Does $J/\psi \to \pi^+\pi^-$ Fix the Electromagnetic Form Factor $F_\pi(t)$ at $t = M^2_{J/\psi}$?

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We show that the $J/\psi \to \pi^+\pi^-$ decay is a reliable source of information for the electromagnetic form factor of the pion at $t = M^2_{J/\psi} = 9.6$ GeV^2 by using general arguments to estimate, or rather, put upper bounds on, the background processes that could spoil this extraction. We briefly comment on the significance of the resulting $F_\pi(M^2_{J/\psi})$.

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It is believed that the pion's electromagnetic form factor $F_\pi(t)$ can be more reliably calculated for $|t| \gg \Lambda^2_{\rm QCD}$ than the corresponding quantities for the nucleon. However, $F_\pi(t)$ is more difficult to measure [1]. In principle $F_\pi(t)$ can be measured for timelike t in e^+e^- colliders via $e^+e^- \to \pi^+\pi^-$. However, for t values of interest, $|t| \approx 10$ GeV^2, the above ratio is rather small and may be difficult to extract from the few $e^+e^- \to \pi^+\pi^-$ events [2].

At the J/ψ resonance the rate of all interactions is vastly enhanced and branching ratios for rare channels such as the G-parity (or isospin) forbidden $J/\psi \to \pi^+\pi^-$ can be measured. This rate could fix $F_\pi(t = M^2_{J/\psi} = 9.6$ GeV^2) if the decay proceeds predominantly via the one-photon exchange amplitude illustrated in Fig. 1(a). The dependence on the charmonium wave function can be eliminated by comparing the obtained branching ratio $B(J/\psi \to \pi^+\pi^-)$ to the leptonic decay rate $B(J/\psi \to e^+e^-)$, from which one obtains that

$$\frac{B(J/\psi \to \pi^+\pi^-)}{B(J/\psi \to e^+e^-)} = \frac{F^2_\pi(M^2_{J/\psi})}{4}. \quad (1)$$

The experimental values [3],

$$B(J/\psi \to e^+e^-) = (6.27 \pm 0.20) \times 10^{-2}\ , \quad (2)$$

$$B(J/\psi \to \pi^+\pi^-) = (1.47 \pm 0.23) \times 10^{-4}\ , \quad (3)$$

would then imply that

$$F_\pi(M^2_{J/\psi}) = 0.098 \pm 0.008. \quad (4)$$

This value of $F_\pi(t = 9.6$ GeV^2), or $tF_\pi(t) = 0.94$, if it is to be believed, is surprisingly large. It exceeds by more than a factor of 2 the corresponding spacelike value $|t|F_\pi(|t|) \approx 0.4$ inferred from π electroproduction data [4] for 3.33 GeV$^2 > -t > 1.18$ GeV^2 (see, however, [1]). On the theoretical side, it is *much* larger than the value obtained using the asymptotic wave function of the pion derived in perturbative QCD for $-t \to \infty$ [5], and is again more than a factor of 2 larger than any result obtained from either QCD sum rule methods [6, 7] or quark model calculations [8, 9]. The relevance of the fact that in the

