Viability of Perturbative Renormalization Factors in Lattice QCD Calculation of the K^0-$\bar{K}^0$ Mixing Matrix

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Validity of perturbative estimation of renormalization factors in weak matrix element calculations in lattice QCD is examined for the K^0-$\bar{K}^0$ mixing matrix by comparing results for gauge invariant and noninvariant operators. A large disagreement found for uncorrected results for the two cases is shown to be removed by the one-loop renormalization factor. This indicates that the large scaling violation in the mixing matrix previously reported is not due to an artifact of prescription of lattice calculations. Our estimate of $\hat{B}_K$ for the continuum in quenched QCD is 0.61(13)–0.83(7).

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The calculation of weak matrix elements is one of the most important tasks of numerical simulation of lattice QCD. In particular the evaluation of the matrix element B_K which appears in K^0-$\bar{K}^0$ mixing is of much practical importance, in that its accurate knowledge is indispensable for exploring the phenomenology of the Cabibbo-Kobayashi-Maskawa matrix, especially of *CP* violation. A few calculations have already been made for this end [1–4], but the most recent one [4] using the Kogut-Susskind quarks has revealed an important problem in such work; the resulting B_K does not show scaling with the lattice spacing, and hence the value depends on that of the coupling constant $\beta=6/g^2$ at which it is evaluated. The question then arises as to whether one-loop perturbation theory is adequate for estimating the renormalization factors needed to match lattice results with the continuum physics; there is no guarantee for its validity. A further doubt was cast [5] on the use of gauge noninvariant operators [3,4] to extract the matrix element, which might cause a large scaling violation of a nonperturbative origin that cannot be corrected by one-loop calculations. These are serious points, since if true, they should invalidate the conventional procedure to calculate matrix elements using lattice QCD.

We have investigated this point by explicitly employing two different operators for B_K, one gauge noninvariant and the other gauge invariant. Indeed, we found a large discrepancy between the matrix element obtained with a gauge invariant operator and that obtained with a conventional gauge noninvariant operator; even the quark mass dependence is significantly different between the two. We found, however, that a one-loop perturbative correction with an appropriate choice of the coupling constant brings the results, including the quark mass dependence, virtually into agreement. This not only wipes out our worry concerning the use of gauge noninvariant operators, but also greatly alleviates our doubt against the use of one-loop renormalization corrections to extract the physics in the continuum.

Our study is made with both quenched and full QCD. For quenched simulations we used 10 configurations separated by 1000 pseudo heat-bath sweeps on a lattice of a size $24^3\times40$ at $\beta=6.0$ and $32^3\times48$ at $\beta=6.3$. For full QCD we analyzed 26 configurations separated by 25 trajectories generated on a 20^4 lattice and duplicated in the time direction at $\beta=5.7$ with two flavors of dynamical Kogut-Susskind (KS) quarks of a mass $m_q a=0.01$ and 0.02, which have already been used for spectroscopic analysis [6]. We work with the KS valence quarks with the mass set to $m_v a=0.01$, 0.02, and 0.03 (0.01 and 0.02 for $\beta=6.3$ in the quenched case).

The K meson B parameter is defined by

$$B_K=\frac{\langle\bar{K}^0|\bar{s}\gamma_\mu(1-\gamma_5)d\,\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle}{\frac{8}{3}\langle\bar{K}^0|\bar{s}\gamma_\mu\gamma_5 d|0\rangle\langle0|\bar{s}\gamma_\mu\gamma_5 d|K^0\rangle}\,. \tag{1}$$

To calculate the numerator we follow the method of Ref. [3] and rewrite the four-quark operator as a sum of four terms $\mathcal{V}_1+\mathcal{V}_2+\mathcal{A}_1+\mathcal{A}_2$ with $\mathcal{V}_1=(V_\mu)_{ab}(V_\mu)_{ba}$, $\mathcal{V}_2=(V_\mu)_{aa}(V_\mu)_{bb}$, $\mathcal{A}_1=(A_\mu)_{ab}(A_\mu)_{ba}$, and $\mathcal{A}_2=(A_\mu)_{aa}\times(A_\mu)_{bb}$. Here $(A_\mu)_{ab}=\bar{q}^a(\gamma_\mu\gamma_5\otimes\xi_5)q^b$ and $(V_\mu)_{ab}=\bar{q}^a\times(\gamma_\mu\otimes\xi_5)q^b$ are the axial-vector and vector currents in the spin-flavor notation for KS fermions with a,b the color indices. Quark fields in the first current in $\mathcal{V}_i$ and $\mathcal{A}_i$ are to be contracted with $\bar{K}^0$ and those in the second with K^0. For the denominator in (1) we use $\langle\bar{K}^0|(A_\mu)_{aa}|0\rangle\langle0|(A_\mu)_{bb}|K^0\rangle$.

In terms of the KS fermion fields the operators above are nonlocal and gauge noninvariant. The previous calculations [3,4] employed these operators. In our work we also use gauge invariant operators constructed by inserting gauge link variables between the quark fields with contracting color indices and summing over all possible shortest paths for the insertion.

