First Experimental Evidence for Two-Phonon Octupole-γ-Vibrational Excitations in Deformed Nuclei

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Systematic nuclear resonance fluorescence experiments, including model independent parity determinations, provided evidence for enhanced $E1$ excitations near 2.5 MeV excitation energy in the deformed nuclei ^{150}Nd, ^{160}Gd, and ^{162,164}Dy. The corresponding $J^\pi = 1^-$ states are interpreted as two-phonon excitations due to the coupling of octupole ($J = 3^-, K = 1$) and quadrupole γ vibrations ($J = 2^+, K = 2$). The results can be explained in the framework of the *sdf* interacting-boson-approximation model and the dynamic collective model describing octupole vibrations and their coupling to quadrupole vibrations in deformed nuclei.

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Multiphonon excitations, i.e., the coupling of two or more fundamental collective excitations in nuclei, have been examined intensively throughout the last years. An outstanding example is the recent discovery of the double giant electric dipole resonance via relativistic Coulomb excitation [1,2]. At lower excitation energies the coupling of two or three quadrupole phonons is a well known phenomenon [3] and has been observed in several spherical nuclei (see, e.g., Ref. [4]). There have been searches for two-phonon octupole-octupole vibrational states too, in particular in the doubly magic nucleus ^{208}Pb [5]. Evidence for two-phonon double γ vibration ($K^\pi = 4^+$) recently has been found in high resolution (n,γ) experiments [6] and in (^{4}He,2n) reactions [7]. Low lying $J^\pi = 1^-$ states in the spherical $N = 82$ isotones have been detected in nuclear resonance fluorescence experiments [8] and were interpreted to be of a $3^- \otimes 2^+$ structure due to the coupling of the octupole 3^- vibration to a quadrupole 2^+ phonon yielding a quintuplet of states with $J^\pi = 1^-, 2^-, \ldots, 5^-$. In inelastic neutron scattering experiments possible candidates for the other members of the quintuplet were found [9]. Recent photon scattering experiments in the neighboring odd mass nuclei [10] give evidence for even more complex two phonon $\otimes$ particle structures. Whereas those studies focused on spherical nuclei, states with $J^\pi = 1^-, K = 0$ were observed in deformed nuclei of the rare earth region in nuclear resonance fluorescence (NRF) experiments, too [11]. The energies of these strong electric dipole transitions with $B(E1)\uparrow$ values of the order of (10–20)$\times 10^{-3}$ $e^2\,\mathrm{fm}^2$ lie close to the energy of the lowest 3^- excitation. Therefore, these states can be explained by a coupling of the 3^- octupole vibration to the quadrupole deformed shape of the nucleus [11].

The aim of the present Letter is to search for the first time for two-phonon excitations in strongly deformed rare earth nuclei caused by the coupling of octupole vibrations to the $K^\pi = 2^+$, γ vibration. Such two-phonon excitations were theoretically already explicitly treated by Donner and Greiner [12] within the dynamic collective model in 1966. The resulting 1^- states can be excited by dipole transitions from the ground state as a result of the coupling of the giant electric dipole resonance to the octupole vibration [12].

Photon scattering (NRF) experiments represent an outstanding tool to search for such 1^- states in even-even nuclei. The low momentum transfer of real photons leads to a high selectivity in favor of inducing dipole transitions from the ground state. The transition energies can directly be measured using modern high resolution γ spectrometers. Transition widths and reduced transition probabilities can be deduced from the measured scattering cross sections. The spins of the excited states can be determined unambiguously from the measured angular distributions in the case of even-even nuclei [13]. Within the validity of the Alaga rules [14] the measured decay branching ratios to the first excited 2^+ state and to the 0^+ ground state (in even-even deformed nuclei) allow one to determine the K quantum numbers of the excited states. The decay branching ratio R_expt is defined as

$$R_\mathrm{expt} = \frac{\Gamma(1\to 2_1^+)}{\Gamma(1\to 0_1^+)}\,\frac{E^3_{\gamma(1\to 0^+)}}{E^3_{\gamma(1\to 2^+)}} = \frac{B(1\to 2^+)}{B(1\to 0^+)}, \quad (1)$$

with Γ and B denoting the corresponding transition widths and reduced transition probabilities, respectively. For the decay of $J = 1$ states into the ground state rotational band of deformed nuclei, one expects $B(1 \to 2)/B(1 \to 0) = 2$ or 0.5 for $K = 0$ or $K = 1$ states, respectively.

Parities can be determined model independently in photon scattering experiments by measuring the linear polarization of the scattered photons using Compton polarimeters [15].

