

Using  $b$  Tagging to Detect  $t\bar{t}$ -Higgs-Boson Production with  $H \rightarrow b\bar{b}$ J. Dai,<sup>1</sup> J. F. Gunion,<sup>2</sup> and R. Vega<sup>3</sup><sup>1</sup>*Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855*<sup>2</sup>*Davis Institute for High Energy Physics, Department of Physics, University of California at Davis, Davis, California 95616*<sup>3</sup>*Department of Physics, Southern Methodist University, Dallas, Texas 75275**and Stanford Linear Accelerator Center, Stanford, California 94305*

(Received 18 June 1993; revised manuscript received 27 September 1993)

We demonstrate that expected efficiencies and purities for  $b$  tagging at the detectors at the Superconducting Super Collider and the CERN Large Hadron Collider may allow detection of the standard model Higgs boson in  $t\bar{t}H$  production, with  $H \rightarrow b\bar{b}$  decay, for  $80 \lesssim m_H \lesssim 130$  GeV, provided  $m_t \gtrsim 130$  GeV.

PACS numbers: 13.85.Qk, 14.80.Gt

Understanding the Higgs sector is one of the fundamental missions of future high energy colliders such as the Superconducting Super Collider (SSC) and the CERN Large Hadron Collider (LHC). However, options for detection of the standard model (SM) Higgs boson  $H$  are limited if  $m_H$  lies in the low end of the intermediate mass range,  $80 \lesssim m_H \lesssim 130$  GeV, i.e., below the region for which the gold-plated  $H \rightarrow Z^*Z \rightarrow 4l$  is robustly viable [1] but above the guaranteed mass reach of the CERN  $e^+e^-$  collider LEP-II. The most discussed detection modes in this region rely on the rare  $\gamma\gamma$  decay channel of the  $H$  [1,2]. Clearly, the establishment of viable techniques for detection of the  $H$  in its main decay mode in this mass region,  $H \rightarrow b\bar{b}$ , would be highly desirable. In this Letter, we demonstrate that if the top quark is not too light, then expected  $b$ -tagging efficiencies and purities at the SSC and the LHC may enable one to obtain viable  $H$  signals in the  $b\bar{b}$  decay mode when the  $H$  is produced in association with  $t\bar{t}$ .

*A priori*, many possible procedures can be envisioned for detection of  $t\bar{t}H$  events. Of these, we have examined the four which appear to be most worthy of investigation. In (1) we require that both  $t$ 's decay to  $lvb$  and search for the  $H$  as a peak in the two-jet ( $2j$ ) mass spectrum of  $ll4j$  final states without  $b$  tagging [3]. In (2)-(4) we require that three or four  $b$ 's be tagged and look for a peak in the  $2b$  mass spectrum. If only two  $b$ 's are tagged, there is little gain against the enormous background from  $t\bar{t}$  related processes which automatically yield two  $b$ 's in the final state. By requiring that three or more  $b$ 's be tagged, such backgrounds can be reduced to the same level as the irreducible background from  $t\bar{t}b\bar{b}$  production. In procedure (2) we require that both  $t$ 's decay to  $lvb$  and tag three  $b$  jets. In procedures (3) and (4), we only require that one (or both) of the  $t$ 's decay to  $lvb$ , and tag three or four  $b$  jets, respectively. In comparing (3) and (4) we shall see that tagging four (or more)  $b$ 's may prove preferable. The event rate is reduced, but the signals are cleaner and statistically not that much worse.

Regarding procedure (1), in which no  $b$ 's are tagged, it is absolutely necessary to specifically demand four energetic jets in the final state in order to eliminate  $t\bar{t}$  and  $t\bar{t}g$  backgrounds. The major backgrounds are then  $t\bar{t}gg$  and  $t\bar{t}q\bar{q}$  ( $q = u, d, s, c, b$ ). An exact computation of  $t\bar{t}gg$  is not

currently available. In our study, we computed with exact matrix elements the  $t\bar{t}H$  signal, and the  $t\bar{t}Z$  and  $t\bar{t}q\bar{q}$  backgrounds. For the cuts of Ref. [3], after multiplying by an approximate QCD correction  $K$  factor of 1.6 the resulting signal for  $m_H = 100$  GeV is of only marginal significance ( $S/\sqrt{B} \sim 4$ ), even though the much larger  $t\bar{t}gg$  background was not included. Thus, our conclusions for the  $2l4j$  mode are considerably more pessimistic than those of Ref. [3].

