Microwave Frequency Dependence of Integer Quantum Hall Effect: Evidence for Finite-Frequency Scaling

L. W. Engel, D. Shahar, Ç. Kurdak, and D. C. Tsui

Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544

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We present ac measurements of the diagonal conductivity σ_{xx}, in the integer quantum Hall regime for frequency f between 0.2 and 14 GHz and temperature $T \gtrsim 50$ mK. Re(σ_{xx}) was obtained from the measured attenuation of a coplanar transmission line on the sample surface. For $f \gtrsim 1$ GHz, σ_{xx} peaks between IQHE minima broaden as f increases, roughly as $(\Delta B) \propto f^{\gamma}$, where ΔB is the peak width. $\gamma = 0.41 \pm 0.04$ for spin-split peaks, and for spin-degenerate peaks $\gamma = 0.20 \pm 0.05$. We interpret our data in terms of dynamic scaling. Our γ, when compared with existing estimates of the localization length exponent ν, is consistent with assigning a dynamic exponent $z=1$.

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Around integer Landau fillings i the integer quantum Hall effect (IQHE) produces wide regions of magnetic field B, in which the transverse resistivity ρ_{xy} is a plateau with quantized value $h/e^2 i$, and the diagonal resistivity ρ_{xx}, and conductivity σ_{xx} are a flat minimum with ρ_{xx}, σ_{xx} vanishing. At low T these plateau regions of B are separated by comparatively narrow transition regions, where ρ_{xy} rises sharply, and ρ_{xx}, σ_{xx} exhibit peaks. The peaks are a manifestation of the delocalization of the electrons in the 2D electron system (2DES), as B approaches B^*, the field at which a Landau level lies on the Fermi energy. This delocalization has been shown to be a critical phenomenon [1], where B^* is a critical magnetic field, and at $T=0$ the localization length ξ diverges as $B \to B^*$. Quite generally, as criticality is approached, the internal dynamics of the system must slow down [2], so that as ξ diverges, a correlation time, τ_c, that characterizes the dynamics of the system, also diverges. In this work we present broadband microwave ac conductivity measurements that for the first time directly exhibit the slowing down of the internal dynamics as B^* is approached, and allow us to determine the critical exponent associated with the divergence of τ_c.

The effect of the critical behavior of ξ or τ_c on the observable magnetotransport at finite T and finite frequency f is considered in scaling theories, which compare ξ [3] or τ_c [4] with the effective sample size L and an effective measuring time τ_m. ξ, τ_c diverge as $B \to B^*$ according to $\xi \sim |B-B^*|^{-\nu}$, $\tau_c \sim |B-B^*|^{-y} \sim \xi^z$, where ν, y are critical exponents and z is the dynamic exponent. In the scaling picture the conductivity is determined by L/ξ, τ_m/τ_c. The scaling regime is reached when at least one of these quantities approaches unity as $B \to B^*$, producing the finite width ΔB of the σ_{xx} peak. Wei *et al.* [1] studied the dc ΔB as a function of T, and found $\Delta B \propto T^{\kappa}$, where $\kappa \approx 0.42$. This power law in T is evidence for scaling behavior, because varying T has the effect of varying L through the T dependence of the inelastic scattering rate. The inelastic scattering rate goes as T^p, so $L \sim T^{-p/2}$, and the measured T exponent $\kappa = p/2\nu$.

In this paper we report on experimental measurements of ac Re(σ_{xx}) in the IQHE regime, for f between 0.2 and 14 GHz, in the low-wave-vector ($q \to 0$) limit, and T between 50 and 470 mK. The measurements were achieved with a coplanar transmission line that was patterned on the sample surface, and coupled capacitively to the 2DES. The IQHE was clearly observable throughout the f range. The width of transitions between IQHE minima in Re(σ_{xx}) was found to increase roughly as $\Delta B \sim f^{\gamma}$, for $f \gtrsim 1$ GHz and temperature $T \sim 50$ mK. For spin-split Landau levels $\gamma = 0.41 \pm 0.04$. For $f < 1$ GHz, ΔB increases with T. The data on the spin-degenerate transitions, though not presently of such quality as to allow precise measurement of γ, are consistent with γ equal to half that for spin-split transitions. The power-law behavior is interpreted as due to dynamic scaling, where the exponent is related to the τ_c exponent as $\gamma = 1/y$. With an estimated ν of 7/3, our measured γ of 0.41 implies (within experimental accuracy) that the dynamic exponent $z=1$. The smaller f dependence for the spin-degenerate levels, together with previous results on scaling in T-dependent dc transport, is consistent with $z=1$ for spin-degenerate as well as spin-split transitions, and with ν being 14/3 for the spin-degenerate levels.