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The NRF experiments have been performed at the bremsstrahlung facility [16] installed at the Stuttgart Dynamitron accelerator [$E_0 = 4$ MeV; $I \approx 0.8$ mA (CW)]. A sectored single crystal Ge Compton polarimeter [17,18] has been used for the parity determinations. Its polarization sensitivity Q has been measured in $(p, p'\vec{\gamma})$ reaction studies and amounts to about 20% at 0.5 MeV and 10% at 4 MeV, respectively.

As an example Fig. 1 shows new results as obtained in a recent hitherto unpublished $^{164}\text{Dy}(\gamma, \vec{\gamma})$ experiment [19]. The measured azimuthal asymmetries ε are plotted as a function of the transition energy. The dashed lines correspond to the asymmetries expected for pure $M1$ (positive ε) and $E1$ transitions (negative ε), respectively. Both $M1$ and $E1$ excitations occur. The strong $M1$ excitations near 3.1 MeV belong to the orbital excitations of the so-called "scissors mode" first observed in ^{156}Gd [20,21]. Furthermore, positive parities are assigned to a group of states near 2.5 MeV, where two-quasiparticle excitations are expected [22]. The attention should be focused on the isolated, strong $E1$ transition at 2.670 MeV. Similiar $E1$ excitations with transition strengths of the order of $B(E1)\uparrow = (3\text{–}5)\times 10^{-3}\ e^2\,\text{fm}^2$ have been observed systematically in the neighboring nuclei ^{150}Nd, ^{162}Dy [23], and ^{160}Gd [24] in our previous photon polarization measurements. The data on these $E1$ transitions are summarized in Table I. A common and important feature of all these $E1$ excitations observed so far in our experiments is given by the fact that the decay branching ratios R_{expt} of the $E1$ transitions from the corresponding 1^- levels do not fulfill the Alaga rules, which may hint at K mixing [25].

In the following the 1^- states near 2.5 MeV are discussed as possible candidates for two-phonon excitations due to the coupling of quadrupole γ vibrations ($J = 2^+, K = 2$) and octupole vibrations ($J = 3^-, K = 1$). We note that it is obviously impossible to generate states with $J{=}1$ by coupling the γ vibration to octupole excitations with ($J = 3^-, K = 0$). Octupole vibrations in deformed nuclei and their strong coupling to the deformed quadrupole spheroid were treated in detail by Donner and Greiner [12]. Within their notations the energy $E_{2\text{ph}}$ of the two-phonon octupole γ excitation with $K{=}1$ and $J_3{=}1$ depends on the parameters C_{31}, C_{30} connected to the energetic position and distances of different octupole bands [12] (J_3 denotes the third component of the angular momentum J of the octupole vibration):

$$E_{2\text{ph}}^{K=1}(1^-) = \left(1 + 0.146\frac{C_{31}}{C_{30}}\beta_0\right) E_3 + E_\gamma^{K=2}(2^+) + 6\epsilon, \tag{2}$$

where $\epsilon = \hbar^2/\Theta$ is given by the moment of inertia Θ and β_0 is the static quadrupole deformation. $E_\gamma^{K=2}$ and E_3 denote the energies of the γ and the unperturbed, decoupled octupole excitation.

It is easy to deduce the exact relation:

$$E_{2\text{ph}}^{K=1}(1^-) = E_{\text{oct}}^{K=1}(1^-) + E_\gamma^{K=2}(2^+). \tag{3}$$

We note that the result that the energy is simply given by the sum of the phonon energies holds in the model only for the examined case where $K{=}1$ and $J_3{=}1$.

In Table II the data available from our systematic photon scattering experiments and from literature are summarized (see [25] and references cited therein). States with uncertain assignments of the bands are given in parentheses. The information on the position of the octupole vibrational bandheads with $K{=}1$ is rather sparse. Therefore we had to assume in most cases that the first $J^\pi{=}1^-$ state following the $K{=}0$ octupole vibrational bandhead is a $K{=}1$ state. In ^{168}Er we took the third 1^- state because it fits into the systematics of octupole excitation energies. As possible candidates for the two-phonon excitation besides the 1^- states in the four isotopes ^{150}Nd, ^{160}Gd, and ^{162,164}Dy investigated so far by polarization measurements we have taken the lowest $J{=}1$ states above the octupole vibrational bandheads exhibiting an uncommon decay branching ratio. An excellent agreement between the excitation energies $E_{\text{expt}}^{K=1}(1^-)$ of the assumed two-phonon excitations and the sum of the $K = 1$ octupole and the γ-vibrational excitations [ac-

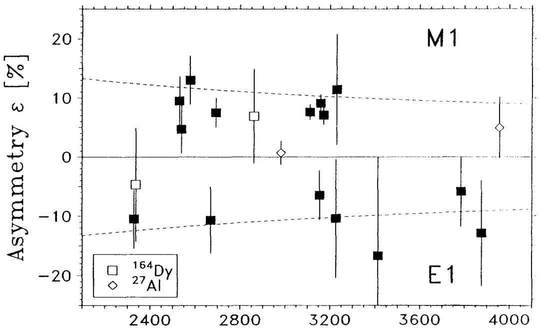

FIG. 1. Experimental results for the azimuthal asymmetries ε measured in the reaction $^{164}\text{Dy}(\gamma, \vec{\gamma}')$. The dashed lines correspond to the asymmetries expected for pure $M1$ ($\varepsilon > 0$) and $E1$ ($\varepsilon < 0$) excitations, respectively. Full symbols correspond to states for which reliable parity assignments were possible.

