Shallow Water Analogy for a Ballistic Field Effect Transistor: New Mechanism of Plasma Wave Generation by dc Current

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We demonstrate that electrons in a ballistic field effect transistor behave as a fluid similar to shallow water. Phenomena similar to wave and soliton propagation, hydraulic jump, and others should take place in this electron fluid. We show that a relatively slow electron flow should be unstable because of plasma wave amplification due to the reflection from the device boundaries. This provides a new mechanism for the generation of tunable far infrared electromagnetic radiation.

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Early theories of ballistic transport in semiconductors considered planar *n-i-n* structures where the electron ballistic motion was somewhat similar to that in electron tubes [1,2]. Experimental studies of ballistic effects primarily involved vertical devices called hot electron transistors [3]. Ideally, in this mode of ballistic transport, electrons travel across the active region of a ballistic device with no collisions.

The purpose of this paper is to show that the situation is quite different in a short field effect transistor where electrons experience practically no collisions with phonons and/or impurities during the transit time [we call such a device a ballistic field effect transistor (FET)], but where a high electron concentration results in many electron-electron collisions. In this case, individual electrons cannot be considered as ballistic particles but the two dimensional (2D) electron gas as a whole will exhibit interesting hydrodynamic behavior. We show that the steady state of a current-carrying ballistic FET is unstable. This instability is the result of the growth of plasma waves at terahertz frequencies which may lead to important practical applications.

As an example, let us consider an AlGaAs/InGaAs high electron mobility transistor similar to one described in [4]. At 77 and 300 K, the momentum relaxation time in a 2D electron gas in InGaAs (where ionized impurity scattering is suppressed) is $\tau_p \approx 10^{-11}$ and 3.5×10^{-13} s, respectively. For the electron drift velocity of 10^7 cm/s, the electron transit time is 10^{-11}, 10^{-12}, and 3×10^{-13} s for a 1, 0.1, and 0.03 μm gate length, respectively. Hence, the electron transit time can definitely be made shorter than the momentum relaxation time at 77 K and perhaps even at 300 K. However, for a typical surface carrier concentration of the 2D electron gas $n_s = 10^{12}$ cm^{-2}, the mean free path for electron-electron collisions is only of the order of the average distance between electrons, i.e., of the order of 100 Å since the average distance between electrons at this concentration is close to the Bohr radius (≈ 100 Å) and hence the electron gas is highly nonideal. Thus, the number of electron-electron collisions during the transit time is large. We also assume that the electron gas is not degenerate since electron-electron collisions are suppressed in a strongly degenerate electron gas (when the Fermi level is more than four thermal energies above the bottom of the subband) because of the Pauli principle. Under such conditions, the electrons behave as a fluid moving in the channel without external friction, and we can describe the electron motion by hydrodynamic equations.

The gate electrode in a ballistic FET (see Fig. 1) is separated from the channel by the gate insulator (which is a doped or undoped wide band gap semiconductor, such as AlGaAs in a typical high electron mobility transistor). The surface concentration n_s in the FET channel is given by

$$n_s = CU/e\,, \tag{1}$$

where C is the gate capacitance per unit area, e is the electronic charge, $U = U_{GC}(x) - U_T$, $U_{GC}(x)$ is the local gate-to-channel voltage, and U_T is the threshold voltage. Equation (1) represents the usual gradual channel approximation [2] which is valid when the characteristic scale of the potential variation in the channel is much greater than the gate-to-channel separation d.

The equation of motion (the Euler equation) is

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{e}{m}\frac{\partial U}{\partial x}\,, \tag{2}$$

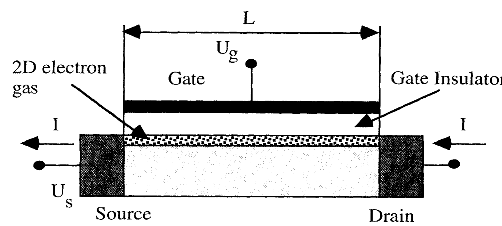

FIG. 1. Schematic structure of the ballistic FET. The gate length L is much smaller than the mean free path λ but much longer than the mean free path for electron-electron collisions, λ_{ee}.

