Electroweak Global Strings with Flux Tubes

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We show that for a large range of parameters in a $SU(2)_L \times U(1)$ electroweak theory with two Higgs doublets there may exist classically stable flux tubes of Z boson magnetic field. In a limit of an extra global $\tilde{U}(1)$ symmetry, these flux tubes become topologically stable. These results are automatically valid even if $\tilde{U}(1)$ is gauged.

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More than twenty years have passed since Nielsen and Olesen discovered that Abrikosov type vertices may appear as classical solutions in spontaneously broken gauge theories [1]. It is expected that these objects, so called strings, play an important role in both particle physics and cosmology [2]. It is of crucial importance to know if strings can exist in the $SU(2)_L \times U(1)$ based electroweak theory. Recently, in an inspiring paper, Vachaspati showed that classically stable stringlike structures can exist even in the standard model [3] (but unfortunately only for unrealistic values of $\sin^2\theta_W$ and Higgs mass [4]). These are the usual Nielsen-Olesen flux tubes embedded in $SU(2)_L \times U(1)$ group and therefore no longer topologically stable.

In this paper we investigate under which conditions Z boson flux tubes could be actually topologically stable. Much to our surprise, it turns out that the price needed to achieve this is just the additional global $\tilde{U}(1)$ symmetry in the Higgs sector. The minimal structure that can lead to extra $\tilde{U}(1)$ is a two Higgs doublet model, with these fields carrying different $\tilde{U}(1)$ charges. As far as our analysis is concerned, this symmetry may or may not be anomalous. Depending on the choice of the fermionic charge assignment, this symmetry can be identified with a Peccei-Quinn [5] or $B-L$ symmetry. We return to the issue of realistic models in another section; now we wish to demonstrate the nature of this phenomenon.

Global symmetry and topologically stable Z flux tubes.—Imagine a $SU(2)_L \times U(1)$ electroweak gauge theory with two doublets Φ_1 and Φ_2 and a potential

$$V = \sum_{i=1}^{2} \frac{\lambda_i}{4}(\Phi_i^\dagger \Phi_i - m_i^2)^2 + \frac{\lambda}{2}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \frac{\lambda'}{2}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \; . \tag{1}$$

Obviously, the above potential is the most general one invariant under $SU(2)_L \times U(1)$ gauge symmetry and an additional global $\tilde{U}(1)$ symmetry. The extra symmetry simply amounts to the freedom of independent phase transformations of the two doublets.

It is clear that in the range of parameters (which includes $\lambda' < 0$), both Φ_i $(i = 1, 2)$ develop vacuum expectation values in one and the same direction in the group space,

$$\langle \Phi_i \rangle = \begin{pmatrix} 0 \\ v_i \end{pmatrix} e^{i\theta_i}, \quad 0 \le \theta_i \le 2\pi \; , \tag{2}$$

which preserves $U(1)_{em}$ much as in the standard model:

$$G \equiv SU(2)_L \times U(1) \times \tilde{U}(1) \underset{v_i \neq 0}{\longrightarrow} U(1)_{em} \equiv H \; . \tag{3}$$

Notice that both v_i have to be nonvanishing in order for $\tilde{U}(1)$ global symmetry to be broken. Obviously, the vacuum manifold is not simply connected $[\pi_1(G/H) \neq 1]$ and therefore, as is well known, there must be a topologically stable string solution in this theory. The origin of the string is clear, since the potential includes no couplings explicitly dependent on the phase difference $\theta \equiv \theta_1 - \theta_2$. Therefore, θ_1 and θ_2 are locally uncorrelated so that θ can wind by $2\pi n$ (n is an integer) around some closed path, ensuring the existence of the string. The stability of the string is guaranteed by the topologically invariant condition

$$\frac{1}{2\pi} \oint dx^\mu \partial_\mu \theta = n \; , \tag{4}$$

where integration is carried over the path enclosing the string at infinity. Naively, one imagines this string to be global since it results from the spontaneous breaking of a global $\tilde{U}(1)$ factor.