To our knowledge, transport in the 2DES has not been explored previously over the full f range of our experiments. Other experiments on rf QHE transport have been performed at the fixed microwave frequency of a waveguide system [5,6] or have been confined to frequencies lower than or overlapping the lower part of our f range [7,8]. In the waveguide measurements of Ref. [5], quantized plateaus were observed in σ_{xy} at 33 GHz and T down to 2.2 K. The results of early work [8] on the IQHE at $f \leq 50$ MHz and $T \geq 0.5$ K do not show any clear f dependence of the transport. Measurements [9] of σ_{xx} using surface acoustic wave attenuation and velocity shift found no evidence for systematic f dependence of the IQHE up to 300 MHz. Other, more recent rf measurements on the 2DES in heterojunctions focused mainly on the regimes of the fractional QHE and of the extreme quantum limit [10]. These investigations include finite-wave-vector σ_{xx} measurements using surface acoustic

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waves [11], and some transmission line measurements using techniques different from ours [12].

Our broadband $\mathrm{Re}[\sigma_{xx}(f)]$ data are calculated from the measured transmission loss of a coplanar transmission line on the front surface of the sample, which is a GaAs/$\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ heterojunction grown by liquid phase epitaxy, and has density $n \approx 4.2\times10^{11}$ cm^{-2}, and mobility $\mu \approx 4\times10^4$ cm^2/V sec. The transmission line we use is a coplanar waveguide [13] (CPW) meander line, and is shown in Figs. 1(a) and 1(b). The signal is applied to the narrow center strip, while the large metal areas on each side are grounded. These ground planes minimize coupling between adjacent periods of the meander line, so we can consider the meander line as if it were a straight CPW of total length $d=14$ mm. The long, narrow transmission line coupled to the 2DES allows broadband measurements of the small conductivities typical of the 2DES at large B, and avoids the detrimental effect of interelectrode capacity, which at such high frequencies would complicate 2DES conductivity measurements made through Ohmic contacts or through simpler capacitive coupling arrangements. The measured loss in the 2DES occurs mainly under the slots of width $w \sim 30$ μm, so the experiment is sensitive to $\sigma_{xx}(f)$ in the long-wavelength limit. We placed Ohmic contacts outside the ground planes, and simultaneously measure the dc resistances R_{xx}, R_{xy} and the relative microwave power transmission P.

We measure the loss of the transmission line with a source and detector located ~ 1 cm from the sample. A simplified schematic of our experimental setup is shown in Fig. 1(c). The microwave source is a resistive splitter in a leveling loop [14] fed from a room-temperature signal generator. The detector is an impedance-matched rectifier operated in the square-law regime. Splitter, sample, and detector load into the mixing chamber of a top-loading dilution refrigerator.

The lowest T for which we present data is nominally 50 mK. The T's given in this paper are those of the bath in which the sample was immersed as the data were taken. Varying the microwave input power indicated that the applied microwaves were not causing heating. However, the dc ΔB for IQHE transitions ceased to narrow with decreasing T between 50 and 100 mK, which means that thermal leakage down the coaxial cable (through which the microwaves were fed to the splitter from room T) caused the sample to be hotter than the bath T of 50 mK.

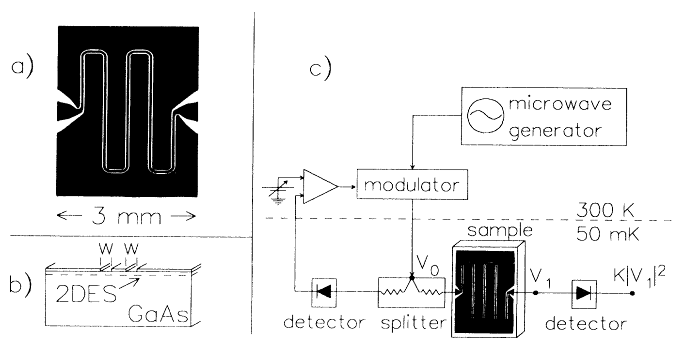

FIG. 1. (a) Top view of coplanar waveguide transmission line, showing metal film only (metal is black). (b) Magnified cross section of sample near the transmission line (not to scale). Slot width $w=30$ μm. (c) Simplified schematic of our experimental setup. Voltage V_0 is fixed by the servo loop, K is the detector sensitivity. Relative transmitted power $P \propto K|V_1/V_0|^2$.

