Disordered Bosons: Critical Phenomena and Evidence for New Low Energy Excitations

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We study the $T = 0$ critical properties of the superfluid-insulator transition in 2D hard core Bose systems with disorder. Using quantum Monte Carlo simulations and finite size scaling on 64×64 lattices we find the dynamical exponent $z = 0.5 \pm 0.1$ and the correlation length exponent $\nu = 2.2 \pm 0.2$. At the transition, the system is metallic with a conductivity $\sigma_c = (1.2 \pm 0.2)(e^*)^2/h$ and a compressibility $\kappa \neq 0$. These conclusions differ from the existing scaling theory as well as from simulations on simplified models argued to be in the same universality class. Our results are suggestive of *new* low lying collective excitations (modified from usual phonons) in the *disordered* system.

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The localization problem for interacting bosons, often called the "dirty boson" problem, has been an active field in recent years beginning with the seminal work of Fisher and co-workers [1]. This problem is very challenging because it contains elements of both disorder and interactions between the particles. In addition, unlike the fermion problem, the system cannot be perturbed about the zero interaction limit, which is singular in the Bose case. The interplay between interaction and disorder has been studied using quantum Monte Carlo [2–4] and other techniques [5]. Besides the theoretical interest, the dirty boson problem in 2D is relevant to a number of experiments. Among them are ^{4}He adsorbed in porous media [6], universal aspects of the superconductor-insulator transition in disordered films [7,8], vortices in type II superconductors [9], and Josephson junction arrays [10].

The experiments on disordered superconducting films, in particular, have attracted much attention, partly because it appears that between a superconducting phase at low disorder (or low magnetic field [11]) and an insulating phase at high disorder (or high magnetic field), the conductivity of bosons with charge e^* approaches a finite value at $T = 0$ of order $\sigma_c \sim (e^*)^2/h \equiv \sigma_Q$, the quantum of conductance. Fisher, Grinstein, and Girvin [12] proposed that in 2D the system is metallic *at* the transition with a *universal* conductivity (analogous to the universal jump in the superfluid density at the Kosterlitz-Thouless-Berezinskii transition). What then is the correct universality class for the transition? It was argued that under certain conditions [8,13], the superconductor-insulator transition is well described by interacting bosons moving in a random potential.

In previous theoretical work a scaling theory [1] was postulated for the disordered superfluid that contained only phonon modes as the relevant low energy excitations. One of the primary conclusions of the scaling theory was that the dynamical exponent z, characterizing the asymmetry between the spatial and temporal correlations, equals d, the spatial dimension. This result was later confirmed by simulations of the quantum rotor model with disorder [14] which was argued to be in the same universality class as the disordered boson model. Our main conclusions are (a) unlike the pure system, in the presence of disorder the boson model and the quantum rotor model are in *different* universality classes (see Fig. 1); and more significantly, (b) we find evidence for *new* low energy excitations in the disordered superfluid not captured by the effective action in Ref. [1].

Our results for the disordered hard core boson model are as follows: (i) We have established the existence of a localized (also called "Bose-glass") phase for the boson model for disorder larger than a critical disorder V_c,

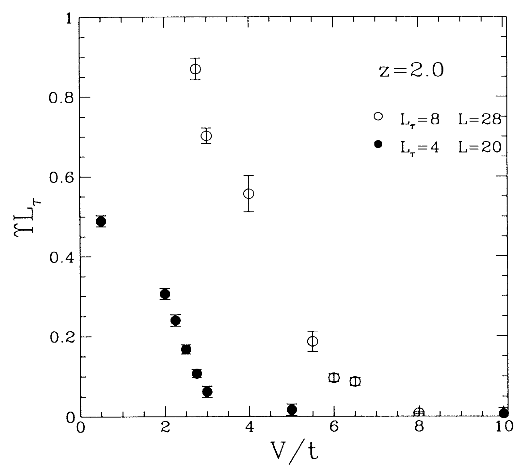

FIG. 1. $\overline{\Upsilon}L_\tau$ vs disorder V for two temperatures such that $L_{\tau 1} = 8/t$ and $L_{\tau 2} = 4/t$ and linear lengths $L_1 = 28$ and $L_2 = 20$ chosen such that the second argument of the scaling function in Eq. (2), $L/L_\tau^{1/z}$ with $z = 2$, is fixed. The value of $z = 2$, suggested by the scaling theory, does not yield an intersection of the curves, indicating the absence of a superfluid-insulator transition for this value of z.

