Decay of Vorticity in Homogeneous Turbulence

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We report on observations of turbulent behavior made without requiring the use of Taylor's "frozen turbulence" hypothesis. Initially, a towed grid generates homogeneous turbulence of grid Reynolds number of order 10^5 within a stationary channel filled with helium II. The subsequent decay in time t of the line density of quantum vortices is measured by second sound attenuation, and the associated rms vorticity ω follows the behavior expected of a *classical fluid* with $\omega \sim t^{-3/2}$, consistent with the notion of a coupled turbulent state of helium II. This technique also yields the time dependence of the Kolmogorov microscale.

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The study of decaying homogeneous isotropic turbulence occupies a unique place in fluid dynamics. The theory was given its modern basis by the influential work of Taylor and Kolmogorov over 50 years ago [1-3]. The experiments, of which there are many, are usually performed in a wind tunnel by blowing air through a grid and studying the way the turbulence decays as it is carried downstream [4-6]. The comparison between theory and experiment specifically assumes the equivalence of the downstream velocity field with the late time velocity field in an isotropic, homogeneous turbulent fluid (the so-called frozen turbulence hypothesis of G.I. Taylor), $\partial/\partial t = -\bar{U}\partial/\partial x$ where $\bar{U}$ is the mean velocity of the airstream. The decay of mean-square velocity fluctuations u'^2 is determined from spatial measurements down the tunnel, and from these data the mean-square vorticity ω^2 can be inferred from general considerations of the rate of turbulent energy decay due to viscosity:

$$\omega^2 \sim -\frac{1}{\nu}\frac{\partial u'^2}{\partial t}, \tag{1}$$

where ν is the kinematic viscosity. The validity of the frozen turbulence hypothesis rests on the assumption $\bar{U} \gg u'$ and its limitations are discussed on p. 46 of Ref. [5]. We emphasize, however, that Eq. (1) is valid independent of Taylor's frozen turbulence hypothesis.

The purpose of this Letter is to report a novel type of experiment, where the turbulence is created by towing a grid through a stationary sample of helium II. This produces a homogeneous turbulent state whose decay can be easily observed. The kinematic viscosity ν of helium II, based on the normal fluid viscosity and total density, is nearly 3 orders of magnitude smaller than air, allowing Reynolds numbers of order 10^5 to be achieved in a small (1 cm × 1 cm) channel. At high Reynolds numbers, both experimental results [7,8] and theoretical arguments [9] suggest that the isothermal flow of helium II is classical, with the normal and superfluid components coupled on sufficiently large length scales. We exploit this observation to measure the rms vorticity directly by observing the attenuation of second sound [7]. As we shall show, this experiment is completely different from studies of turbulence in helium II created by thermally induced counterflow of the normal and superfluids, which also have a history of over 50 years [7].

In Taylor's and Kolmogorov's original scaling arguments [1,2], the turbulent eddies have their energy distributed over a wide range of scales l. The largest correspond in our case to the size of the channel. Most of the energy of the flow is concentrated in energy-containing eddies whose scale l_e is not much smaller than the channel size. At high enough Reynolds numbers $Re_M = Ul_M/\nu$, where U is the grid velocity (typically 50 cm/sec), a long cascade of eddy scales are excited and only on the smallest scales is the shear high enough to cause viscous dissipation. This scale η, the Kolmogorov length, is defined by a Reynolds number $Re_d = v\eta/\nu \equiv 1$, where v is the characteristic velocity of eddies on the scale η. The energy dissipation per unit mass ε occurs around this scale and is supplied by the energy-containing eddies at a scale l_e. Dimensional reasoning then suggests that [1-3]

$$\varepsilon = \varepsilon_0 u'^3/l_e, \tag{2}$$

where ε_0 is expected to be a constant of order unity [2,10,11].

During the decay of turbulence, eddies grow in time with a power law close to one-half [3,10] until the integral scale l_e is of the order of the characteristic dimension of the channel (1 cm). Thus, even if l_e were initially not of order the dimensions of the channel, the accepted growth law of Ref. [10] (assuming it is applicable), and the parameters of the experiment given in Table I, would imply that after about 4 sec the growth of l_e will saturate. A detailed analysis shows that the resulting time dependence of ω would be indistinguishable from our data in the time regime discussed below. Thus in order to provide a simple expression for the time dependence of the

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TABLE I. Quantities deduced from our decay experiment. $T=1.647$ K, $\mathrm{Re}_M=93000$ ($U=50$ cm/sec), $\nu=8.97\times10^{-5}$ cm^2/sec.