We create K^0 and $\bar{K}^0$ mesons by two wall sources placed at the edges of the lattice. Gauge link variables are fixed to the Landau gauge throughout the entire lattice. Quark propagators are calculated with the Dirichlet

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(periodic) boundary condition in the time (space) direction. Fits to extract the numerator and the denominator of (1) are made over the time slices $12 \le t \le 28$ for lattices with the temporal size $T=40$ and over $16 \le t \le 32$ for those with $T=48$. Errors of B_K are estimated by a jackknife procedure.

Our raw results for B_K are shown in Fig. 1 for the quenched calculation at $\beta=6.0$. The lower branch of data corresponds to those with gauge noninvariant operators, which show a good agreement with the results of Refs. [3,4]. On the other hand, the upper branch, which gives B_K with the gauge invariant operators, grossly disagrees with B_K from the gauge noninvariant operators; even the quark mass dependence differs substantially between the two calculations.

In order to interpret the lattice results in the continuum theory, wave function renormalization corrections are generally necessary. For the axial-vector current $(A_\mu)_{aa}$ in the denominator of (1) it is given by $Z_A = 1 - 12.233 \times g^2/16\pi^2$ for the gauge noninvariant current in the Landau gauge [7,8] with Z_A defined as $A_\mu^{\rm cont} = Z_A A_\mu^{\rm lat}$, while for the gauge invariant current $Z_A = 1$.

The renormalization factor for the four-quark operators may be written as

$$\mathcal{O}_i^{\rm cont}(\mu) = (\delta_{ij} + (g^2/16\pi^2)(\gamma_{ij}\ln\mu a + c_{ij}^{\rm cont} - c_{ij}^{\rm lat}))\mathcal{O}_j^{\rm lattice}, \tag{2}$$

where $i,j = 1,\ldots,4$ corresponds to $\mathcal{V}_1$, $\mathcal{V}_2$, $\mathcal{A}_1$, and $\mathcal{A}_2$ and μ denotes the renormalization scale for the continuum operators. The matrix γ_{ij} is given by

$$\gamma_{ij} = \begin{pmatrix} 9 & -3 \\ 0 & 0 \end{pmatrix} \otimes 1 + \begin{pmatrix} -7 & -3 \\ -6 & 2 \end{pmatrix} \otimes \sigma_1$$

in a 2×2 block representation, and $c_{ij}^{\rm cont}$ and $c_{ij}^{\rm lat}$ are the finite renormalizations in the continuum and on the lattice. In the continuum, using a finite mass for gluons to regularize infrared divergences, we find $c_{ij}^{\rm cont} = \frac{11}{12}\gamma_{ij}$ for massless quark for naive dimensional regularization (NDR) with the modified minimal subtraction ($\overline{\rm MS}$) subtraction scheme. The dimensional reduction with the $\overline{\rm EZ}$ subtraction scheme [9] yields $c_{ij}^{\rm cont} = \frac{7}{12}\gamma_{ij}$. The difference in the corrected values of B_K for the two schemes is small (2%), and we use the NDR scheme for the numerical results below.

For the gauge noninvariant operator in the Landau gauge the lattice finite part $c_{ij}^{\rm lat}$ takes the values [7]

$$c_{ij}^{\rm lat} = \begin{pmatrix} 37.446 & -2.913 & -5.253 & -2.251 \\ 0 & 28.706 & -4.502 & 1.501 \\ -5.253 & -2.251 & 37.976 & -4.504 \\ -4.502 & 1.501 & 0 & 24.464 \end{pmatrix}, \tag{3}$$

where a finite gluon mass is used to regularize infrared divergences as in the continuum. The results agree with those of Ref. [10]. For the gauge invariant operator we obtained [7]

$$c_{ij}^{\rm lat} = \begin{pmatrix} -18.915 & -4.772 & -5.253 & -2.251 \\ 0 & -60.000 & -4.502 & 1.501 \\ -5.253 & -2.251 & -19.513 & -2.977 \\ -4.502 & 1.501 & 0 & 0 \end{pmatrix}. \tag{4}$$

The elements in the second and fourth row are already known and our results coincide with those in the literature [11] after correcting for the difference in the method of regularizing infrared divergences. The values in (3) and (4) above are for massless quark. Corrections due to finite quark masses are negligibly small for the range of quark masses used for our analyses.

In evaluating perturbative corrections there is uncertainty as to which gauge coupling constant and which value of μ should be used. We take the mean-field improved $\overline{\rm MS}$ coupling constant at the scale $\mu = \pi/a$, evaluated by $1/g^2_{\overline{\rm MS}}(\pi/a) = P/g^2 + 0.02461$ with P the plaquette expectation value [12]. With this scheme the scale is naturally given by $\mu = \pi/a$. The result is shown in Fig. 2. We see that the two calculations now show a good agreement with each other.