TABLE I. Experimental data of strong electric dipole transitions near 2.5 MeV from the present NRF experiments including polarization measurements.

Nucleus	E (MeV)	Γ_0 (meV)	R_{expt}	$B(E1)\uparrow$ ($10^{-3}\ e^2\,\text{fm}^2$)
^{150}Nd	2.414	14.9±2.0	0.86±0.09	3.0±0.4
^{160}Gd	2.471	16.4±2.6	1.56±0.21	3.1±0.5
^{162}Dy	2.520	30.2±4.0	1.31±0.08	5.0±0.4
^{164}Dy	2.670	27.0±4.7	1.14±0.24	4.1±0.7

TABLE II. Compilation of experimental data on $J^\pi = 1^-$ states in deformed nuclei from NRF experiments and data on collective bandheads from literature. States with uncertain band assignments are given in parentheses (see text). In the case of ^{156,158}Gd a strong K mixing of the octupole vibrational states has been observed. Therefore we took an average energy for the expected position of the two-phonon state. In ^{168}Er the 1936 keV level is the third 1^- state. R_{expt} is the branching ratio defined in Eq. (1).

Nucleus	$E_{\mathrm{oct}}^{K=0}(1^-)$ (keV)	$E_{\mathrm{oct}}^{K=1}(1^-)$ (keV)	$E_\gamma^{K=2}(2^+)$ (keV)	R_{epxt}	$E_{\mathrm{exp}}^{K=1}(1^-)$ (keV)	$E_{\mathrm{oct}}^{K=1}(1^-) + E_\gamma^{K=2}(2^+)$ (keV)
^{150}Nd	853	(1283)	1062	0.86±0.09	2414	2345
^{156}Gd	(1243)	(1367)	1154	1.49±0.31	2539	2459
^{158}Gd	(1264)	(977)	1187	0.88±0.20	2447	2307
^{162}Dy	1276	1637	888	1.31±0.08	2520	2525
^{164}Dy	1675	(1808)	762	1.14±0.24	2670	2570
^{166}Er	1663	(1830)	786	1.53±0.25	2599	2616
^{168}Er	1786	(1936)	821	1.01±0.09	2728	2757
^{172}Yb	1599	(1155)	1466	0.77±0.14	2617	2621

cording to Eq. (3)] can be stated. Figure 2 illustrates this fact in graphical form.

In order to gain more information about the observed 1^- states we performed a number of calculations in the framework of the *sdf* interacting-boson-approximation (IBA) model [26–29]. The model Hamiltonian consisted of three parts:

$$\hat{H} = \hat{H}_{sd} + \hat{H}_f + \hat{V}_{sd,f} = [\kappa \hat{Q}_{sd} \cdot \hat{Q}_{sd} + \eta \hat{L} \cdot \hat{L}] + [\epsilon_f \hat{n}_f] + [\tilde{\kappa} \hat{Q}_{sd} \cdot \hat{Q}_f + \alpha \hat{L}_d \cdot \hat{L}_f + \beta \hat{O}_{df} \cdot \hat{O}^*_{df}] , \tag{4}$$

where $\hat{Q}_{sd} = s^\dagger \tilde{d} + d^\dagger s + \chi (d^\dagger \tilde{d})^{(2)}$ and $\hat{Q}_f = f^\dagger \tilde{f}^{(2)}$ are the usual quadrupole operators and $\hat{O}_{df} \cdot \hat{O}^*_{df} = (d^\dagger \tilde{f})^{(3)} \cdot (f^\dagger \tilde{d})^{(3)}$ is an octupole-octupole interaction.

We computed energies, $B(E1)$ and $B(E3)$ values, and K quantum numbers. For the $B(E1)$ values we used the two body operator of Ref. [30]. In order to compute states with good K quantum number, a calculation was performed using the SU(3) value for χ. K quantum number assignment was then based on the computed band structures built on the 1^- states.