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where $\partial U/\partial x$ is the longitudinal electric field in the channel, $v(x,t)$ is the local electron velocity, and m is the electron effective mass. Equation (2) has to be solved together with the usual continuity equation which [taking Eq. (1) into account] can be written as

$$\frac{\partial U}{\partial t}+\frac{\partial (Uv)}{\partial x}=0\,. \tag{3}$$

We notice that Eqs. (2) and (3) coincide with the hydrodynamic equations for shallow water (see, for example, [5]). This means that the 2D electron fluid in a ballistic FET should behave like shallow water. The reason is that the surface charge in the channel is proportional to U [see Eq. (1)] and not controlled by Poisson's equation as in the three dimensional case. In this hydrodynamic analogy, v corresponds to the fluid velocity, and eU/m corresponds to gh where h is the shallow water level and g is the free fall acceleration.

This analogy has profound consequences for understanding of the complicated and interesting physics of 2D electrons in the ballistic FET. Phenomena similar to wave and soliton propagation, hydraulic jump, and the "choking" effect [5,6] should take place in this hydrodynamic electron fluid. The effects of collisions, surface scatterings, changes in the channel cross section, and others may be also understood using this analogy.

In this paper, we limit ourselves to the consideration of the plasma waves and show that these waves may grow due to the reflections from the device boundaries under the boundary conditions specific for a ballistic FET.

Let us first assume that the gate voltage swing is fixed at U_0 and the channel current is zero. The wave dispersion law, $k=\pm\omega/s$, corresponding to the well known shallow water waves, is readily obtained from the linearized system of Eqs. (2) and (3). Here ω is the frequency, k is the wave vector, and $s=(eU_0/m)^{1/2}$ is the wave velocity. These waves are just the plasma waves in the FET channel [7-10]. Allen, Tsui, and Logan observed infrared absorption [11] and Tsui, Gornik, and Logan observed weak infrared emission [12] related to such waves as silicon inversion layers.

If the electrons move with a velocity v_0 corresponding to the electron flux per unit width $j=n_s v_0=CU_0v_0/e$, this dispersion relation becomes $k=\omega/(v_0\pm s)$. This change in the dispersion relation clearly means that the waves are carried along by the flow.

We now consider the situation when the source and drain are connected to a current source and the gate and source are connected to a voltage source, U_{GS}. This corresponds to the constant value of $U=U_0$ at the source ($x=0$) and to the constant value of the current at the drain ($x=L$). The ac variation of the electric current at the source side of the channel is possible even for a constant external current since the ac current at the source is short circuited to the gate by the dc voltage source. These boundary conditions correspond to a zero impedance at the source and an infinite impedance at the drain and are analogous to those for a transmission line, short circuited at one end and open at the other end. However, in contrast to the transmission line, the wave velocities in our system differ for the opposite directions of propagation.

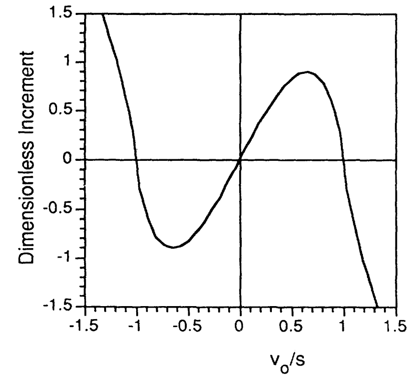

FIG. 2. Dimensionless plasma wave increment, $2\omega''L/s$, as a function of Mach number, $M=v_0/s$. The steady electron flow is unstable when $0<v_0<s$ and $v_0<-s$.

Now we will show that this velocity difference leads to the instability of the steady electron flow with respect to plasma wave generation. To this end, we will study the temporal behavior of a small fluctuation superimposed on a steady uniform flow.