In other words, this string is not expected to carry any magnetic flux associated with $SU(2)_L \times U(1)$ group. However, this premise is false as we now demonstrate.

The crucial point is that besides the usual topologically invariant boundary condition (4), one must also demand the single valuedness of the vacuum expectation values (vev's) v_i; i.e., the phases θ_i must return to their original values while encircling the string. More precisely, this implies topological constraints

$$\frac{1}{2\pi} \oint dx^\mu \partial_\mu \theta_i = n_i, \tag{5}$$

where n_i are integers, and so from (4) one has $n = n_1 - n_2$. In what follows, we consider a single string solution, i.e., $n = 1$ case.

Using the equations of motion for the Z boson field, one easily obtains the behavior far away from the core of

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the string,

$$Z_\mu = \frac{\cos\theta_W}{g} \frac{v_1^2\partial_\mu\theta_1 + v_2^2\partial_\mu\theta_2}{v_1^2 + v_2^2} . \qquad (6)$$

Integrating (6) over the same loop as before, we obtain

$$\oint Z_\mu dx^\mu = \frac{\cos\theta_W}{g} \frac{v_1^2 n_1 + v_2^2 n_2}{v_1^2 + v_2^2} . \qquad (7)$$

Once again, since n_i are integers, the right-hand side of (7) can never vanish, which implies the nonvanishing Z boson flux, trapped in the string. This is the key observation of our paper.

Note that the Z flux in (7) differs from the original one of Nielsen and Olesen, since as long as $v_1^2 \neq v_2^2$ (as expected in realistic situations) it will not be an integer. However, it is topologically stable, due to the topological nature of the scalar configuration behind its existence.

We wish to note here that our expression (6) and the nonintegral nature of the Z flux is reminiscent of the situation encountered by Hill *et al.* in their study of so called frustrated strings [6]. Their philosophy was, however, orthogonal to ours since they attributed the existence of strings to the spontaneous breaking of a local gauge symmetry and made no use of a global symmetry (or at least show no awareness of it). Even more important, in their work there is no mention of possible relevance of their results for the physically interesting case of electroweak Z flux tubes.

Realistic models.—This is all very nice, but unfortunately the resulting Goldstone boson due to $\tilde{\mathrm{U}}(1)$ breaking is necessarily coupled to fermions, independently of whether both or just one Higgs doublet is coupled to fermions. Therefore, stellar objects would copiously produce such particles and radiate their energy away unless their couplings to light fermions were strongly suppressed, typically being less than 10^{-10} [7]. As is well known, the way out is to introduce a $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ singlet field S, by attributing breaking of $\tilde{\mathrm{U}}(1)$ to its large expectation value $v_S \equiv |\langle S\rangle| \geq 10^9$ GeV. A typical example is a celebrated invisible axion scenario [8] with $\tilde{\mathrm{U}}(1)$ being a Peccei-Quinn symmetry. The way to define $\tilde{\mathrm{U}}(1)$ charge of S is through the explicit phase dependent coupling in the potential

$$\Delta V(\text{phase}) = \mu(\Phi_1^\dagger\Phi_2 S + \text{H.c.}) = \mu v_1 v_2 \cos(\theta_S - \theta) . \qquad (8)$$

Clearly, as soon as S picks up a nonzero vev, it breaks $\tilde{\mathrm{U}}(1)$ and a global string is formed through the winding of θ_S by 2π (again we restrict ourselves to $n = 1$ case). Next when Φ_i develop vev's and break $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ their phases are locally correlated to θ_S through the coupling in (8), i.e., $\theta = \theta_S$, or $n_1 - n_2 = 1$. As a consequence, our previous discussion becomes automatically valid. Notice an important point that Z flux in (6) is completely independent of the global symmetry breaking scale.

Now that we have seen that our solution is for real, let us discuss some natural candidates for $\tilde{\mathrm{U}}(1)$.