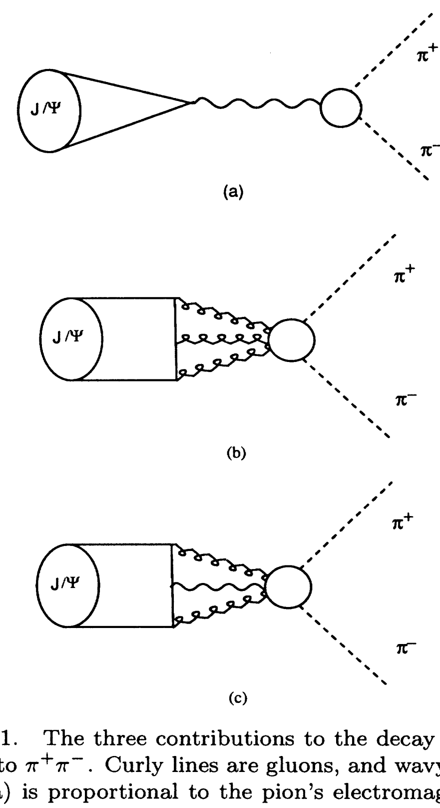

FIG. 1. The three contributions to the decay of charmonium into $\pi^+\pi^-$. Curly lines are gluons, and wavy lines photons. (a) is proportional to the pion's electromagnetic form factor. (b) and (c) are background processes not proportional to $F_\pi(M^2_{J\psi})$.

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case of the J/ψ one is at large timelike and not large spacelike t will be returned to in our conclusion. First, though, we would like to justify the use of Eq. (1) to extract $F_\pi(t = 9.6 \text{ GeV}^2)$.

There are two additional mechanisms contributing to $J/\psi \to \pi^+\pi^-$:

$$A^{J/\psi\to\pi^+\pi^-} = A^\pi_\gamma + A^\pi_{ggg} + A^\pi_{\gamma gg}. \quad (5)$$

A^π_{ggg} is taken to mean the contributuion to the amplitude of a purely hadronic process, which perturbatively would be initiated via a three-gluon state and hence the nomenclature. Likewise, $A^\pi_{\gamma gg}$ is a mixed hadronic-electromagnetic contribution that would be initiated via a two-gluon, one-photon intermediate state [Figs. 1(b) and 1(c), respectively]. In the following we will estimate A^π_{ggg} and $A^\pi_{\gamma gg}$ and show that both amplitudes fall considerably short of explaining the observed $J/\psi \to \pi^+\pi^-$ decay rate, thus justifying Eq. (4) above.

(I) A^π_{ggg}: Because the $J/\psi \to \pi^+\pi^-$ violates isospin, this purely hadronic process [10] can proceed only via the isospin breaking parameter $m^0_d - m^0_u$ which appears explicitly in the QCD Lagrangian [11]. Such an amplitude should therefore be suppressed by the small dimensionless factor $\epsilon_I = (m^0_d - m^0_u)/Q$ with Q some typical momentum in the problem. Rather than rely on any explicit, model dependent calculation, we present the following more general argument by comparing with the SU(3) analog process $J/\psi \to K\overline{K}$. Since the $J/\psi \to K\overline{K}$ decay violates SU(3) symmetry [12], the corresponding purely hadronic decay amplitude A^K_{ggg} will have in this case the explicit small SU(3) breaking suppression factor $\epsilon_{SU(3)} = (m^0_s - m^0_{d,u})/Q$. Consequently we expect that

$$\frac{A^\pi_{ggg}}{A^K_{ggg}} \approx \frac{\epsilon_I}{\epsilon_{SU(3)}} = \frac{m^0_d - m^0_u}{m^0_s - m^0_{d,u}} \approx 0.02 - 0.03, \quad (6)$$

where in the spirit of the Vafa-Witten theorem [11] we used the values of Lagrangian or "current" quark masses in estimating the above ratio. There are two $K\overline{K}$ decay modes, $J/\psi \to K^0\overline{K}^0$ (or $K^0_s K^0_L$) and $J/\psi \to K^+K^-$. The amplitude A^K_{ggg} is simply given by the former,

$$A^K_{ggg} \approx A^{J/\psi\to K^0_S K^0_L}. \quad (7)$$

The point is that the one photon and γgg contributions to the $J/\psi \to K^0_S K^0_L$ decay also vanish in the SU(3) limit due to the canceling contribution of $s, \overline{d}$ quarks of opposite charge [13]. Thus the amplitudes A^K_γ and $A^K_{\gamma gg}$ are suppressed by both explicit α_E and $\epsilon_{SU(3)}$ factors and are hence negligible. Multiplying Eq. (3) and Eq. (6) with the observed branching rate

$$B(J/\psi \to K^0_S K^0_L) = (1.1 \pm 0.14) \times 10^{-4} \quad (8)$$

implies that

$$A^\pi_{ggg} \approx \frac{1}{30} A^{J/\psi\to\pi^+\pi^-} , \quad (9)$$

so that it can be safely ignored.

