the same bulk disorder η, one with the extra line or plane potential and one without. For a line potential, the simulation is repeated for 1000 to 2000 realizations of η to reduce the error bar on $\Delta(t)$ to within (3–4)%. In the case of a plane potential, averaging along the plane reduces the fluctuation in $\Delta(t)$ significantly, so that fewer realizations are necessary.

Figure 3 shows $u(t) \equiv \Delta(t)/\delta(t)$ against t from simulations of the $d = 2$ single-step model at $p = \frac{1}{2}$ and (from

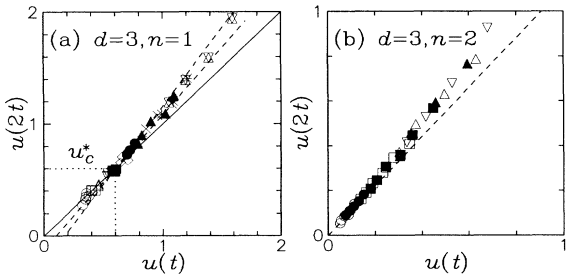

FIG. 4. $u(2t)$ versus $u(t)$ from simulations of the hypercube-stacking model at $p = \frac{1}{2}$ and a varying set of $\tilde{p} < p$. In each case the same plotting symbol is used for data at a given $\tilde{p}$. (a) Columnar defect. Lines of slope 1 (solid) and 1.2 and 1.4 (dashed) are drawn. (b) Planar defect. Dashed line has the predicted slope 1.1.

bottom to top) a decreasing set of $\tilde{p}$ between 0.47 and 0.4. (The case $\tilde{p} < p$ corresponds to an attractive potential.) Here $\delta(t)$ is the root-mean-square fluctuation of the surface at time t. In all cases $u(t)$ grows (on average) with t, indicating the absence of a transition. The inset shows $u(2t)$ against $u(t)$, yielding a data collapse consistent with Eq. (7). The coefficient c is approximately 0.24. The extremely slow increase of $u(t)$ for $\tilde{p}$ close to p may account for the spurious diverging localization length at a $\tilde{p} < p$ as reported in previous numerical studies [2,11]. Take, for example, the bottom-most curve at $\tilde{p} = 0.47$ which passes $u \simeq 0.085$ at $t = 100$. To reach $u(t_c) = 1$, the length of the DP should be of the order of $t_c \sim 100 \times 2^{1/0.085c} \simeq 10^{17}$. The corresponding transverse localization length reaches $\xi_c = t_c^{\zeta} \sim 10^{10}$ at this strength of the line potential.

Figures 4(a) and 4(b) show $u(2t)$ against $u(t)$ for attractive line ($n = 1$) and plane ($n = 2$) potentials, respectively. Again p is fixed at $\frac{1}{2}$. In the case $n = 1$, we found that $\Delta(t)$ saturates to a finite value before t reaches 4096 for $\tilde{p} \geq 0.35$, while for $\tilde{p} \leq 0.20$ it grows linearly with t after a transient period. The data shown in Fig. 4(a) are consistent with an unstable fixed point at $u_c^* \simeq 0.6$, which corresponds to a localization transition at $\tilde{p}_c \simeq 0.29 \pm 0.01$. The slope $\phi'(u_c^*)$ of the data set at u_c^* is between 1.2 and 1.4. This gives $\nu_{\|} = 2.9 \pm 0.9$ and $\nu_{\perp} = \zeta\nu_{\|} = 1.8 \pm 0.6$. In the case $n = 2$, $u(t)$ increases with t for all values of $\tilde{p}$ investigated up to $\tilde{p} = 0.48$, indicating localization at any $\tilde{p} < p$. The slope of the data set shown in Fig. 4(b) close to the origin is consistent with the predicted value $2^{1-\omega-\zeta} = 1.10$. Thus the exponents take the predicted values $\nu_{\|} = 7.14$ and $\nu_{\perp} = \zeta\nu_{\|} = 4.43$ as given by (8).

To conclude, we have shown that a RG scheme based on phenomenological scaling ideas gives a simple description of DP localization by an extended defect in the presence of bulk disorder. The validity and working hypotheses of the method have been demonstrated on the hierarchical lattice. For an attractive line potential in two dimensions, our simulations have shown that the DP is localized at any finite depth of the potential well. In three dimensions, an attractive planar defect of any strength localizes the DP, while an attractive columnar defect does so only when the potential it provides is sufficiently strong.

We wish to thank J. Krug, T. Nattermann, and D. E. Wolf for useful discussions. One of us (L.T.) gratefully acknowledges the use of computing facilities at Forschungszentrum Jülich. The work is supported in part by the Deutsche Forschungsgemeinschaft under SFB 341.

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