We let $v=v_0+v_1\exp(-i\omega t)$, $U=U_0+U_1\exp(-i\omega t)$, linearize Eqs. (2) and (3) with respect to v_1 and U_1, and use the boundary conditions $U_1(0)=0$ and $\Delta j(L)=0$ [i.e., $U_0v_1(L)+v_0U_1(L)=0$] as discussed above. This procedure leads to the following expressions for the real and imaginary parts of $\omega=\omega'+i\omega''$:

$$\omega'=\frac{|s^2-v_0^2|}{2Ls}\pi n\,, \tag{4}$$

$$\omega''=\frac{s^2-v_0^2}{2Ls}\ln\left|\frac{s+v_0}{s-v_0}\right|, \tag{5}$$

where n is an odd integer for $|v_0|<s$ and an even integer for $|v_0|>s$. Equation (5) (see also Fig. 2) shows that for positive v_0, the steady flow is unstable if $v_0<s$ and stable if $v_0>s$. If v_0 is negative (or, in other words, if the boundary conditions at the source and drain are interchanged), the flow is stable if $|v_0|<s$ and unstable if $|v_0|>s$. The wave increment in units of $s/2L$ depends only on the Mach number, $M=v_0/s$. For $M\ll 1$, $\omega''=v_0/L$ which is the inverse electron transit time.

In this work, we will only consider the practically more important case $s>v_0>0$. In this case, the steady electron flow is unstable at low electron velocities. [Supercritical velocities ($|v_0|>s$) may be difficult to achieve since the drift velocity normally saturates due to the emission of optical phonons at about 2×10^7 cm/s. Thus, in order for $|v_0|$ to be greater than $s=(eU_0/m)^{1/2}$, the value of U_0 should not exceed 15 meV.] The reason for the instability becomes clear if we consider the wave reflection from each boundary. The solution of linearized Eqs. (2) and (3) shows that the reflection does not change the wave amplitude at $x=0$ (where the voltage is fixed), while at $x=L$ (where the current is fixed) the amplitude ratio of the reflected and oncoming waves is $(s+v_0)/(s-v_0)$. Hence, the reflection from the boundary with the fixed current results in the wave amplification for $v_0<s$. Let $\tau=L/(s+v_0)+L/(s-v_0)$ be the time during which the wave travels from the source to the drain and back. During time t, the wave amplitude grows in $[(s+v_0)/(s-v_0)]^{t/\tau}$ times since t/τ is the number of wave round passages during time t. Equating $[(s+v_0)/(s-v_0)]^{t/\tau}$ to $\exp(\omega'' t)$, we obtain Eq. (5). Thus, the proposed new mechanism of plasma wave generation is based on the amplification of the wave during its reflection from the boundary where the current is kept fixed.

We are unaware of any observations of such an instability of low velocity flows in shallow water. (The required boundary condition of a fixed flow at the drain end of a water channel is an unusual one.) However, a similar effect seems to account for the self-excitation of jets and organ pipes [13,14].

There are two decay mechanisms which oppose the wave growth: external friction related to electron scattering by phonons or impurities, and internal friction caused by the viscosity of the electron fluid. The external friction can be accounted for by adding the term $-v/\tau_p$ into the right-hand side of Eq. (2). This leads to the addition of the $-1/2\tau_p$ term to the wave increment. Hence, the wave grows only if the number of scattering events during the transit time is small. The viscosity ν of the electron fluid causes an additional damping with the decrement of νk^2 where k is the wave vector. Hence, the viscosity is especially effective in damping higher order modes. Comparing ω'' with νk^2 for the first mode, we find that the effect of the viscosity for $v_0 \leq s$ is small when the Reynolds number $\mathrm{Re}=Lv_0/\nu$ is much greater than unity. In a highly nonideal electron gas where the Bohr energy, thermal energy, and Fermi energy are of the same order which roughly corresponds to the surface electron concentration of 10^{12} cm $^{-2}$ at 77 K, the viscosity of the electron fluid, $v_F\lambda_{ee}$, is on the order of $\hbar/m$ which is approximately 15 cm^2/s (comparable to that of castor oil or glycerin at room temperature). The Reynolds number of our electron fluid may be estimated as $\mathrm{Re}=mv_0L/\hbar \approx 12$ for $v_0=10^7$ cm/s and $L=0.2$ μm.