To obtain $\mathrm{Re}(\sigma_{xx})$ from the measured P, we analyze the CPW in the quasi-TEM approximation [15], which is reasonable since the total width of the line is much less than 1/4 the wavelength of microwave radiation at our highest f. The notations Z_0, v_{p0} will throughout indicate respectively the CPW's characteristic impedance and propagation velocity, both evaluated in the limit where $\sigma_{xx} \to 0$. The total width of the CPW relative to the slotwidth w is fixed to give $Z_0 = 50$ Ω, $v_{p0} = 1.16\times10^{10}$ cm/sec. The quasi-TEM analysis of the CPW gives $P = \exp[-\mathrm{Re}(\sigma_{xx})Z_0 d/w]$ when (1) the measuring system produces no reflections at the ends of the line [14], (2) $|\sigma_{xx}| v_{p0} Z_0 / w\pi f \ll 1$, (3) rf current in the 2DES is well confined to the regions under the slots. Condition (3) holds when $(|\sigma_{xx}|/\pi f C_c)^{1/2} \ll w$, where C_c is the capacitance per unit area between the 2DES and the top metal. We obtain $\mathrm{Re}(\sigma_{xx})$ from P, according to $\mathrm{Re}[\sigma_{xx}(f)] = C(f,P) w |\ln P| / Z_0 d$. $C(f,P)$ corrects for the experimental conditions when statements (1)–(3) are not all true. C is largest at low f and large $\mathrm{Re}(\sigma_{xx})$, and is derived from computer simulations of the measuring system, including source and detector reflection, and capacitive coupling to the 2DES. The P dependence of C results from conditions (2) and (3) containing σ_{xx}, which determines P, and does not reflect any nonlinearity in the sample. For all the $\mathrm{Re}(\sigma_{xx})$ data that will be presented below, $1 \le C(f,P) \le 1.4$, and for $\mathrm{Re}(\sigma_{xx})$ at the value used to determine the widths of peaks between IQHE minima, $1 \le C(f,P) \le 1.14$. Reflections are small in our measuring system and result in $\sim \pm 1$ dB oscillations of P with varying f. These oscillations give rise to an f-dependent systematic error of $\sim \pm 25\%$ in the $\mathrm{Re}(\sigma_{xx})$ data that we present. Typically our system is ~ 10 times more sensitive to the real part of σ_{xx} than to the imaginary part.

Figure 2 shows the dc transport coefficients R_{xx}, R_{xy}, and the 14 GHz P, for $T=50$ mK. Odd-integer plateaus with $i>3$ are not observed. IQHE plateau regions correspond to maxima in P since microwave loss increases with σ_{xx}. The $i=1,2,4$ P maxima are all flat, with no apparent microwave loss due to the 2DES, and we use the $i=2$ maximum to calibrate $P \equiv 1$ at each f. The identical maximal P on all the well-developed plateaus indicates that the mode that carries the microwave power through the CPW is unaffected by σ_{xy}. The transitions between the 14 GHz IQHE P maxima are clearly wider than the corresponding transitions in the dc transport traces.

Figure 3 shows the f dependence of $\mathrm{Re}(\sigma_{xx})$ vs B at 50

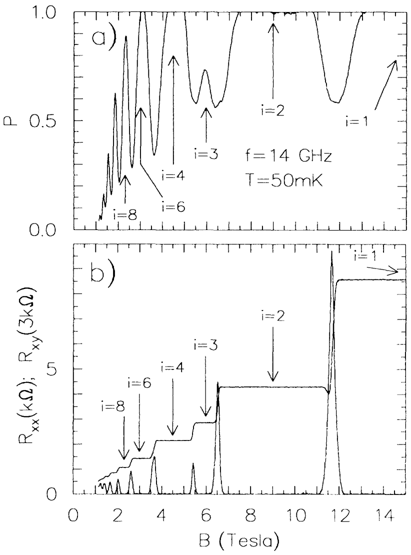

FIG. 2. (a) Microwave relative transmission, $f=14$ GHz, (b) dc resistances, $T=50$ mK.