(II) $A^\pi_{\gamma gg}$: It is very suggestive from a perturbative framework that this process is suppressed by a factor of α_s/π as it involves an extra gluon loop in comparison with the corresponding expression for A^π_γ. Indeed recent detailed calculations [14] using a range of pion wave functions [5, 6] indicate that

$$R = \frac{A^\pi_{\gamma gg}}{A^\pi_\gamma} = \frac{\alpha_s}{\pi} \begin{Bmatrix} 0.45 \\ 0.23 \end{Bmatrix} \approx \begin{Bmatrix} \frac{1}{20} \\ \frac{1}{40} \end{Bmatrix} , \quad (10)$$

where the smaller R value corresponds to the use of the more realistic, nonasymptotic pion wave function [6] allowing for a larger $F_\pi(t)$ [which, however, still falls short by more than a factor of 2 of explaining $B(J/\psi \to \pi^+\pi^-)$].

In order, however, not to rely too heavily on detailed model calculations we would like to obtain a more general, "phenomenological" estimate for $A^\pi_{\gamma gg}$. Let us therefore for the moment assume that only $A^\pi_{\gamma gg}$ contributes to the decay $J/\psi \to \pi^+\pi^-$.

Consider first the total inclusive radiative decay of J/ψ into noncharmed hadrons: $B(J/\psi \to \gamma + \text{hadrons})$. This process can be viewed as $J/\psi \to \gamma gg$ with the subsequent hadronization of the two-gluon system, in the same way that $J/\psi \to$ hadrons proceeds via a three-gluon initial perturbative state. Thus the ratio

$$\frac{B(J/\psi \to \gamma + \text{hadrons})}{B(J/\psi \to \text{hadrons only})} \approx \frac{B(J/\psi \to \gamma + gg)}{B(J/\psi \to ggg)} = \frac{16}{5}\frac{\alpha_E}{\alpha_s} = 0.07 - 0.09 \quad (11)$$

is readily [15] computed reflecting simply color and symmetrization factors (and where we have taken $1/3 \geq \alpha_s \geq 1/4$). Note that the symmetrization factors *enhance* the case with the final state photon by a factor of 3. Such an enhancement would in general be absent in the case that the bosons were not final state particles but were instead found in a virtual intermediate state, as we will be using below. Nevertheless, in order to be as conservative as possible, we will use Eq. (11) in our estimates without furthur modification.

For the three-gluon system the incorporation of the gluons or the quark pairs (to which they may convert) into hadrons is guaranteed by the basic hypothesis of quark and gluon confinement. However, we are for our purposes interested in the case where the γgg intermediate state converts into hadrons *only.* For this to happen, the virtual photon must convert into a $q\overline{q}$ pair which will cost an explicit extra factor of α_E [16]:

$$B(J/\psi \to \gamma gg \to \text{hadrons}) = B(J/\psi \to \gamma gg \to q\overline{q}gg \to \text{hadrons}) \approx \alpha_E B(J/\psi \to \gamma + \text{hadrons}) = (5 - 7) \times 10^{-4} . \quad (12)$$

We are focusing on a particular exclusive channel, namely, a final $\pi^+\pi^-$ state. Thus we need to estimate the probability f that the $q\bar{q}gg$ state in Eq. (12) hadronizes specifically into a $\pi^+\pi^-$ state. While it is uncertain how reliably one can directly compute f, we will infer an estimate for f from the probability that such a $q\bar{q}gg$ will hadronize into an analog $\pi\rho$ state; i.e., we will take that

$$f \equiv B(q\bar{q}gg|_{J/\psi} \to \pi^+\pi^-) \approx \frac{1}{2}B(q\bar{q}gg|_{J/\psi} \to \pi\rho), \tag{13}$$

where the factor of 1/2 reflects the two transverse polarizations of the ρ included in the $\pi\rho$ final state.

