Experimental Evidence for Self-Affine Roughening in a Micromodel of Geomorphological Evolution

A. Czirók and E. Somfai
Department of Atomic Physics, Eötvös University, Budapest Puskin u 5-7, 1088 Hungary

T. Vicsek
Department of Atomic Physics, Eötvös University, Budapest Puskin u 5-7, 1088 Hungary and Institute for Technical Physics, Budapest, P.O.B. 76, 1325 Hungary
(Received 2 March 1993)

An experimental evidence for kinetic roughening in a micromodel of mountain ranges is presented. We have observed that during the watering of an initially smooth ridge made of a mixture of silica sand and earthy soil the surface evolves into a shape analogous to actual mountain profiles with self-affine geometry. For the exponents describing respectively the temporal and the spatial scaling of the surface width $\beta \approx 0.9$ and $\alpha = 0.78 \pm 0.05$ have been obtained. The latter value is in very good agreement with $\alpha = 0.8 \pm 0.1$ we have calculated for genuine transect profiles.

PACS numbers: 92.40.Gc, 05.40.+j, 64.60.Ht

Although mountain ranges were among the first objects to visually illustrate the fractal aspects of nature's geometry [1], their actual self-affine properties have been demonstrated only recently on a few selected examples [2,3]. Now that we know that these most common geographical patterns can be described in terms of fractals, the fundamental question about the mechanisms producing fractal surfaces in geomorphological evolution arises in a natural way. This is a difficult question to answer because the various processes resulting in the rough surface of mountainous regions are very complex and have a wide range of origins. One of the difficulties is that the formation of mountain ranges takes place over an extremely long period of time, thus direct observations of erosion mechanisms leading to self-affine ridges are hardly feasible. It is expected that model experiments on the erosion of mountain ridges should contribute significantly to our understanding of the emergence of fractal structures in geomorphological phenomena.

The evolution of a self-affine surface can be described in terms of two *exponents* α and β corresponding respectively to the *spatial and temporal scaling of the surface roughness* [4]. In this picture the width or the standard deviation $w(L,t)$ of the surface of linear extent L scales as L^α for long times and as t^β at the early stages of the process. According to the corresponding dynamic scaling theory the width behaves as $w(L,t) \sim L^\alpha f(t/L^{\alpha/\beta})$, where $f(x)$ is some scaling function. The fractal dimension D associated with the rough surfaces can be expressed through α as $D = d - \alpha$, where d is the embedding dimension. Kinetic roughening has attracted great interest recently and the framework of dynamic scaling has widely been used to successfully interpret the related theoretical, simulational, and experimental results [4]. As concerning the geomorphological aspects, for a limited number of cases the exponent α and the fractal dimension (or closely related quantities) have been determined from mountain transect profiles or elevation contour lines [2,3]. The measured α values are in the range between 0.72 and 0.88 for structures of intermediate size [2]. A value 0.57 was obtained [3] from large scale height correlations (from the elevation data for extended regions of Switzerland).

It is widely accepted that, at least on some length scales, the most relevant mechanism responsible for the ragged surface of mountains is *erosion.* However, erosion can affect the surface morphology through various mechanisms and it is to be clarified which of these mechanisms are dominant in the process of fractal roughening. A number of computer models have been suggested to account for the morphological consequences of erosion, in particular, for the evolution of river networks (see, e.g., Ref. [5]). As concerning experiments, in an early work by Purdy [6] the roughening of the surface of limestone blocks was observed when the blocks were sprayed with acid. In this case the very inhomogeneous dissolution of the substrate due to the networks of cracks was the main reason for the appearance of the rough surface. Watering sand in a recent experiment by Wittmann *et al.* [7] resulted in fractal dendritic river patterns and a rough surface. In neither of these works was the self-affine nature of the resulting surface investigated. On the basis of our observations the above experiments represent a limited selection of possible erosion mechanisms. We expect that on macroscopic length scales a relevant mechanism of erosion from the point of fractal roughening is the following. Breaking, dissolution, freezing, and other local effects lead to the disintegration of the material forming the mountain, and this results in a coherent motion of larger masses in the form of landslidelike mass gravity movements of widely scattered magnitude.