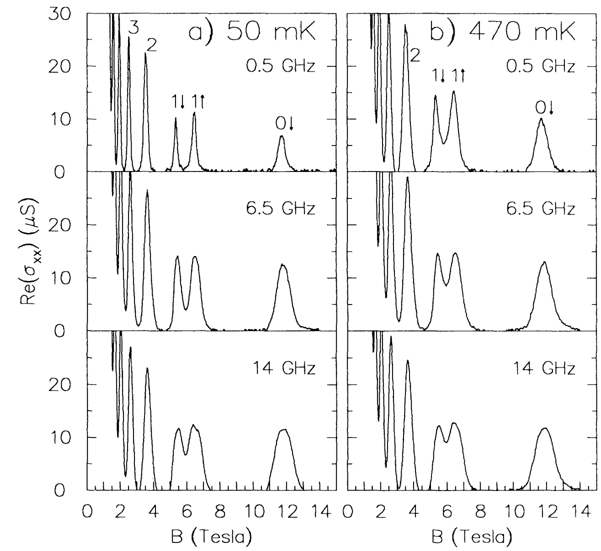

FIG. 3. $\mathrm{Re}(\sigma_{xx})$ vs B at three frequencies and two temperatures. Peaks are marked with Landau level index N and spin.

and 470 mK. We are mainly concerned with the width of the Landau level peaks in $\mathrm{Re}(\sigma_{xx})$. At 50 mK, the peaks of the $N=0\downarrow,1\uparrow,1\downarrow,2,3$ Landau levels ($i=1\rightarrow 2,2\rightarrow 3,3\rightarrow 4,4\rightarrow 6,6\rightarrow 8$ transitions) all broaden with each successively higher f on Fig. 3(a), as is seen most clearly for the $0\downarrow$ peak. (Arrows after Landau orbital quantum numbers N indicate the spin direction, and an N without an arrow refers to a peak in which the spin splitting is not resolved.) The $i=3$ QHE minimum becomes markedly less well developed as f increases. The 470 mK peaks are broader than the corresponding 50 mK peaks at 0.5 GHz, but at the higher f's the widths of the 470 and 50 mK peaks are nearly the same.

Figure 4 summarizes the f dependence of the widths ΔB of $\mathrm{Re}(\sigma_{xx})$ peaks between IQHE minima. The ΔB are measured between the extrema of the derivatives of $\mathrm{Re}(\sigma_{xx})$ vs B. Systematic errors of $\pm 25\%$ in the data are due to reflections in the measuring system. Figure 4(a) shows ΔB vs f at $T=50$ mK for the $\mathrm{Re}(\sigma_{xx})$ peaks for several different Landau levels. The ΔB's increase with f for $f \gtrsim 1$ GHz, but at lower f appear to be f independent. The lines drawn in Fig. 4(a) are least-squares fits to the data with $f \geq 0.97$ GHz to the power law $\Delta B \propto f^{\gamma}$. For the $N=0\downarrow,1\uparrow,1\downarrow$ peaks the fits give $\gamma=0.43$, 0.38, 0.42, with errors of $\pm 10\%$ estimated from scatter. The agreement of γ between the different spin-split Landau levels is good, as is the agreement of our measured γ with the temperature exponent κ measured by Wei *et al.* [1]. Fits to $\Delta B \propto f^{\gamma}$ for the spin-unresolved $N=2,3$ peaks, though rough, are consistent with a spin-unsplit f exponent γ of half that for the spin-split levels, with scatter-estimated error of $\pm 25\%$.

Figure 4(b) shows ΔB vs f for the $0\downarrow$ peaks and $T \approx 50$, 200, and 470 mK. The line drawn in Fig. 4(b) is the same least-squares fit as is shown in Fig. 4(a) for the $f \gtrsim 1$ GHz, 50 mK data. For $f > 6$ GHz, $\Delta B(f)$ is apparently the same at all three T's, and follows the power law $\Delta B \propto f^{\gamma}$. Likewise, at all three T's, ΔB vs f levels off at low f. The ΔB in the low-f limit is T dependent, as is the f at which ΔB begins to increase with f. The data suggest a change at $f \sim k_B T/h$, from an f-determined ΔB at low T, high f, to a T-determined ΔB at low f, high T.