For a sample with $L=0.2$ μm at 77 K, assuming $\tau_p \approx 10^{-11}$ s, the increment v_0/L exceeds the decrement $1/2\tau_p$ caused by the collisions when $v_0>10^6$ cm/s. For the same sample, the decrement caused by viscosity, $\nu(2\pi/L)^2/16$, is smaller than the increment of v_0/L when $v_0>\pi^2\nu/4L \approx 1.8\times10^6$ cm/s. Hence, the threshold velocity for the instability is well below the peak velocity in GaAs.

Once the electron velocity exceeds the threshold, the plasma waves grow. Since no other steady states exist for $v_0<s$ [15], this growth should lead to oscillations for which the plasma wave amplitude is limited by nonlinearity. Presumably, the amplitude of these nonlinear oscillations should be comparable to U_0 if the flow velocity is substantially larger than the threshold value.

Let us now discuss possible applications of this instability. The plasma oscillations result in a periodic variation of the channel charge and the mirror image charge in the gate contact, i.e., to the periodic variation of the dipole moment. This variation should lead to electromagnetic radiation. The device length is much smaller than the wavelength of the electromagnetic radiation, λ_R, at the plasma wave frequency. (The transverse dimension W may be made comparable to λ_R.) Hence, the ballistic FET operates as a point or linear source of electromagnetic radiation. Many such devices can be placed into a quasioptical array for power combining. The maximum radiation intensity is limited by the gate voltage swing. The energy stored in the gate capacitor is CU_0^2WL. This energy can be radiated during the time larger than the inverse wave increment which is equal to the electron transit time L/v_0 for $v_0 \ll s$. Hence, the maximum radiation power P can be estimated as $CU_0^2Wv_0\alpha^2$ where $\alpha=U_1/U_0$ where U_1 is the wave amplitude. Assuming $d\approx350$ Å, $\varepsilon=13$, $U\approx0.5$ V, and $v_0\approx10^7$ cm/s, $W=100$ μm, $L=0.2$ μm, $\alpha\approx0.5$, we obtain $s=1.15\times10^8$ cm/s, $n_s\approx10^{12}$ cm^{-2}, $P=2$ mW at the frequency $f=\omega/2\pi \approx s/4L \approx 1.5$ THz. This estimate represents only an upper bound since the radiation efficiency may be fairly small (dependent on the antenna design and other factors). Nevertheless, it demonstrates a promising potential of a ballistic FET for the terahertz frequency range applications. This device decouples the operating frequency (which can be tuned in a wide frequency range by varying U_0) from the electron transit time limitation. The maximum modulation frequency is still limited by the transit time (≈ 2 ps in our example).

Tsui, Gornik, and Logan [12] observed the emission of a very weak (10^{-9} W) electromagnetic radiation caused by plasma wave excitation in silicon inversion layers with metal gratings on the gate at 4.2 K in weak electric fields. The mechanism of plasma wave excitation was completely different from the effect considered above. Their samples had a transit time on the order of nanoseconds and a scattering time on the order of picoseconds so that our criteria for the instability were not satisfied. The possible mechanism of the weak plasmon excitation observed in [12] may be related to the incoherent emission by individual electrons [16].

In summary, we have demonstrated that hydrodynamic equations for shallow water precisely describe the behavior of electrons in a ballistic FET, with the water level corresponding to the gate-to-channel voltage. We have shown that a steady dc current flow in a ballistic FET may be unstable, and a relatively low drain current should induce plasma oscillations in the terahertz frequency range. This provides a new mechanism for the generation of tunable far infrared electromagnetic radiation.

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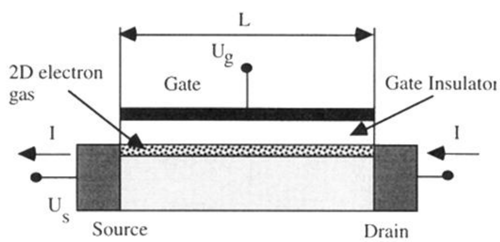

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