Line density at 1 sec	L	4×10^5 cm^{-2}
Vorticity	$\omega=\kappa L$	400 sec^{-1}
Dissipation	$\varepsilon=\nu\omega^2$	14.4 cm^{2}sec^{-3}
Quantities at the dissipation scale		
Kolmogorov microscale	$\eta=(\nu^3/\varepsilon)^{1/4}$	4.74×10^{-4} cm
Velocity scale	$v=(\nu\varepsilon)^{1/4}$	0.189 cm sec^{-1}
Time scale	$\tau=(\nu/\varepsilon)^{1/2}$	2.5×10^{-3} sec
Kinetic energy density	$q=(3/2)v^2$	0.054 cm^{2}sec^{-2}
Taylor microscale	$\lambda=(15\nu/\varepsilon)^{1/2}u'$	2.35×10^{-2} cm
Microscale Reynolds number	$R_\lambda=u'\lambda/\nu$	634
Quantities at the energy-containing eddy scale		
Eddy scale	l_e	1 cm
Velocity scale	$u'=(l_e/\eta)^{1/3}v$	2.42 cm sec^{-1}
Kinetic energy density	$q=(3/2)u'^2$	8.78 cm^{2}sec^{-2}
Eddy viscosity	$\nu_{\mathrm{turb}}=l_e u'$	2.42 cm^{2}sec^{-1}
Time scale	$\tau=l_e/u'$	0.413 sec

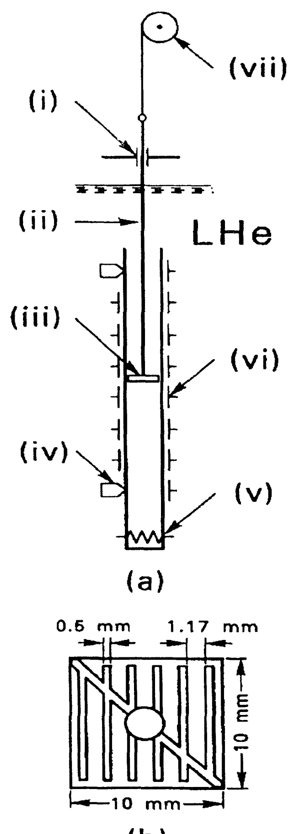

FIG. 1. (a) Layout of apparatus used to study grid turbulence. (i) Vacuum seal, (ii) 5/16 rod, (iii) grid, (iv) germanium thermometer, (v) counterflow heater, (vi) second sound transducer pair, and (vii) stepper motor. (b) Detail of grid construction.

vorticity, without making further assumptions about the applicability of Ref. [10] to our experiment, we will make the approximation that l_e is a constant, in which case the vorticity is expected from (1) and (2) to be well described by

$$\omega=\omega_0\left[1+\frac{\nu t}{2}\left(\frac{2}{3}\frac{\omega_0\varepsilon_0}{l_e\nu}\right)^{2/3}\right]^{-3/2}\underset{t\to\infty}{\approx}\left(\frac{2}{\nu}\right)^{1/2}\frac{3l_e}{\varepsilon_0}t^{-3/2}. \tag{3}$$

We will see that this simple formula is a good representation of the data, providing a useful, self-consistent interpretation. Given the uncertainties in the precise initial conditions and the theory of the decay of turbulence, we did not feel a more complicated analysis was warranted.

Our apparatus, shown schematically in Fig. 1(a), consists of a brass channel with a 1 cm square cross section, suspended vertically in a bath of liquid helium II. The channel is closed at the bottom end, and instrumented along its length with pairs of vibrating superleak second sound transducers. One of our grids, depicted in plan view in Fig. 1(b), was machined from a 1.5 mm thick brass wafer, and the resulting tines are 1.67 mm apart. The cross-sectional area of the grid riding within the channel amounts to less than 0.35 cm^2 and is suspended at the end of a 5/32 in. rod. The rod passes out of the top of the channel and exits the cryostat via a sliding vacuum seal. The end of the rod is attached to a cable and drum assembly linked to a stepper motor. Besides position accuracy, the stepper motor provides the capability to preselect velocity and acceleration as grid motion parameters. Thus, we could draw the grid along the channel and observe the decay in the resulting turbulent flow.