The only means for suppressing the large  $t\bar{t}$  backgrounds is to employ multiple  $b$  tagging. Indeed, if four  $b$ 's are tagged, only the irreducible backgrounds from the  $t\bar{t}Z$  (with  $Z \rightarrow b\bar{b}$ ) and  $t\bar{t}b\bar{b}$  processes would remain for a sufficiently high purity of  $b$  tagging. In choosing canonical values for the  $b$ -tagging efficiency and purity we have been guided by the results obtained by the Solenoidal Detector Collaboration (SDC) [4].

We now outline our procedure at the parton level. To trigger on the events of interest, we require that at least one of the  $t$  quarks decay to an isolated lepton ( $e$  or  $\mu$ ). At the parton level, the trigger lepton is required to have  $p_T \geq 20$  GeV and  $|\eta| < 2.5$  and to be isolated by  $\Delta R \geq 0.3$  from all other jets or leptons. The  $W$  from the other  $t$  quark is allowed to decay either leptonically or hadronically. If both  $W$ 's decay leptonically, then we allow for either of the leptons to provide the required isolated lepton trigger. Next, any  $b$  jet with  $|\eta| < 2$  and  $p_T > 30$  GeV is assumed to have a probability of 30% (independent of  $p_T$ ) to be vertex tagged, provided there is no other vertex within  $\Delta R_V = 0.5$ . The probability to tag (i.e., misidentify) a light quark or gluon jet as a  $b$  jet under these same conditions is assumed to be 1%, while that of mistagging a  $c$  jet is taken to be 5%. Jets with  $p_T$  below 30 GeV are assumed not to be tagged. Finally, in order for a  $b$  jet to be included in our invariant mass distributions we require that it be separated from all other jets (including other  $b$ 's) by at least  $\Delta R_C = 0.7$ . This is experimentally possible since the vertex of the  $b$  jet will be visible and a neighboring jet will be apparent via its energetic charged tracks that do not track to the tagged  $b$ -jet vertex.

Of course, it is quite possible that our assumptions regarding  $b$ -tagging efficiency and purity are too conservative. We shall also demonstrate how much easier  $H$

detection becomes if the  $b$ -tagging probability can be brought up to 40% while decreasing the misidentification probability to 0.5% over the stated kinematic range.

The final critical ingredient in our analysis is the mass resolution that can be achieved for the combined mass of two  $b$  jets. For a purely hadronic jet the SDC Technical Design Report [4] quotes energy resolutions (depending upon design and integration time) that are typically no worse than  $\delta E/E = 0.5/\sqrt{E}(\text{GeV}) + 0.03$  (leading to roughly an 8% jet-pair invariant mass resolution for 30 GeV jets). For leptons the energy resolution is typically of order  $\delta E/E = 0.2/\sqrt{E}(\text{GeV}) + 0.01$ . The first step of our parton-level analysis is to explicitly smear the energies of all leptons and jets using these resolutions. In particular, our  $m(b\bar{b})$  distributions will automatically reflect the jet energy resolution. Since  $b$  jets have hard fragmentation functions the above hadronic resolution may be somewhat pessimistic for purely hadronic  $b$  decays. However, semileptonic  $b$  decays will have worse resolution. For the result to be presented here, we have used standard hadronic resolutions as a compromise.

Two distinct more detailed approaches to the semileptonic  $b$  decays are possible. First, it might be experimentally possible to greatly limit the presence of such decays by requiring that a  $b$  vertex not have an associated lepton with energy larger than some appropriate lower bound. Our results would then be unaltered if the  $b$ -tagging efficiency we employ is reinterpreted as including the probability for a purely hadronic primary  $b$  decay. Alternatively, semileptonic decays can be explicitly included in the analysis. We have done this without correcting for the average missing neutrino momentum, i.e., by using only the combined  $c$  and  $l$  momenta in constructing the  $b\bar{b}$  mass for decays of this type. Keeping all other efficiencies constant, one finds roughly a doubling in the times (quoted later) required to see a signal of a given statistical significance. However, if  $b \rightarrow cl\nu$  decays are retained, at least 5% would be added to the (30–40)% tagging efficiency by explicitly looking for the associated lepton [5]. In the case of  $4b$  tagging, this would restore much of the statistical significance obtained prior to including  $b \rightarrow cl\nu$  decays. This type of analysis will be pursued at greater length elsewhere [6].