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where V_c is nonzero. Such a result was known in 1D; however, it was not clear previously, given the considerably weaker localizing effects in 2D, whether quantum fluctuations would destabilize a localized phase, i.e., push V_c to infinity. (ii) The exponents characterizing the transition are $z = 0.5 \pm 0.05$ and $\nu = 2.2 \pm 0.2$. The dynamical exponent z is defined by $\xi_\tau \sim \xi^z$, where ξ_τ is the correlation "length" in the imaginary time direction and ξ is the boson order-parameter correlation length. The correlation length exponent ν is given by $\xi \sim \delta^{-\nu}$, where δ is the deviation from the critical point. (iii) The compressibility at the transition is finite. (iv) The conductivity at the transition is $\sigma_c = (1.2 \pm 0.2)\sigma_Q$. These results should be compared with the quantum rotor model that includes only the phase degrees of freedom. For short-range repulsive interactions it is found in Refs. [14,15] that $z_{\rm QR} = 2$, $\nu_{\rm QR} = 1.0 \pm 0.1$, and $\sigma^*_{\rm QR} = (0.14 \pm 0.03)\sigma_Q$. From the scaling theory [1], the compressibility $\kappa \sim \delta^{\nu(d-z)}$, where d is the dimensionality of the system. Since we find $z < d$, this would imply that κ should vanish at the transition. We, however, find that the compressibility is nonzero at the transition. We therefore conclude that the existing scaling theory fails to include all the relevant degrees of freedom in the presence of disorder.

We consider a model of hard core bosons on a 2D square lattice $N = L \times L$, described by the Hamiltonian H given by

$$H = \frac{-t}{2} \sum_{\langle i,j \rangle} (a_i^\dagger a_j + \text{H.c.}) + \sum_i (V_i - \mu) n_i. \qquad (1)$$

Here a_i ($a_i^\dagger$) is a boson annihilation (creation) operator at a site i and $\langle i,j \rangle$ are nearest neighbor sites. The random potential is modeled by site energies V_i chosen from a uniform distribution $[-V, V]$ and the chemical potential μ is chosen to fix the density. We shall study the transition from a superfluid to an insulating phase driven by increasing disorder.

The algorithm consists of dividing the "imaginary time" interval $\beta = 1/T \equiv L_\tau$ in the partition function $Z = \text{Tr}(e^{-H/MT})^M$ into M time slices of width $\Delta\tau = L_\tau/M$ and inserting complete sets of states (in the occupation number basis) between the adjacent time slices. This leads to a world line algorithm which is parallelized by distributing regions of the lattice among the nodes of a concurrent processor [16]. We impose periodic boundary conditions in the spatial and temporal directions. The results are obtained on an $L^2 = 64 \times 64$ lattice with up to $M = 48$, significantly larger than those used in previous simulations. We average over 32 to 2048 realizations of the random potential.

In order to distinguish between the superfluid and localized phases, we calculate the stiffness Υ, which is a measure of the rigidity of the many-body wave function to a twist in the boundary conditions and is defined as $\Upsilon = \langle W_x^2 + W_y^2 \rangle$. Here $W_\mu = 1/L \sum_{\ell=1}^{N_b} [r_{\ell_\mu}(0) - r_{\ell_\mu}(\beta)]$ is the winding number [2,17] along $\mu = x$ or y, N_b is the average number of bosons, and $r_{\ell_\mu}(\tau)$ is the position of the ℓth boson at time τ. We define a reduced stiffness by $\overline{\Upsilon} = \Upsilon/2\rho\beta t$, where $\rho = N_b/L^2 = 1/2$ in the simulations. From a single simulation in the zero winding number sector on a 64×64 lattice, we obtain the stiffness on lattice sizes ranging from 4×4 to 32×32 by measuring the winding numbers on various sublattices (which are not constrained to $W = 0$).

Near the superfluid-insulator transition (at a critical disorder $V = V_c$ and $T = 0$), the spatial correlation length diverges as $\xi \sim \delta^{-\nu}$ and the correlation time diverges as $\xi_\tau \sim \xi^z \sim \delta^{-z\nu}$, where $\delta = \mid V - V_c \mid /V_c$. On a finite space-time lattice ($L^2 \times L_\tau$), the diverging correlation lengths are cut off by the system size. Near the transition, we make the following finite size scaling ansatz for the stiffness $\overline{\Upsilon}(L, L_\tau, \delta) = (L^{2-d}/L_\tau)\mathcal{F}[L/\xi, L_\tau/\xi_\tau]$.