The protocol was to pull the grid-rod assembly at various velocities from the bottom to the top of the channel and observe the decay in the resulting turbulent field at some sensor pair. The kinematics were calculated and grid motion initiated in such a way that the grid passed a reference position exactly 2 sec after the instruments began recording data. This was defined to be the point $t=0$ in the data stream. The determination of the reference position and many other details of the experiment are described in a recent thesis [12].

Second sound attenuation allows the measurement of the length L of quantized vortex line per unit volume [7]. In a homogeneous field, the rms vorticity of the superfluid component is given by

$$\omega_s=\kappa L\,, \tag{4}$$

where $\kappa=h/m=9.97\times10^{-4}$ cm^2/sec is the quantum of circulation and m is the mass of the helium atom. We will assume that on sufficiently large scales (greater than the Kolmogorov length of the normal fluid), ω_s is approximately equal to that of the normal fluid ω_n; then ω_s can be taken to be the rms vorticity ω in the path of the second sound beam. With this assumption, a self-consistent picture of the vorticity decay is obtained. Our assumption is based on the following considerations. There is a body of experimental data which suggests that the two fluids are coupled together at high Reynolds numbers [7,8]. It follows that $\kappa L \geq \omega_n$: If this inequality did not hold then the superfluid could not have enough vorticity to match that of the normal fluid.

The equality $\omega_n = \omega_s$ holds if the length of line per unit volume is the minimum necessary to create an average superfluid vorticity equal to ω_n. In principle, ω_s could be greater than ω_n, but will match it dynamically if any excess superfluid vorticity decays on a time scale τ_s comparable with the quickest time scale τ_n associated with dissipation in the normal fluid, which occurs on the Kolmogorov scale η, namely, $\tau_n = \eta^2/\nu$. The decay of line density in the superfluid is known to proceed at a rate given by $dL/dt \cong \kappa L^2$ [13]. The time constant associated with this decay is $\tau_s \approx 1/\kappa L = l^2/\kappa$, where l is the vortex line spacing, of order η. Thus $\tau_s/\tau_n \cong \sqrt{\nu/\kappa} \cong 1$ [14]. Hence we conclude *a priori* that our assumption is reasonable.

Since we were guided by classical theory to seek power law behavior, we performed many of our initial investigations by plotting the decay results in log-log form. Three distinct regions stand out in the averaged data plotted in Fig. 2, and are typical of the data. The rounding seen in the lower panel of Fig. 2 at small times is consistent with

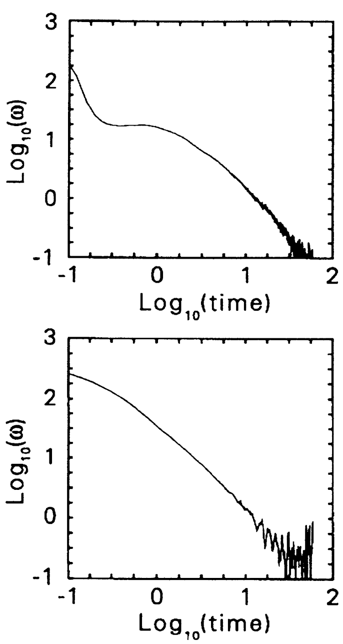

FIG. 2. An illustration of the dramatic differences in decay of two turbulent flows of the same initial vorticity produced by a counterflow (upper curve) and towed grid (lower curve). The experimental channel was identical in both cases except the grid was removed for the counterflow experiment. The form of the upper curve has been discussed by Schwarz and Rozen [15] and by Smith [12]. At long times, the decays appear to coincide.

Eq. (3). There follows a region of clear power law behavior. At large times, the decay typically evolves into a third regime as the vorticity drops below approximately 4 $\sec^{-1}$. This third regime of decay is characterized by small turbulent intensities, and the measurements are especially affected by instrument noise. We do not fully understand this region which could involve such matters as uncoupling of the normal and superfluid components.