We have employed the SM predictions for the  $H \rightarrow b\bar{b}$  branching ratio. At  $m_H = 80, 100, 120,$  and  $140$  GeV we find  $B(H \rightarrow b\bar{b}) = 0.857, 0.842, 0.748,$  and  $0.437$ , respectively. Not surprisingly, we will find that by  $m_H = 140$  GeV the branching ratio has fallen sufficiently that the  $H$  detection in the  $b\bar{b}$  mode becomes fairly difficult.

We use exact matrix element calculations for the signal reaction  $t\bar{t}H$  [7] (with  $H \rightarrow b\bar{b}$ ), the irreducible backgrounds  $t\bar{t}b\bar{b}$  and  $t\bar{t}Z$  (with  $Z \rightarrow b\bar{b}$ ), and the reducible backgrounds  $t\bar{t}c\bar{c}$  (with one or two  $c$ 's mistagged as a  $b$ ) and  $t\bar{t}(g)$  (with one or two of the non- $b$  jets mistagged as a  $b$ ). For all reactions we have included correlations in the three-body decays of the top quarks. This turns out to be important for the  $t\bar{t}(g)$  background in which one or

more of the mistagged jets comes from the decay of a  $W$ . Uncorrelated decays would lead to a roughly 25% overestimate of this background.

All the production reactions we consider are dominated by  $gg$  collisions. We have employed distribution functions for the gluons evaluated at a momentum transfer scale given by the subprocess energy. It is well known that QCD corrections are substantial for  $gg$  initiated processes. For example, for the  $gg \rightarrow t\bar{t}$  process the QCD correction “ $K$ ” factor has been found to be of the order of 1.6 for our choice of scale [8]. Thus, rates for the  $t\bar{t}H$ ,  $t\bar{t}Z$ ,  $t\bar{t}b\bar{b}$ , and  $t\bar{t}c\bar{c}$  processes have been multiplied by a QCD correction factor of 1.6. In the case of  $t\bar{t}(g)$  we have incorporated the  $K$  factor as follows. We have generated events without an extra gluon ( $t\bar{t}$  events) and have also generated events with an extra gluon ( $t\bar{t}g$  events) requiring that the  $p_T$  of the extra gluon be  $> 30$  GeV. For this cutoff one finds  $\sigma(t\bar{t}g) \sim 0.6\sigma(t\bar{t})$ . Thus, if the two event rates are added together without cuts an effective  $K$  factor of 1.6 is generated. Explicitly allowing for an appropriate number of  $t\bar{t}g$  events is important in properly estimating the background from this source due to mistagged non- $b$  jets. Our procedure should yield an upper limit on the number of events with an extra gluon having  $p_T > 30$  GeV and therefore potentially vertex taggable. The gluon distribution functions we have employed are those of Harriman, Martin, Stirling, and Roberts [9]. Results for other distribution function choices will be presented in Ref. [6].

In our analysis we consider all pairs of tagged  $b$  jets (or mistagged non- $b$  jets) as potentially coming from the  $H$  decay. Consequently, even the signal process has a combinatoric background arising from pairs in which one or both of the  $b$ 's come from  $t$  decays. The event rate for the  $H$  signal is obtained by subtracting this combinatoric background from the  $H$  peak in the  $m(b\bar{b})$  mass distribution. The background rate we employ includes this signal combinatoric background as well as the full backgrounds (including all combinatorics) for the true irreducible and reducible backgrounds. The number of  $b\bar{b}$  pairs that are included in the combinatoric backgrounds is determined on an event basis according to the appropriate probabilities for tagging or mistagging  $b$  or non- $b$  jets, respectively.

We shall present results for two  $b$ -tagging scenarios specified by giving (i) the efficiency for tagging a  $b$  jet,  $e_{b\text{-tag}}$ ; and (ii) the probability of mistagging a light quark or gluon jet,  $e_{\text{mistag}}$ . In case (a) [(b)], we take  $(e_{b\text{-tag}}, e_{\text{mistag}}) = (30\%, 1\%)$  [(40%, 0.5%)]. In both cases we have taken the probability of mistagging a  $c$  jet as being 5%. (In fact, for mistagging probabilities of this general level the  $t\bar{t}c\bar{c}$  background is not significant.) Case (a) we regard as a lower bound, and case (b) we think is quite achievable. For further comparison, we shall also give SSC results at  $m_t = 140$  GeV obtained using the  $p_T$ -dependent  $e_{b\text{-tag}}$  values of Ref. [4] for  $p_T > 20$  GeV, for  $e_{\text{mistag}} = 0.5\%, 1\%,$  and  $1.5\%$ .