(a) Peccei-Quinn (axion) case: In this case the doublets Φ_i couple separately to up and down quarks. Resulting axion strings have been studied at length and as is well known, due to instanton induced explicit breaking of $\mathrm{U}(1)_{\mathrm{PQ}}$, these strings become boundaries of domain walls below the scale of QCD phase transition [9]. The structures decay rapidly before dominating the energy content of the Universe (unless there are truly stable domain walls) [9].

(b) Majoron case: Another natural candidate for $\tilde{\mathrm{U}}(1)$ that comes to mind is $B-L$ symmetry, which is automatically preserved in the standard model. The point is that $B-L$ symmetry is free from anomalies which destabilize the Z flux tubes.

Unfortunately, the simplest and most popular Majoron scheme leaves no room for the considered structures since it is based on a single Higgs doublet picture [10]. One simply adds a singlet S and a right-handed neutrino ν_R, and defines the $B-L$ of S through the coupling

$$L_\gamma(s) = h_R S \nu_R^T C \nu_R + \text{H.c.} , \qquad (9)$$

where the $B-L$ property of the right-handed neutrino is defined through

$$L_\gamma(\nu) = h_\nu (\bar{\nu}\bar{e})_L \Phi^* \nu_R + \text{H.c.} \qquad (10)$$

Clearly, one Higgs doublet suffices and it carries no $B-L$ quantum number. In turn the $B-L$ strings are purely global and devoid of any Z flux. We show how this can be easily modified in a two doublet case.

(c) Anomaly free two Higgs doublet model: There is a unique version of the two doublet model which has no global anomaly, one with one of the doublets decoupled from the quarks. In addition, this automatically ensures natural flavor conservation in neutral currents.

The global symmetry in this case can be identified in the fermion sector as the global hyperchange rotation which leaves the decoupled Higgs doublet, say Φ_2, invariant. Clearly, this symmetry is anomaly free. However, since it is chiral and the corresponding Goldstone boson has diagonal couplings to light fermions, once again one is led to introduce a singlet coupled as in (8). As before, consistency of the model requires $v_S \geq 10^9$ GeV. The distinguishing feature with respect to the axion model is, as we emphasized before, the topological stability of the Z flux tubes due to the absence of anomalies. The strings, therefore, will never become boundaries of domain walls.

The nice feature of this scenario is that the global freedom of the model can take the meaning of $B-L$. All one has to do is to couple S to ν_R in (9), and the charge becomes a liner superposition of hypercharge Y and $B-L$,

$$\tilde{Q} = 2Y - 5(B-L) . \qquad (11)$$

The unusual feature of this picture is that the Majoron is now coupled at the tree level to both the electron and the light quarks, but its couplings are suppressed by v_S^{-1}.

Ũ(1) breaking and Z flux tubes.—The essential ingredient in our construction of Z flux tubes was the existence of a $\tilde{\mathrm{U}}(1)$ symmetry. What happens when this symmetry is explicitly broken? Here we wish to show that Z flux can remain classically stable in a certain range of parameters of the theory even if $\tilde{\mathrm{U}}(1)$ global symmetry is explicitly violated and there is no ultralight pseudoscalar boson in the spectrum. In this case Z flux will flow along the boundary of the domain wall that terminates in its tube.

To see what happens, let us switch on an explicit $\tilde{\mathrm{U}}(1)$ breaking in the two Higgs doublet model

$$\Delta V = \frac{\gamma}{4}[(\Phi_1^\dagger\Phi_2)^2 + \mathrm{H.c.}] = \frac{\gamma}{2}v_1^2v_2^2\cos 2\theta\,. \qquad (12)$$

For simplicity we restrict ourselves to the above term only, which automatically preserves a discrete Z_2 symmetry: $\Phi_1 \to -\Phi_1$, $\Phi_2 \to \Phi_2$ (or vice versa), needed to ensure natural flavor conservation in the neutral currents. Now, as soon as $\gamma \neq 0$, the phases θ_1 and θ_2 become locally correlated. For example, for $\gamma < 0$ (12) is minimized for $\theta = 0$; however, θ cannot vanish everywhere, since it has to wind up around the string. In other words, while enclosing the string, one is forced to pass through a region $\theta \neq 0$ indicating the existence of a domain wall attached to a string. This string-wall system will remain classically stable as long as there exists a potential barrier that prevents the unwinding of this configuration.