Note that the actual branching ratio,

$$B(J/\psi \to \pi^+\rho^-) \approx 0.4\% \,, \tag{14}$$

appears to be anomolously large in comparison with the branching ratio to other two body channels. Indeed it has triggered the speculation of the existence of a glueball state in the vicinity of the J/ψ [17]. While such speculation is controversial [6], there is general uniform agreement that the $\pi^+\rho^-$ branching ratio is unusually large. Hence irrespective of the correct explanation for $B(J/\psi \to \pi^+\rho^-)$, its usage to estimate $A^{\pi}_{\gamma gg}$ must lead to a conservative upper bound. On the other hand, if the glueball resonance scenario is correct, we would be severely overestimating $A^{\pi}_{\gamma gg}$ since such a resonance would clearly not couple to the γgg channel.

Finally, in order to estimate $A^{\pi}_{\gamma gg}$, we will (conservatively) ignore the possible unusual behavior of the $\pi\rho$ final state and note that a general two body, light meson exclusive state is expected to be a short distance event. Hence, in order to generate the same $q\bar{q}gg$ state in Eq. (13), we need to convert one gluon into a $q\bar{q}$ pair, and thus we will take that

$$\frac{B(J/\psi \to \gamma gg \to q\bar{q}gg)}{B(J/\psi \to ggg \to q\bar{q}gg)} = \frac{1}{\alpha_s}B(J/\psi \to \gamma gg \to \text{hadrons}), \tag{15}$$

which will again enhance our estimate for $A^{\pi}_{\gamma gg}$ by $1/\alpha_s$. Combining Eqs. (13) and (15) and inserting (12) and (14), we obtain that $A^{\pi}_{\gamma gg}$ alone would contribute a branching

$$B^{\gamma gg}(J/\psi \to \pi^+\pi^-) = (3-6)\times 10^{-6}, \tag{16}$$

which is at least 25 times smaller than the observed value. We hence conclude that even under the most unfavorable scenarios, $A^{\pi}_{\gamma gg}$ is less than a 20% correction so that

$$A^{J/\psi\to\pi^+\pi^-} = A^{\pi}_{\gamma} + A^{\pi}_{ggg} + A^{\pi}_{\gamma gg} \approx A^{\pi}_{\gamma}. \tag{17}$$

Having established that the $J/\psi \to \pi^+\pi^-$ data imply a fairly large value of $F_\pi(t = M^2_{J/\psi})$, we briefly turn to some concluding remarks:

(i) Recent results from E760 at Fermilab [18] indicate that the proton's electromagnetic form factor in the large timelike region is also unusually large (by about a factor of 2 over the spacelike data). A substantial imaginary part to hadronic form factors in the timelike region could account for this apparently systematic enhancement.

(ii) We expect that $A^K_\gamma \approx A^\pi_\gamma$ as the kaon's and pion's electromagnetic form factors should be rather similar at $t = M^2_{J/\psi}$. Since $A^K_{\gamma gg}$ can be argued to be small along similar lines presented for $A^{\pi}_{\gamma gg}$ and using our previous value for A^K_{ggg}, Eq. (7), we obtain that

$$A^{J/\psi\to K^+K^-} \approx A^{J/\psi\to\pi^+\pi^-} + A^{J/\psi\to K^0_S K^0_L}. \tag{18}$$

Considering that $A(J/\psi \to ggg \to K\bar{K})$ is expected to have a substantial imaginary part (see [14] for an explicit calculation of an analogous case), there could in general be a large relative phase between the two terms in Eq. (18). Thus the latter is quite consistent with the observed branching

$$B(J/\psi \to K^+K^-) = (2.4 \pm 0.3)\times 10^{-4}. \tag{19}$$

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[12] Recall that in the SU(3) flavor symmetry limit, $A(J/\psi \to K^+K^-) = A(J/\psi \to \pi^+\pi^-)$.

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