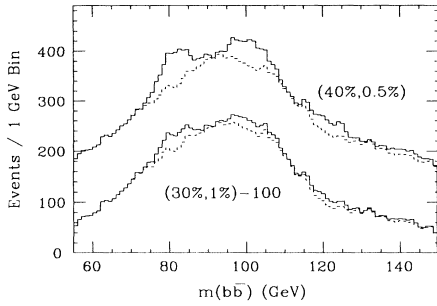


FIG. 1. Events per 1 GeV bin in  $m(b\bar{b})$  for signal plus background (solid) compared to background alone (dashes). Results for the SSC with  $L = 10 \text{ fb}^{-1}$ , and  $b$ -tagging efficiency and purity given by  $(e_{b\text{-tag}}, e_{\text{mistag}}) = (30\%, 1\%)$  (lower histograms) and  $(40\%, 0.5\%)$  (upper histograms), are shown for  $3b$  tagging. Note that the lower histograms for the former case have been shifted downwards by 100 events per bin in order to clarify the display. The Higgs-boson signals at  $m(b\bar{b}) = 80, 100, 120,$  and  $140 \text{ GeV}$  are displayed after removing the combinatoric background from the  $t\bar{t}H$  reaction itself. See discussion in the text for details.

In this Letter, we present results for procedures (3) and (4) in which only one lepton is required for triggering and either three (or more) or four (or more), respectively, jets must be tagged as  $b$  jets. Procedure (2), in which two leptons are triggered on and three jets are required to be tagged as  $b$ 's, yields smaller statistical significance for the Higgs-boson signals than procedures (3) and (4).

In order to give a rough idea of the levels of the Higgs-boson signals and the various backgrounds we first present plots of event rates as a function of  $m(b\bar{b})$  in Figs. 1 and 2, for procedures (3) and (4), respectively, at  $m_t = 140 \text{ GeV}$ . In each figure, the two  $b$ -tagging  $(e_{b\text{-tag}}, e_{\text{mistag}})$  cases of  $(30\%, 1\%)$  and  $(40\%, 0.5\%)$  are compared. All jet energies have been smeared using the resolution quoted earlier. Results presented are for the SSC with integrated luminosity of  $L = 10 \text{ fb}^{-1}$ . Since the combinatoric background from the  $t\bar{t}H$  reaction itself is  $m_H$  dependent, we have adopted the following procedure for displaying all the Higgs-boson mass peaks on one plot. For each  $m_H$  case, we have taken only the bins that lie

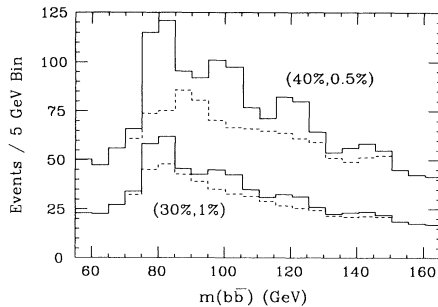


FIG. 2. As in Fig. 1, except for  $4b$  tagging. Event numbers are given per 5 GeV bin.

within  $\pm 10 \text{ GeV}$  of the Higgs-boson peak in the  $m(b\bar{b})$  distribution. From the event numbers in each of these central bins we have subtracted an approximate combinatoric background determined by averaging the distribution values for a representative set of bins immediately below and immediately above the central bins. The remainder in each of the central bins is then added to the event number distributions coming from the sum of all other processes. Thus, the upper (solid) histogram corresponds to the sum of event rates given by

$$[N(t\bar{t}H) - N(t\bar{t}H)_{\text{comb}}] + N(t\bar{t}Z) + N(t\bar{t}) + N(t\bar{t}g) + N(t\bar{t}b\bar{b}). \quad (1)$$

$N(t\bar{t}H)_{\text{comb}}$  is roughly  $\frac{1}{4}$  of the average value of  $N(t\bar{t}H) - N(t\bar{t}H)_{\text{comb}}$  in the bins within  $\pm 5 \text{ GeV}$  of the Higgs-boson peak. Also histogrammed is the event rate for the  $t\bar{t}Z + t\bar{t} + t\bar{t}g + t\bar{t}b\bar{b}$  background. Additional graphs of signal and individual backgrounds as a function of  $m(b\bar{b})$  will appear elsewhere [6]. For now, we note that similar plots for  $m_t = 180$  exhibit quite dramatic Higgs-boson peaks for all but  $m_H = 140 \text{ GeV}$ .