In this work we present an experimental evidence for fractal roughening in a micromodel of mountain ranges. Our approach allows us to study the *dynamic scaling* of fractally rough surfaces in processes analogous to *geomorphological phenomena.* We have observed how by watering an initially smooth ridge made of a mixture of silica sand and earthy soil with organic matter the surface evolves into a shape analogous to actual mountain profiles

0031-9007/93/71(13)/2154(4)$06.00

with a self-affine type scaling of the surface width over 2 orders of magnitude. For the dynamic exponent β the estimate ≈ 0.9 has been obtained, while our measured $\alpha = 0.78 \pm 0.05$ is in very good agreement with the value $\alpha = 0.8 \pm 0.1$ we have calculated for the transect profile pictures taken in the Dolomites, Italy.

The experiments were carried out using a micromodel with the following specifications. Channels were adjusted to the four edges of a table of linear size 90 cm. Watering was accomplished by implementing a suitably modified commercial sprayer (we drilled additional holes into the sprayer originally designed for spraying insecticide, and applied a combination of periodic and random motion of the sprayer over the surface). Before each run of the experiment a ridge was erected across the table from a mixture of sand and soil. We sifted the sand and the soil and used mixtures with a well defined sand to soil ratio close to one (ranging from 0.4 to 0.6). The average grain size of the sand (regular silica sand used at constructions) was about 150 μm, while the soil (earthy soil mixed with organic matter, id. Floravin 1991/87131) was purchased at a florist's shop. To have a better knowledge of the substances we used, the *size distribution of the particles* in the mixture has been determined by taking scanning electron micrographs from the samples and using digital image processing techniques to evaluate the pictures. Several other, better defined substances such as high quality clay and large blocks of sugar have also been examined in our experiments; however, quite remarkably, much better results could be obtained with the above simple mixture. The shape of the ridge was such that it had one rectangular shaped and two trapezoid shaped cross sections. The approximate elevation, length, and width of the rectangular shaped plateau were respectively 18, 60, and 4 cm (the actual sizes scattered around these average values). During the experiments water was sprayed with an intensity 1500–3000 cc/min over the surface which gradually roughened as parts of the ridge were taken away by a combination of "*landslides*" and the *flow* of fine grain suspension.

We took many pictures of the *profile* of the experimental surfaces as they eroded under the influence of watering. When digitizing the pictures we used a scanner to achieve a high (3000×1500 pixels) resolution. The evaluation of the images provided us with a function $h(x)$ corresponding to the position (height) of the profile above the table at the point x along the horizontal direction. We also digitized a number of photos published in a book on the Italian Dolomites [8].

Figures 1(a) and 1(b) illustrate the actual appearance of two model "mountains" obtained by spraying the ridge respectively for 20 and 10 min. (To get a better resolution, the pictures which were used for actual evaluation were taken by applying a strong background light providing a much sharper contrast than in the gray-scale photos shown here.) Figure 1(c) shows the digitized and rescaled (reduced in the horizontal direction by a factor of 6) image obtained from the profile of the picture in Fig. 1(b). For comparison [Fig. 1(d)] the digitized profile of one of the mountains in the Dolomites is also displayed. For the first glance the two images Figs. 1(b) and 1(d)