Choosing, say, $v_1 \gg v_2$ tells us that the Z flux tube will be classically stable, as long as nonvanishing v_2 is energetically favored everywhere in space (including the vicinity of the domain wall), or in other words, as long as the (mass)^2 of $|\Phi_2|$ is not always negative. The contribution to this term comes from the potential and the gradient energy (the latter is significant only inside the wall). The effective mass term has the form

$$m_{\mathrm{eff}}^2 = -\lambda_2 m_2^2 + (\lambda + \lambda' + \gamma\cos 2\theta)v_1^2 + \left|\frac{\partial\theta}{\partial x}\right|^2 , \qquad (13)$$

where x is the coordinate transverse to the wall. Obviously, the string-wall system can be unstable only if m_{eff}^2 becomes positive at some point inside the wall pushing $|\Phi_2|$ over the top of the Mexican hat potential.

Note that the $\tilde{\mathrm{U}}(1)$ violating expression (12) gives a mass to a would have been Goldstone boson

$$m_a^2 \simeq \gamma v_1^2\,, \qquad (14)$$

where the subscript refers to an "axionic" nature of the particle.

Furthermore, since $\theta(x)$ changes by $\Delta\theta = \pi$ through the wall whose thickness is $\delta \sim m_a^{-1}$ one gets $|\frac{\partial\theta}{\partial x}| \simeq \pi m_a$. In terms of physical fields, the condition $m_{\mathrm{eff}}^2 < 0$ implies that the "axion" field is lighter than the radial mode of the string.

Of course, although classically stable this system can decay through quantum mechanical tunneling via the hole formation in the wall sheet [11]. However, this decay rate will be exponentially suppressed by the ratio of the mass of the radial mode to the mass of the axion. Clearly, we end up predicting that the axion mass should lie below M_W in order for the considered structure to be stable.

In addition, if we switch on the other possible phase dependent couplings in the potential of the type $\Phi_1^\dagger\Phi_2$, Z_2 symmetry gets explicitly broken and the two $\Delta\theta = \pi$ domain walls attached to the string will collapse into the single one with $\Delta\theta = 2\pi$.

For simplicity, you can assume the coefficients to be real. This offers an even more interesting possibility of spontaneous CP violation. Therefore, previous analysis shows an intriguing possibility of CP domain walls terminating into Z flux tubes.

As we mentioned earlier, Vachaspati has shown that contrary to the conventional wisdom, there may exist classically stable stringlike structures in the standard electroweak model, but, unfortunately not in the realistic range of parameters. The main problem in his construction lies in the fact that the usual Nielsen-Olesen flux tubes embedded in $\mathrm{SU}(2)_L\times\mathrm{U}(1)$ are not topologically stable. This leads one naturally to pursue the situation when there is a global $\tilde{\mathrm{U}}(1)$ symmetry in the theory, for then the existence of (global) strings is ensured by topological considerations. And if the relevant scalar fields interact with the Z boson, this will lead to the trapping of the Z flux on these stringlike structures. This is the central message of our work.

If the reader is at all uneasy about the existence of a global symmetry, we invite her to consider gauging it [12], since it changes none of our analysis. Of course, this picks up the anomaly free $\tilde{\mathrm{U}}(1)$, say, $B-L$. The advantage in this case is that the scale of symmetry breaking of $B-L$ may be kept much lower, all the way to TeV energy or so. Furthermore, the $B-L$ version can be naturally embedded in $L-R$ models or SO(10) grand unified theory.

Finally, for laboratory purposes the situation with $\tilde{\mathrm{U}}(1)$ explicitly broken may be even more interesting. Although in this case the objects that carry Z flux are only classically stable, they may be both long lived enough and light enough to be produced in the supercolliders. Also this is the minimal Higgs structure that leads to the above phenomenon and it can result in Z flux tubes being boundaries of CP domain walls. For this to work, there must be a pseudoscalar particle with a mass definitely smaller than M_W.

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