In order to quantify the observability of the Higgs-boson signals, such as those illustrated in Figs. 1 and 2, we have computed the number of SSC years required for a  $5\sigma$  significance of the signal, defined by  $S \equiv N(t\bar{t}H) - N(t\bar{t}H)_{\text{comb}}$ , summed over bins within  $\pm 5 \text{ GeV}$  of a given Higgs-boson mass peak. The background at each value of  $m_H$  is computed as  $B \equiv N(t\bar{t}H)_{\text{comb}} + N(t\bar{t}Z) + N(t\bar{t}) + N(t\bar{t}g) + N(t\bar{t}b\bar{b})$  summed over these same central bins. Results for the SSC are given in Table I, for  $b$ -tagging scenarios (a) and (b) and both  $3b$  and  $4b$  tagging. Also given (in parentheses) is the associated number of signal events ( $S$ ). The associated number of background events ( $B$ ) can be obtained from the relation  $B = S^2/25$ . So that trends can be clearly illustrated, we have not made an arbitrary cutoff in the number of years allowed for our entries. In addition, it should be kept in

TABLE I. Number of  $10\text{-fb}^{-1}$  years (signal event rate) at the SSC required for a  $5\sigma$  confidence level signal in four cases: (I),(II)  $3b$  tagging with  $(e_{b\text{-tag}}, e_{\text{mistag}}) = (30\%, 1\%), (40\%, 0.5\%)$ ; and (III),(IV)  $4b$  tagging with  $(e_{b\text{-tag}}, e_{\text{mistag}}) = (30\%, 1\%), (40\%, 0.5\%)$ .  $m_t$  and  $m_H$  are in GeV.

Case	$m_t \backslash m_H$	80	100	120	140
I	110	2.1(331)	3.5(411)	6.0(458)	29.1(845)
	140	1.4(324)	3.0(512)	4.5(486)	22.4(954)
	180	0.3(96)	0.8(175)	2.9(353)	13.6(747)
II	110	0.5(191)	1.0(249)	1.6(275)	7.6(522)
	140	0.3(170)	0.6(243)	1.1(266)	5.3(503)
	180	0.1(48)	0.2(97)	0.6(171)	3.1(375)
III	110	10.8(135)	15.9(144)	33.0(186)	170.0(386)
	140	3.5(95)	5.0(97)	10.4(122)	55.0(247)
	180	1.0(39)	1.7(45)	4.2(68)	25.5(160)
IV	110	2.4(94)	4.1(116)	8.6(153)	46.0(330)
	140	0.6(54)	1.1(66)	2.6(96)	13.3(189)
	180	0.2(23)	0.4(33)	0.9(46)	5.5(110)

mind that (at least at the SSC) the ultimate luminosity that can be achieved and managed by the detectors might be much larger than currently assumed.

From the table it is immediately apparent that for the two larger  $t$  masses, especially for  $m_t=180$  GeV, detection of the  $H$  using  $b$  tagging should be possible within a few SSC years for the more optimistic scenarios, so long as  $m_H \lesssim 130$  GeV. By  $m_H \gtrsim 140$  GeV the  $H \rightarrow b\bar{b}$  branching ratio has dropped to too small a value due to the onset of the  $WW^*$  decay modes. The  $m_t=110$  GeV entries illustrate the fact that  $H$  detection using  $b$  tagging will be difficult for lighter top quark masses. The  $t\bar{t}(g)$  background is much larger and the signal rates somewhat smaller than for the larger  $m_t$  values considered. We estimate that  $m_t=130$  GeV is the boundary below which detection difficulty increases dramatically. On a statistical basis,  $3b$  tagging is superior to  $4b$  tagging. But this is only true if the background shapes can be reliably simulated by Monte Carlo techniques. In the absence of a reliable prediction for the background shape,  $4b$  tagging could prove the preferable procedure. Of course, the  $3b$  tagging and  $4b$  tagging procedures are complementary and the data should be analyzed in both ways.