From the log-log plot, the middle period of decay suggests power law behavior between 1 to 5 sec. To extract the decay exponent m in the simple power law $\omega \approx t^{-m}$, we plotted $\omega^{-1/m}$ vs t adjusting m until the data were as close as possible to a straight line. This eliminated any effect on m due to a small systematic uncertainty in the onset of decay. Values of m vs grid Reynolds number are plotted in Fig. 3. We have included data corresponding to the different grid designs, as well as data taken at different operating temperatures. These data indicate that the vorticity decays in a manner consistent with Eq. (2) providing an attractive means of estimating l_e/ε_0. Namely, if a is the slope of the straight line fit to the data plotted as described above then l_e/ε_0 is

$$\frac{l_e}{\varepsilon_0} = a^{-3/2}\frac{1}{3}\left(\frac{\nu}{2}\right)^{1/2}. \tag{5}$$

Over the range of Reynolds numbers investigated at 1.647 K, $l_e/\varepsilon_0 \sim 0.127 \pm 0.023$ cm. Using $l_e = 1$ cm, we obtain $\varepsilon_0 \cong 7$.

Once one has the vorticity, the average energy dissipation per unit mass comes from

$$\varepsilon = \nu\omega^2 \tag{6}$$

(Ref. [3], Sec. 39). The Kolmogorov microscale is given by

$$\eta = (\nu^3/\varepsilon)^{1/4} = \sqrt{\nu/\omega}\,. \tag{7}$$

Thus, we may extract from $\omega(t)$ the time dependence for the eddies at the dissipation end of the cascade, as plotted

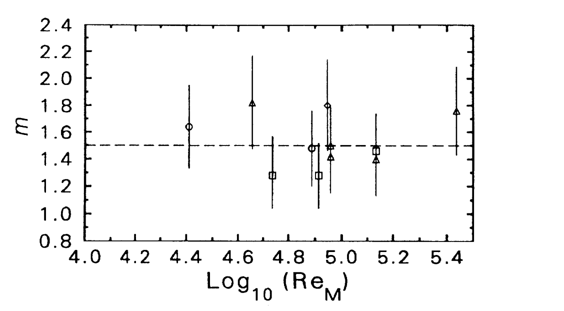

FIG. 3. Decay exponent m as a function of grid Reynolds number. The dashed line shows the simple result from Eq. (3). The diamonds and triangles were taken with the grid design shown in Fig. 1(b) at $T = 1.52$ and 1.647 K. Circles correspond to another, rectangular grid design described in Ref. [12] at 1.647 K. The average of all data shown is $\langle m \rangle = 1.5 \pm 0.2$.

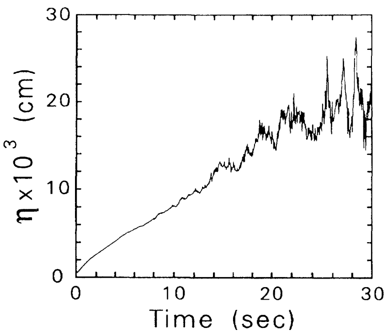

FIG. 4. Kolmogorov length η as a function of time during decay.

in Fig. 4. The Kolmogorov length grows as a function of time because the smallest eddies are continuously dissipated by viscosity. In the interval 1 to 3 sec, our observations of ω imply that $\eta \sim t^{3/4}$.

We conclude by showing in Table I the wide range of quantities which may be deduced from our simple experiment. Scaling suggests that if $\Delta u = |u(x+l) - u(x)|$, l being the separation distance, then $\varepsilon = (\Delta u)^3/l$ on scales $l_e > l > \eta$. When l becomes of order l_e, $(\Delta u)^3 \sim u'^3$ and we recover (2). When $l \sim \eta$ then $\Delta u \sim v$. Our results give ω, ε, η, τ, v, and q (the kinetic energy density) on the microscale. We then calculate $u' = (l_e/\eta)^{1/3} v$ and hence q and τ on the scale l_e.

In summary, we have used the quantized vortices of helium II as a quantitative visualization device for turbulent eddies. It is perhaps not too surprising that the turbulent behavior of helium II seems to be classical: Well above the critical velocity for the onset of flow with dissipation there is a proliferation of quantum vortices which couple the normal and superfluid components, so that the entire flow is dominated by inertia. We believe that our experiment indicates the potential usefulness of helium II for future studies of high Reynolds number turbulence.

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