We performed calculations for the nuclei ^{156,158}Gd, ^{168}Er, and ^{172}Yb which have similar structures. In Fig. 3 we show the results obtained for the nucleus ^{172}Yb, but the other three nuclei displayed essentially the same behavior. The left part shows all $J^\pi = 1^-$ states of the computed level scheme with $E1$ transitions from the

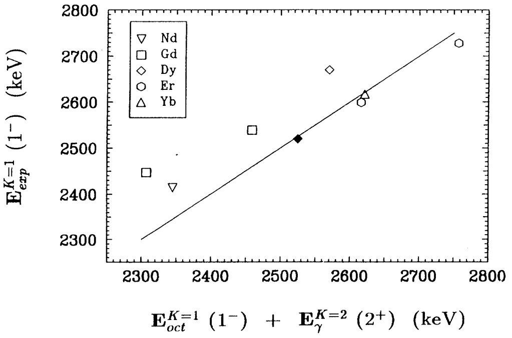

FIG. 2. Experimental excitation energies $E_{\mathrm{expt}}^{K=1}(1^-)$ of states attributed to a two-phonon excitation versus the sum of the $K = 1$ octupole and the γ-vibrational excitations $[E_3^{K=1}(1^-) + E_\gamma^{K=2}(2^+)]$. The full line corresponds to the *exact* fulfilment of Eq. (3). In the case of ^{162}Dy (full symbol) all needed energies are experimentally known (see Table II).

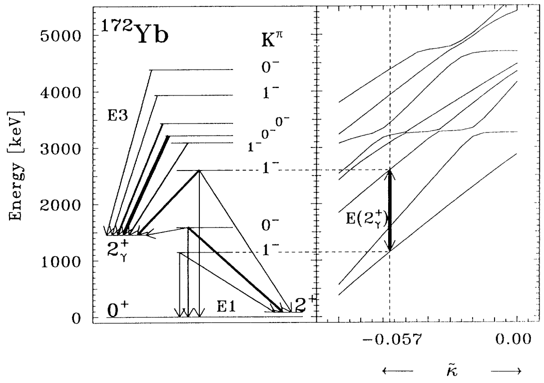

FIG. 3. The left part of the figure shows an excerpt from the computed level scheme of ^{172}Yb with $E1$ and $E3$ transitions; the thickness of the arrow is proportional to the $B(E1)$ or $B(E3)$ values. In the right part of the figure the energies of the lowest 1^- states are shown versus the main parameter $\tilde{\kappa}$ of the interaction $\hat{V}_{sd,f}$. The energy difference between the first, third, and seventh states with $J^\pi = 1^-$ is independent of the interaction strength $\tilde{\kappa}$ and amounts to the excitation energy of the 2^+_γ bandhead. All three states have $K = 1$, as predicted for the two-phonon states. The physical value for $\tilde{\kappa}$ for which the calculations in the left part of the figure were made is marked with a dashed line.

dipole excitations to the ground state and to the first 2^+ state and with $E3$ transitions to the 2^+ bandhead of the γ band. The third 1^- level is the level for which we proposed the $2^+_\gamma \otimes 3^-_{K=1}$ structure. It shows a large $B(E3; 1^-_3 \rightarrow 2^+_\gamma)$ value consistent with the interpretation in terms of a coupled octupole excitation. We note, however, that the $B(E1)$ value calculated in the *sdf* IBA to the K=1, 1^- state is orders of magnitude too small. In the right part of the figure the energies of the lowest 1^- states are shown versus the main parameter $\tilde{\kappa}$ of the potential $\hat{V}_{sd,f}$ describing the interaction between quadrupole and octupole bosons. The energy difference between the lowest $J^\pi = 1^-$ states with $K = 1$ is independent of the interaction strength $\tilde{\kappa}$ and amounts to the excitation energy of the 2^+_γ bandhead. The same behavior is repeated at higher energies: The energy difference between the third and seventh 1^- state is again the excitation energy of the 2^+_γ bandhead and independent of $\tilde{\kappa}$. The physical value of $\tilde{\kappa}$ yielding the experimental energies is marked with the dashed line.

In conclusion, the observed strong $E1$ excitations near 2.5 MeV in deformed nuclei exhibiting an uncommon decay branching may be attributed to a two-phonon excitation caused by the coupling of the octupole and γ-quadrupole vibrations. This conclusion is based on the nearly quantitative agreement of the experimental excitation energies with the sum of the $K = 1$ octupole and $K = 2$ γ vibration as suggested by the collective model and on the results of *sdf* IBA calculations which reproduce the experimental energies and the structure of the states. The *sdf* IBA fails to account for the enhanced $B(E1)$ values. To obtain more information on the structure of these states it is important to get additional experimental data on the octupole vibrational bands as well as on higher lying dipole excitations. Therefore it would be of interest to perform further parity measurements for candidates of such excitations in other rare earth nuclei and in particular to investigate in detail the decay of those states to lower lying states in different collective bands.

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