String Theory and the Principle of Black Hole Complementarity

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String theory provides an example of the kind of apparent inconsistency that the *principle of black hole complementarity* deals with. To a freely infalling observer a string falling through a black hole horizon appears to be a Planck size object. To an outside observer the string and all the information it carries begin to spread as the string approaches the horizon. In a time of order the "information retention time" it fills the entire area of the horizon.

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The paradox of information loss in black hole evaporation [1] is essentially concerned with the localization of information and how it is perceived by different observers. According to the *principle of black hole complementarity* [2,3] no inconsistency follows from the following two assumptions.

(1) To a freely falling observer, matter falling toward a black hole encounters nothing out of the ordinary upon crossing the horizon. All quantum information contained in the initial matter passes freely to the interior of the black hole.

(2) To an observer outside the black hole, matter, upon reaching the "stretched horizon" [2], is disrupted and emitted as thermalized radiation before crossing the horizon. All quantum information contained in the initial matter is found in the emitted radiation.

In this paper an example will be described in which information appears to be localized in extremely different ways to infalling and outside observers. The example relies on the peculiar zero-point fluctuations of fundamental strings [4,5]. The result is closely related to the Regge behavior of string scattering amplitudes.

Let us first recall a standard argument about why string theory should not influence the discussion of black holes and information loss. It is widely understood that strings are extended objects and therefore may introduce a bit of nonlocality. However, the argument goes, the extended objects have a size of order the Planck length. On the other hand, the horizon of a massive black hole is very flat on this scale so that strings can be effectively replaced by point particles. We shall see that this logic is correct for the freely falling observer but completely incorrect for external observers.

The size of strings.— One of the oldest known and widely ignored properties of strings is that their physical size is not well defined unless a "resolution time," ϵ, is prescribed [4,5]. The time ϵ is a smearing time over which the internal motions of the string are averaged. If the resolution time is measured in Planck units then the spatial extent of the wave function of the string in Planck units satisfies

$$R^2_{\text{string}} \sim \ln \frac{1}{\epsilon}\,, \tag{1}$$

for $\epsilon \ll 1$. Thus as the string is examined with better and better time resolution it appears to slowly grow. For the purposes of low energy physics, resolution times are always large and this phenomenon is not important.

Before deriving (1) let us recall that (1) is closely related to the well known Regge behavior of string scattering amplitudes. If a string of energy $E \gg 1$ collides with a target at rest, the scattering amplitude for momentum transfer q is given by [4]

$$A(E,q^2) \sim F(q^2)(E)^{-(q^2+c)} = F(q^2)e^{-(q^2+c)\ln E}\,, \tag{2}$$

where c is a constant. Fourier transforming to find the amplitude as a function of impact parameter shows that the radius of the scattering event grows like $(\ln E)^{1/2}$. If we now assume (correctly) that the scattering event averages over a time of order $\epsilon = E^{-1}$ we recover (1). The growth of strings with energy is their oldest known property.

To derive (1) consider a string in the light cone frame. The normal mode expansion for the transverse coordinate of a point σ is

$$X^i(\sigma) = X^i_{\text{c.m.}} + \sum_{l>0} [X^i_l \cos(l\sigma) + \bar{X}^i_l \sin(l\sigma)]\,. \tag{3}$$

Consider the mean square transverse distance between the center of mass and the material point σ

$$\langle [X(\sigma) - X_{\text{c.m.}}]^2 \rangle\,. \tag{4}$$

If the string is in the ground state this reduces to

$$\langle [X(\sigma) - X_{\text{c.m.}}]^2 \rangle = \sum_l \frac{1}{l}\,, \tag{5}$$

which diverges for every point σ. The same divergence is found in the mean square distance between any pair of points σ_1, and σ_2.

If the observation of the string lasts a time ϵ in the strings rest frame the contribution of modes with $l > 1/\epsilon$ is averaged out. The result is

$$\langle [X(\sigma) - X_{\text{c.m.}}]^2 \rangle_\epsilon \equiv R^2_\epsilon \sim \ln \frac{1}{\epsilon}\,. \tag{6}$$

Another quantity which diverges as $\epsilon \to 0$ is the total length of the projection of the string on the transverse

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plane [5]. This is defined by

$$L=\int_0^{2\pi} d\sigma \left[\frac{\partial X}{\partial \sigma} \frac{\partial X}{\partial \sigma} \right]^{1/2} . \tag{7}$$

When the resolution time is accounted for one finds that L increases like $1/\epsilon$. Because the mean radius R grows so much slower than the total length L the string must trace over the same region of transverse space many times. As $\epsilon \to 0$ the string becomes space filling. In [5] a particularly graphic illustration of these facts was obtained by Monte Carlo sampling of the probability functional of the string. We refer the reader to that reference for pictures of typical string configurations corresponding to decreasing resolution time.

Here we simply remark that as $\epsilon \to 0$ not only does the string wave function spread but the information which distinguishes different states of the string is also diffused over the area $\sim R_\epsilon^2$.

Now consider a string falling toward a black hole. An observer falling with the string does measurements which we shall suppose involve ordinary energies and time scales. In other words the resolution time in the infalling frame is not significantly smaller than the Planck time. The string and all its information is localized in a transverse size of order unity.

Now let us consider an observation of the string done by a distant fiducial observer whose clocks register Schwarzchild time. Suppose the measurement again averages over a time ~ 1. But now because of the redshift factor, this corresponds to a time in the string frame which is much smaller. It is easily seen that the resolution time in the string frame is of order

$$\epsilon \sim \exp\left[-\frac{t}{4M} \right] . \tag{8}$$

Accordingly the transverse size of the string seen in such a measurement is given by (6). This becomes

$$R^2 \sim t/M . \tag{9}$$

In other words the distant observer sees the string, upon passing through the stretched horizon, start to spread. In fact the spreading appears to behave as if the string were diffusing away from its original transverse location.

Eventually from the outside point of view the string will fill an area comparable to the whole horizon. This occurs when $R^2 \sim M^2$ or $t \sim M^3$.

It is interesting that the information retention time defined in [3] is also of order M^3. This time is defined as follows. Suppose at time $t=0$ the stretched horizon is in some pure state. At that time a particle in some state $|i\rangle$ is absorbed at the stretched horizon. The resulting states of the stretched horizon are initially orthogonal for different $|i\rangle$. However, after some time the density matrix of the stretched horizon loses memory of $|i\rangle$, the lost information having been radiated in the Hawking radiation. The time for this to occur is called the information retention time. In [3] it is argued that this time is of order M^3. This suggests the following speculative picture. The information in a particle is absorbed at the stretched horizon. According to an outside observer it begins to spread as it sinks toward the event horizon. At a time M^3 it is spread over the entire horizon and can no longer expand. By roughly that time the information must be radiated away.

The same event is viewed by the infalling observer who simply sees a microscopic string fall past the horizon with nothing to disrupt it until it approaches the singularity.

The above description has ignored the splitting and joining of strings which can take place near the horizon. We hope to return to this point at a later time.

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[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); Phys. Rev. D **14**, 2460 (1976).
[2] L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D **48**, 3742 (1993). See also G. 't Hooft, Nucl. Phys. **B335**, 138 (1990).
[3] L. Susskind and L. Thorlacius, Stanford University Report No. SU-ITP-93-19 (to be published).
[4] L. Susskind, Phys. Rev. D **1**, 1182 (1970).
[5] M. Karliner, I. Klebanov, and L. Susskind, Int. J. Mod. Phys. A **3**, 1981 (1988).