By redefining variables, it can be shown that in $d = 2$

$$\overline{\Upsilon}(L, L_\tau, \delta) = \frac{1}{L_\tau} \mathcal{G}(\delta {L_\tau}^{1/z\nu}, L/L_\tau^{1/z}) \ . \qquad (2)$$

To begin, let us assume that $z = 2$ as suggested by the scaling theory. In this case for the size of the simulations we have studied $L^z > L_\tau$ and the divergence of the correlation length is limited by L_τ rather than by L. In the spatial direction this implies that the correlations cannot grow beyond $L_\tau^{1/z}$. In Fig. 1 we plot $\overline{\Upsilon} L_\tau$ vs V for two values of L_τ and correspondingly for two different L such that the second argument $L/\sqrt{L_\tau}$ of $\mathcal{G}$ is fixed. A signature of a superfluid to insulator transition with $z = 2$ should show up as an intersection of the two curves indicating the size independence at the critical point. The curves fail to show any sign of an intersection. The possibility of an intersection of the curves improves considerably if z is reduced below unity and provides the first hint that $z < 1$.

In order to extract the value of z we perform an equivalent finite size scaling analysis in the regime $z < 1$ by noting that now L provides the cutoff for the diverging correlation length and $L_\tau > L^z$. The scaling function $\mathcal{F}$ can be written as $\mathcal{F}(x, y) = yx^{-z} f(x, x^z/y)$, where $x = L/\xi$, $y = L_\tau/\xi_\tau$, and $f(x_1, x_2)$ is *analytic* at $(0,0)$. We approach the finite size scaling regime, given by the limit $x \to 0$, $y \to 0$, by keeping the argument $x^z/y \ll 1$ (or equivalently $L_\tau \gg L^z$). Keeping only the zeroth order term in the Taylor expansion of $f(x_1, x_2)$ around $x_2 = 0$, the dependence on L_τ drops out, and we obtain

$$\overline{\Upsilon}(L, \delta) L^z = g(\delta L^{1/\nu}) \ . \qquad (3)$$

In Fig. 2 we plot $\overline{\Upsilon}(L) L^z$ vs V for various lattice sizes at a fixed $T = 0.25t$. We have tried various values of z and find that we get a very distinct intersection for $z = 0.5 \pm 0.1$ (we use $z = 0.5$ in Fig. 2). We caution that the scaling analysis is difficult because we have to determine two exponents. Nevertheless, based on the analysis in Figs. 1 and 2 we rule out $z = 2$, which was assumed in simulations of the quantum rotor model [14]. Linearizing Eq. (3) around the critical point $V_c = 2.5t$,

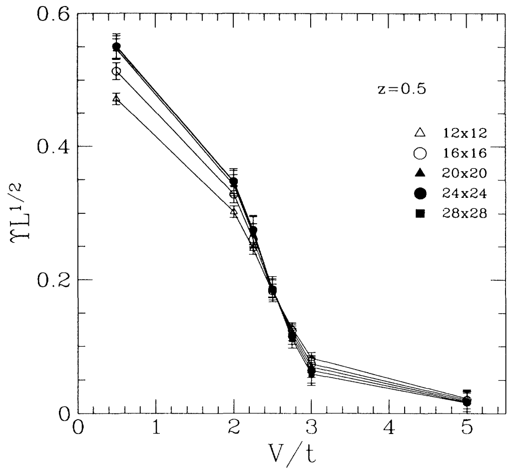

FIG. 2. From Eq. (3) we obtain the dynamical exponent z by plotting $\overline{\Upsilon}(L)L^z$ vs the disorder V for $L = \{12, 16, 20, 24, 28\}$. The value of z used is 0.5 and $T = 0.25t$. The curves cross at $V = 2.53t$ which identifies the critical disorder because at this point $\overline{\Upsilon}(L)L^z \equiv g(0)$ becomes independent of the lattice size.

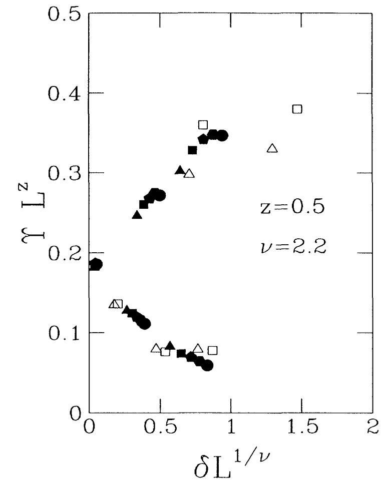

FIG. 3. Scaled plot of $\rho_s(L)L^z$ vs $L^{1/\nu}\delta$ for different lattice sizes and for two temperatures $T = 0.25t$ and $T = 0.125t$. Here $z = 0.5$ and $\nu = 2.2$ for both temperatures; $V_c = 2.5t$ at $T = 0.25t$ and $V_c = 5.2t$ at $T = 0.125t$. The collapse of the data on a single scaling curve shows that we are in the critical regime for quantum fluctuations.