It may first seem difficult that the discrepancy between the two calculations can be removed by the renormalization factor which depends little on m_q. It is important to

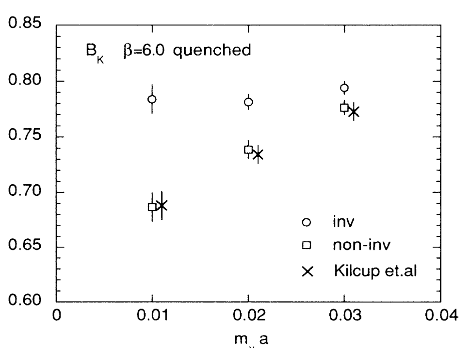

FIG. 1. Raw results for B_K for the quenched calculation at $\beta=6.0$. Open circles (squares) are for gauge invariant (noninvariant) operators. Crosses represent the results of Refs. [3,4] for gauge noninvariant operators.

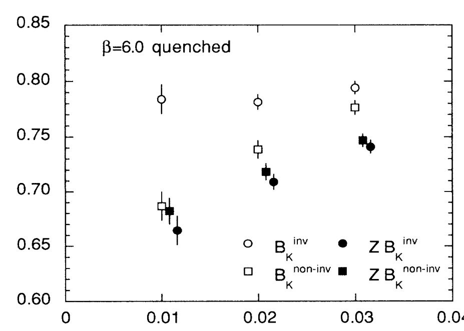

FIG. 2. Comparison of B_K before (open symbols) and after (filled symbols) renormalization correction.

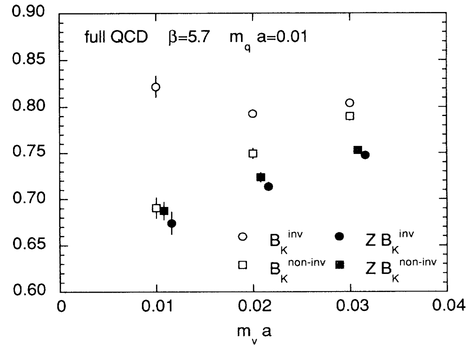

FIG. 3. Same as Fig. 2 for two-flavor full QCD with the sea quark mass $m_q a = 0.01$ at $\beta = 5.7$.

observe in this context that the two matrices (3) and (4) have completely different structures. The matrix (3) is close to diagonal with the diagonal entries similar in value to the correction factor $(Z_A)^2$ for the denominator of B_K. Thus it gives rise only to a slight shift of B_K after correction, as was already noted in Ref. [3]. On the other hand, the matrix (4) for the gauge invariant operator is largely deviated from the unit matrix, which subtly affects the summation of the four terms and brings the curve with the gauge invariant operators into agreement with that with the gauge noninvariant operators.

In Fig. 3 a similar figure is shown for full QCD at $\beta = 5.7$ with the sea quark mass $m_q a = 0.01$. The large disagreement seen between the bare values [13] (open symbols) are brought into a very good agreement after the renormalization correction (filled symbols).

We present in Fig. 4 the renormalization group invariant quantity $\hat{B}_K = \alpha_{\overline{\rm MS}}(\pi/a)^{-2/9} B_K(\pi/a)$ as a function of the lattice spacing a which is determined from the ρ meson mass. The values of $B_K(\pi/a)$ are extracted by an interpolation of simulation results to the physical K meson mass. We observe that the quenched values,

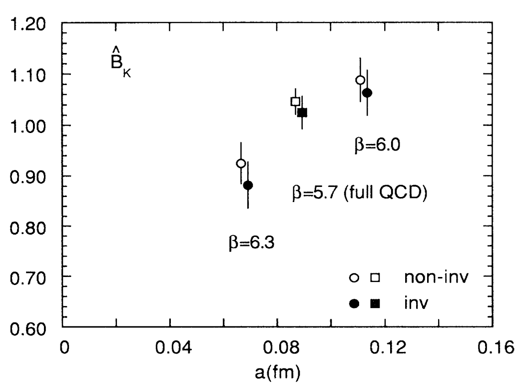

FIG. 4. Renormalization group invariant $\hat{B}_K = \alpha_{\overline{\rm MS}}(\pi/a)^{-2/9} \times B_K(\pi/a)$ as a function of the lattice spacing a fixed by the ρ meson mass. Circles are for quenched QCD and squares for two-flavor full QCD with the sea quark mass of $m_q a = 0.01$.

which agree between the gauge invariant (filled circles) and noninvariant (open circles) operators, exhibit a substantial decrease of about 15% between $a = 0.11$ fm at $\beta = 6.0$ and $a = 0.07$ fm at $\beta = 6.3$. Thus a large scaling violation originally indicated by the results of Ref. [4] is not an artifact of their using the gauge noninvariant operators. Making an empirical extrapolation of the form $\hat{B}_K(a) = \hat{B}_K + ca^n$ with $n = 1$ or 2 we obtain $\hat{B}_K = 0.61(13)$–$0.83(7)$ as our estimate of $\hat{B}_K$ in the continuum limit in quenched QCD.

Let us finally remark that including dynamical quarks does not seem to lead to a noticeable change of the value of the $\hat{B}_K$ parameter. In fact the full QCD results plotted by squares in Fig. 4 lie close to the interpolation of quenched data, indicating that the bulk of sea quark effects is absorbed into a renormalization of scale.

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