We interpret the low T, high f data, where $\Delta B \sim f^{\gamma}$, as dynamic scaling, with $\tau_m^{-1} \sim 2\pi f$. In this case our measured exponent γ is just $1/y$, where y is the exponent describing the divergence of τ_c as $B \rightarrow B^*$. The agreement of the γ's measured for the three different spin-split Landau levels is consistent with the identification of γ as a universal critical exponent. Reference [1] demonstrates the dependence of the scale of the system on T, so the appearance of the $\Delta B \propto f^{\gamma}$ behavior only above a T-dependent f makes sense in the dynamic scaling interpretation. A theoretical discussion [4] on the interplay of dynamic and length scaling is also consistent with $\Delta B \propto f^{\gamma}$ setting in only for $f \gtrsim k_B T/h$.

Our measured value of the f exponent, $\gamma=0.41 \pm 0.04$ (for spin-split levels) can be combined with existing estimates of the ξ exponent ν to estimate the dynamic exponent z. Theoretical estimates of ν from analytic calculations [16] and numerical simulations [17,18] all result in ν in the range of 2.3 to 2.4. dc transport experiments [19] with small samples were also interpreted as agreeing with this ν. $\nu=7/3$, together with $1/y=\gamma=0.41$ implies that $z=1.0$, within our experimental error of 10%. In a theoretical study [20] of superconductor-insulator phase transitions, $z=1$ was also suggested.

Our ΔB vs f data on IQHE transitions indicate that the critical exponents are not the same for spin-split and

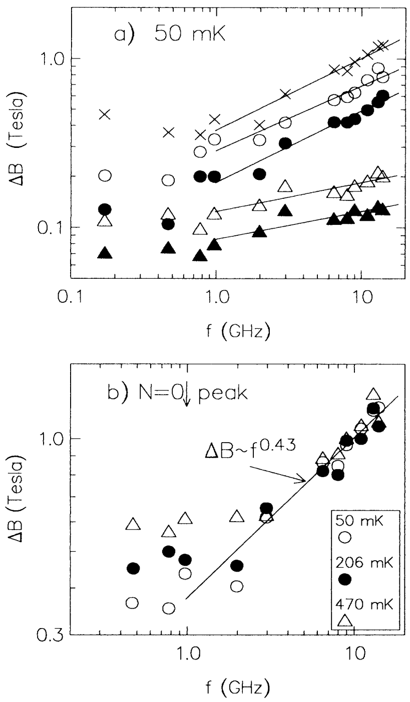

FIG. 4. (a) Peak width between extremal $d\,\mathrm{Re}(\sigma_{xx})/dB$, vs frequency, for three different spin-split Landau levels ($\times, N=0\downarrow$; $\circ$, $N=1\uparrow$; $\bullet$, $N=1\downarrow$) and two spin-degenerate levels ($\triangle$, $N=2$; $\blacktriangle$, $N=3$). $T=50$ mK. ΔB for $N=2$ and $N=3$ are divided by 2 for clarity. Lines are least squares fits to data with $f \geq 0.97$ GHz. (b) Peak width vs frequency at three different temperatures, for $N=0\downarrow$ Landau level.

spin-degenerate levels, a result that can be applied to interpreting dc ΔB vs T data [21] on spin-degenerate levels. For these levels the T exponent κ is 0.21, half that for spin-split transitions. Given $\kappa = p/2\nu$, the different κ's can be explained by either $1/p$ or ν being twice as large for the spin-degenerate transitions. In our experiment, the f-exponent $\gamma = 1/z\nu$ and does not depend on p. Our observation that $\gamma = \kappa$ for both types of transitions shows directly that ν is different for the spin-split and spin-degenerate levels, while z and p are the same. A unified, consistent explanation of our results together with those of Refs. [1] and [21] is then (1) $z=1$ for both spin-degenerate and spin-split levels, (2) ν is nearly 7/3 for the spin-split levels and 14/3 for the spin-degenerate levels, and (3) $p=2$ for both kinds of level for the samples in the references.

In summary, we have used a transmission line to study the IQHE in a new frequency range. For $f \gtrsim 1$ GHz $\sim k_B T/h$, we observe the widths of three different Landau level peaks in $\mathrm{Re}(\sigma_{xx})$ to go as $\Delta B \sim f^{\gamma}$, with $\gamma = 0.41 \pm 0.04$. This power law can be interpreted consistently as due to dynamic scaling, and if $\nu = 7/3$, it follows from the value of our measured γ that, within about 10% error, $z=1$. The different ΔB vs f curves seen at different T's also fit in qualitatively with the dynamic scaling picture. Our data also indicate that $z=1$ and $\nu \sim 14/3$ hold for spin-degenerate transitions.

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