we get $\overline{\Upsilon}L^z = g(0) + g'(0)L^{1/\nu}\delta$ · This equation allows us to extract $\nu = 2.2 \pm 0.2$, which satisfies the bound of $\nu \geq 2/d = 1$ in $d = 2$ [18]. Thus the exponent for the stiffness, $\Upsilon \sim \delta^\zeta$, where $\zeta = (d - 2 + z)\nu \approx 1$. In order to ascertain that the temperature for the data in Fig. 2 is low enough to probe the quantum fluctuations, we have repeated the simulations at $T = 0.125t$ and find that though V_c changes to $5.2t$, the exponents at both temperatures agree within error and the data from both temperatures can be collapsed onto a single scaling function $g(x)$ as shown in Fig. 3. We therefore believe that the transition is governed by the $T = 0$ fixed point. The change in the critical disorder with temperature is given by the relation $T^{1/z\nu} \sim |V_c(T) - V_c(0)|$, where the crossover exponent $z\nu \sim 1.1$. Using a linear extrapolation we deduce that $V_c(0) \sim 8$.

Based on a Lagrangian describing the long wavelength phonon excitations in the superfluid phase, Fisher and co-workers [1] obtained the compressibility $\kappa \sim \delta^{\nu(d-z)}$. Using our value of z, one would conclude that the compressibility vanishes at the transition. We directly compute the compressibility from the density fluctuations $\kappa = \langle\rho^2\rangle - \langle\rho\rangle^2$ and find that it is indeed finite at the transition, contrary to the scaling conjecture of Ref. [1]. Also the bound $z \geq 1$ derived on the assumption of long wavelength phonon excitations is violated.

The conductivity at the transition can be shown [19] to be given by $\sigma_c = \sigma_Q 2(L_\tau^2/N)\langle j(\beta/2)j(0)\rangle$, where $j(\tau)$ is the current operator. Using the value of $\langle j(\beta/2)j(0)\rangle$ at $V = V_c$ from the 64 × 64 lattice data, we find that $\sigma_c = (1.2 \pm 0.2)\sigma_Q$. This is in good agreement with the data on disordered superconducting films [7,8].

Our simulation results of a finite κ at the transition in conjunction with a dynamical exponent $z < 2$ suggests that the low lying excitations are modified from usual phonons in the presence of disorder. Such a conjecture is further supported by the low temperature specific heat obtained deep within the superfluid phase, which is found to deviate from the expected $C_v \sim T^2$ behavior for phonons in 2D [20]. The anomalous specific heat behavior could arise from diffusive density fluctuation modes in the disordered system [21]. Another possible source are remnant localized states in the superfluid [22].

Our results have bearing for the pinning of vortices (equivalent to the boson world lines) by extended-twin boundary or columnar defects (equivalent to the time-independent disorder potential) in disordered type II superconductors in a magnetic field [9]. The Bose-glass phase can be interpreted as a localized vortex phase [11] below a finite temperature $T = T^*$ for the disordered superconductor in which the linear resistivity ρ_L is indeed zero. Following the scaling ansatz in Ref. [23] we obtain $\rho_L \sim |T - T^*|^{\nu(z_D - z)}$ and at $T = T^*$ the electric field scales with the current density as $E \sim J^{(1+z_D)/(1+z)}$, where z_D is a new exponent describing the time scale to relax fluctuations at the transition. Using $\rho_L \sim |T - T^*|^{6.5}$ and $E \sim J^3$ from the data [24] and $z = 1/2$ from our simulations, we obtain $z_D = 3.5$ and $\nu = 2.17$, where the value of ν is consistent with our results. For $z = 2$, the corresponding values are [25] $z_D = 8$ and $\nu = 1.08$.

In conclusion, we have shown that with increasing amounts of disorder, a superfluid gets localized in 2D. We have obtained the exponents characterizing the transition that have provided a window into the nature of the excitations in the disordered system. Our primary result is that, unlike previously believed, an effective action written in terms of the phase of the order parameter and containing only linearly dispersing phonon modes is in a *different* universality class from the disordered boson model. This has led us to propose that the coupling between the amplitude and phase degrees of freedom could lead to *new* low lying excitations in the *disordered* system which are also important at the transition.

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