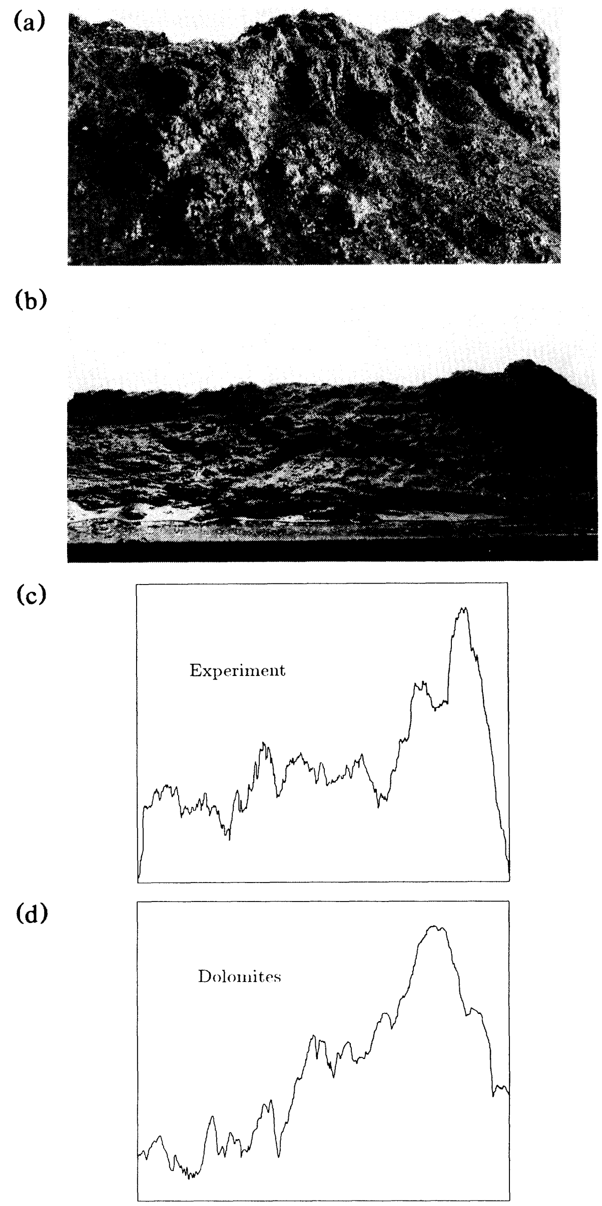

FIG. 1. (a),(b) Two representative morphologies obtained by spraying the model ridges of length 65 cm for 20 and 10 min, respectively. In (c) we display the digitized and rescaled (reduced in the horizontal direction by a factor of 4) image obtained from the profile of the picture in (b). (d) For comparison the digitized profile of one of the mountains in the Dolomites is also shown.

seem quite different, however, the remarkable *similarity* of Figs. 1(c) and 1(d) reveals that it is only the typical slopes which are distinct in the two pictures.

The roughness of the digitized surfaces was characterized using two methods. We calculated (a) the height correlations $c(\Delta x)=\langle|h(x+\Delta x)-h(x)|\rangle_x$ and (b) the surface width $w(\Delta x)=\langle(\langle h^2\rangle-\langle h\rangle^2)^{1/2}\rangle_{x'}$ [where the internal averaging is made over $h(x)$ values in the interval $[x', x'+\Delta x]$].

The results of the experiments are illustrated in Figs. 2 and 3. In Fig. 2 the spatial scaling of the eroded surface is examined by plotting the surface width as a function of Δx on a double logarithmic plot for the digitized profiles shown in Figs. 1(c) and 1(d). Very similar scalings have been obtained for the four further experimental ridges we have investigated by plotting both the widths and the correlation functions. The self-affine nature of the experimental profiles is well demonstrated by the scaling region (a nearly straight part of the corresponding curve) *extending over two decades.* The roughness exponent corresponding to Fig. 1(b) is 0.81. From averaging over five plots analogous to Fig. 2 we find for the roughness exponent of the experimental profiles $\alpha=0.78\pm0.05$ in a surprisingly *good agreement* with the estimate 0.8 ± 0.1 we obtained for the pictures taken from ridges in the Dolomites [8]. This value is also consistent with other measurements [2]. We would like to note that our study is concerned with ridges with a typical length in the range of a few miles, while in the Switzerland study [3] leading to $\alpha\simeq0.57$ the behavior on larger length scales was determined. In fact, Fig. 3 of Ref. [3] demonstrates that for the Switzerland data the behavior of the roughness on smaller length scales crosses over to a scaling with an exponent considerably higher than 0.57.