For canonical luminosities the LHC is somewhat superior to the SSC for a given scenario. For instance, the case I results for  $\sqrt{s}=16$  TeV and integrated luminosity of  $L=100 \text{ fb}^{-1}$  at  $m_t=140$  GeV are No. of years ( $S$ )=1.0 (402), 2.4 (669), and 4.6 (706), for  $m_H=80, 100,$  and  $120$  GeV, respectively. However, the efficiency of  $b$  tagging at the LHC, given the many overlapping events expected, may not be as great as assumed here. Overlapping events will also reduce the probability that the  $b$ 's from the Higgs-boson decay in a signal event will be isolated by  $\Delta R_C$  from other jets.

The rapid worsening of  $H$  detectability in the  $t\bar{t}b\bar{b}$  channel as we move from optimistic to pessimistic ( $e_{b\text{-tag}}, e_{\text{mistag}}$ ) scenarios illustrates the importance of having as good a vertex tagging capability as possible at the SSC and LHC. Ability to employ a smaller minimum value of  $\Delta R_C$  (for which there is good separation of the calorimetric energy of a  $b$  jet from that of a neighboring jet) would also be very beneficial.

To further define the  $b$ -tagging efficiency and purity that will be required for observable signals, we also give results obtained by employing the  $p_T$ -dependent  $e_{b\text{-tag}}$  values of Ref. [4] down to  $p_T > 20$  GeV. These  $e_{b\text{-tag}}$  values are somewhat lower than 30% for  $p_T \lesssim 60$  GeV, but reach  $\sim 40\%$  for  $p_T \gtrsim 120$  GeV. The numbers of SSC years required for a  $5\sigma$  signal at  $m_t=140$  GeV for  $e_{\text{mistag}}=(0.5, 1, 1.5)\%$ , and for  $m_H=80, 100, 120,$  and  $140$  GeV, respectively, are (0.8, 1.3, 1.9), (1.8, 3.1, 4.5), (3.3, 5.4, 7.5), and (12.1, 20.0, 29.8) for  $3b$  tagging, and (1.8, 3.0, 5.1), (2.8, 4.0, 6.1), (6.5, 8.2, 11.0), and (39.3, 48.3, 63.4) for  $4b$  tagging. Comparing the  $e_{\text{mistag}}=1\%$  numbers to those in Table I (rows I and III) for the uniform  $e_{b\text{-tag}}=0.3$  value, we see little change for  $3b$  tagging and some improvement for  $4b$  tagging. The larger  $p_T$

range and higher  $e_{b\text{-tag}}$  values at large  $p_T$  at the very least compensate for the smaller  $e_{b\text{-tag}}$  values at moderate  $p_T$ . Comparing results for the three  $e_{\text{mistag}}$  cases shows clearly how critical the achievable  $b$ -tagging purity is. An average  $e_{\text{mistag}}$  value significantly above 1% would make observation quite difficult for  $m_H \gtrsim 100$  GeV.

As a final remark, we note that although the lowest  $m_H$  value studied is 80 GeV, the trends make it clear that  $t\bar{t}H \rightarrow lb\bar{b}b\bar{b}$  detection is, if anything, more viable at still somewhat lower  $m_H$  values. This is in sharp contrast to the  $H \rightarrow \gamma\gamma$  discovery modes which rapidly deteriorate [due to decreasing  $B(H \rightarrow \gamma\gamma)$ ] for  $m_H < 80$  GeV.

In summary, we have demonstrated that at the SSC and LHC expected vertex tagging capabilities for a typical detector should be sufficient to allow  $H$  detection in the  $t\bar{t}H \rightarrow lb\bar{b}b\bar{b}X$  final state for a range of larger  $m_t$  values and moderate  $m_H$ . The precise region of viability depends critically upon the efficiency and purity of  $b$  tagging, but, allowing for several years of running, should extend at least from  $m_H=80$  (and below) to  $m_H \sim 110$  GeV for  $m_t \geq 140$  GeV. For  $m_t \sim 180$  GeV and good efficiency and purity,  $m_H$  values up to  $\sim 130$  GeV can be probed in just one SSC year.

This work has been supported in part by Department of Energy Grants No. DE-FG03-91ER40674 and No. DE-AC03-76SF00515, and by Texas National Research Laboratory Grants No. RGFY93-330 and No. RCFY93-229. J.F.G. is grateful to A. Seiden, B. Hubbard, and S. Wilenbrock for helpful conversations. J.F.G. also benefited from early discussions with T. Weiler. J.D. would like to thank M. Peskin and T. Han for discussions, and D. Wu for explaining the work of Ref. [3].

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