Comment on "Solid-on-Solid Rules and Models for Nonequilibrium Growth in 2+1 Dimensions"

Recently, Das Sarma and Ghaisas (DG) [1] investigated a number of solid-on-solid growth models. By measuring the growth exponents α and β for the surface width and comparing these exponents to those given by several Langevin equations, they assigned the models to various universality classes. In this Comment we *(1) show that several of the DG models do not obey conventional scaling and (2) claim that those of the DG models which do scale are expected to be governed by the Edwards-Wilkinson (EW) equation on long length scales.* Our conclusions are based on a study of the structure factor (SF) $S(k,t) \equiv \langle h_k(t) h_{-k}(t) \rangle$, which provides more information about a model than the width of the interface.

We illustrate the lack of conventional scaling for the case of model 1+ of Ref. [1] in $d' = 1$ substrate dimensions, but have found similar behavior for models 2+ and $d2+$ in $d' = 1$, and for models 1+ and $uu4\pm$ in $d' = 2$. Figure 1 shows $S(k,t)$ for $t \gg L^z$, where L is the system size. From this and short-time studies, we find that $S(k,t) \sim L^{\phi} k^{-\gamma}$ for $t \gg L^z$, and $S(k,t) \sim t^{\phi/(\gamma+\phi)} k^{-\gamma}$ for $k^{-z} \ll t \ll L^z$, with $\gamma \approx 2.4$ and $\phi \approx 1.59$. The fact that ϕ is nonzero demonstrates that model 1+ cannot be described in terms of a linear Langevin equation that yields stable growth, at least for this range of L and t. Our value of γ implies that the behavior of the interface width when measured over a distance x for a *fixed* system size L is $W(x, t \to \infty) \sim x^{\alpha}$ with $\alpha = (\gamma - d')/2 \approx 0.70 \pm 0.02$. The hyperscaling relation [2] $z = \gamma$ then implies $\beta \approx 0.29 \pm 0.01$. These estimates are in marked contrast to $\alpha = 1.45 \pm 0.01$ and $\beta = 0.375 \pm 0.001$ measured by DG. The discrepancies are explained by noting that because of the L- and t-dependent prefactors in $S(k,t)$, measurements of the width, $W(L,t) = \sum_k S(k,t)$, as a function of *system size* and time yield $W(L) \sim L^{(\gamma+\phi-d')/2} \sim L^{1.5}$ for $t \gg L^z$ and $W(t) \sim t^{\beta+d'\phi/[2z(z+\phi)]} \sim t^{0.375}$ for $t \ll L^z$ [3], in good agreement with the values measured in Ref. [1].

A study of the surface diffusion currents on a tilted substrate [4] allows us to classify the behavior of all the models studied by DG. We find that those models which do not scale have surface currents which are uphill or very nearly zero [5]. The other eight models in Ref. [1] all have unambiguous downhill currents, linearly dependent on the slope for small slopes. They also show excellent scaling of the SF. The $2\pm$, $4\pm$, and $ru4\pm$, which have very large downhill currents, were all identified by DG as exhibiting EW scaling. However, we predict that the rest of these models, which have more modest downhill currents, also cross over to EW scaling. Indeed, by studying the SF at large length scales (small k), we have confirmed that such a crossover is occurring in the 3+, $d4+$, and $ud4+$ models, with α decreasing (and clearly less than 0.5) at the smallest k that we can reach [6].

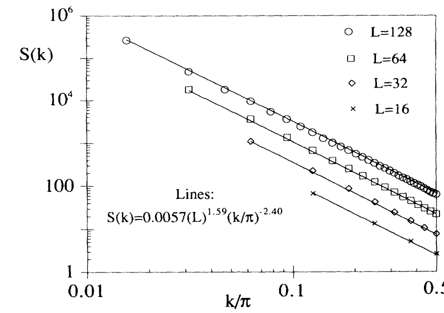

FIG. 1. Steady-state structure factor $S(k, t \gg L^z)$ for model 1+ in $d' = 1$.

We can imagine two possible scenarios for those models which do not exhibit scaling: Either the models have an instability and scaling is never satisfied, in which case it is probably best to consider the exponents α and β as undefined; or scaling is satisfied for large enough times and system sizes. If the models do eventually scale, it is our judgement that our estimates of the exponents are more reliable than those of Ref. [1] since theirs appear to be an artifact of the anomalous behavior of the SF for the time scales and system sizes investigated. The assignment of these models to universality classes is still an open question. In our opinion, the only universality class that has been unambiguously identified is the EW class.

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Received 4 March 1993
PACS numbers: 61.50.Cj, 05.40.+j, 05.70.Ln, 68.55.Bd

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[2] D. E. Wolf and J. Villain, Europhys. Lett. **13**, 389 (1990); Z-W. Lai and S. Das Sarma, Phys. Rev. Lett. **66**, 2348 (1991). We have confirmed this hyperscaling estimate of z by studying the decay of fluctuations in the steady-state regime [M. Siegert and M. Plischke, Phys. Rev. Lett. **68**, 2035 (1992)].
[3] The first of these relations is obvious; the second will be discussed in a longer publication.
[4] J. Krug, M. Plischke, and M. Siegert, Phys. Rev. Lett. **70**, 3271 (1993); see also D. A. Kessler and B. G. Orr (to be published).
[5] The 2+ and $d2+$ (Wolf-Villain) models in $d' = 1$ do have downhill currents, but they decrease rapidly in magnitude with increasing system size. It is difficult to determine if they are converging to zero or to a small nonzero value.
[6] The crossover to EW behavior in the $d4+$ model has already been noted by M. Kotrla, A. C. Levi, and P. Smilauer, Europhys. Lett. **20**, 25 (1992).

0031-9007/93/71(15)/2509(1)$06.00