The question of stability of the result $\alpha\simeq0.78$ against changing the experimental conditions rises in a natural way. We would like to underline that according to our observations the exponent we have found is determined by the actual roughening mechanism and not by the nature of the composite used for the experiments. We arrived at this conclusion by carrying out a long series of experiments with a wide selection of materials. We have observed two major classes of surface development: (i) roughening through landslides and (ii) other processes. Remarkably, on the length scale of our experiments, the more interesting class (i) occurred only for a range of the sand to soil ratio. Once the roughening took place by landslides, the various sand to soil ratios (or further composites made of sand and various sticky materials) resulted in the same exponent α. As concerning the other surface evolution processes (ii) we have found that no well pronounced roughening could be observed using composite ratios very different from one to one or by using various other substances such as clay or even a block of sugar. Pure sand was simply washed away. Clay and a large brick of bulk sugar do not let the water penetrate the main body of the object; in that case surface evolution was entirely due to dissolution effects along the surface. Because of wetting and surface tension effects, however, the characteristic size of the grooves on the surface was too large (in the range of 10–20 cm) for producing a rough surface on the scale of our experiment. In contrast, when the material of the simulated mountains underwent landslides of widely scattered sizes the scale of the smallest structures was about 1 mm.

We also studied the scaling of the width of the entire profile as a function of time in order to get an estimate of the exponent β. The elapsed time was associated with the difference between the initial height and the actual average height of the profiles. At the beginning of the experiment (until the model mountain became completely wet, and the first landslides could occur) the surface remained smooth. Then, the dynamics of the surface roughening was dominated by a combination of gradual roughening

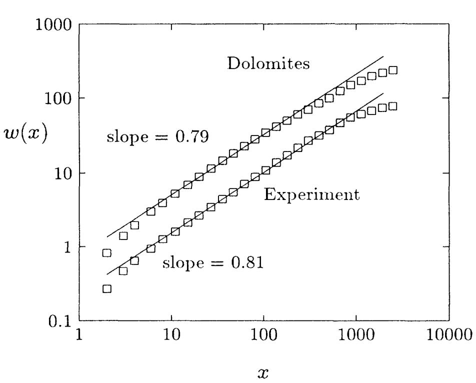

FIG. 2. The surface width (w) for the digitized profiles displayed in Figs. 1(c) and 1(d). The self-affine nature of the experimental profile is well demonstrated by the almost perfect scaling region extending over two decades. The slopes indicate the roughness exponents corresponding to the given profiles.

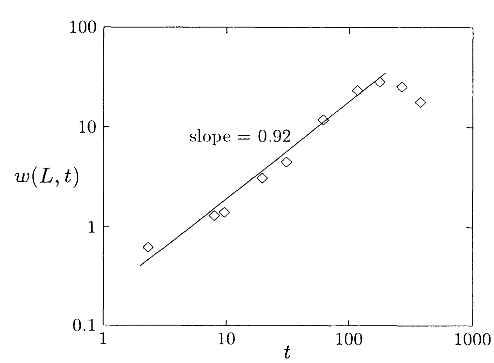

FIG. 3. Temporal scaling of the total width of the experimental profiles. These data, averaged over five independent experiments, suggest a scaling in time with an exponent β near to 0.9. The data curve down for large t values because after some time the total height of the ridge becomes small due to erosion and the height fluctuations are bound to decrease.

and massive landslides resulting in large perturbations of the surface width in time. Because of the lack of an extra averaging the total width $w(t)$ fluctuates much more as a function of time t than, for example, the average widths $w(\Delta x, t \gg 1)$ as a function of Δx. Correspondingly, the scaling in Fig. 3 is less well pronounced than in Fig. 2. However, Fig. 3 indicates a temporal scaling of the total surface width according to an exponent ≈ 0.9. After some time the total width saturated at a level depending on L. We plan to carry out further experiments to investigate in detail the time dependent correlations during the stationary regime of the roughening process.

These results, obtained for our micromodel of geomorphological evolution, demonstrate that the erosion of mountainous regions may lead to self-affine fractal ranges and the whole roughening process can be described in terms of dynamic scaling. As concerning the actual mechanisms for the kinetic roughening of the profiles we observed the following relevant processes: (a) suspended material in the ridge region was carried away by the flowing water, (b) small to very large landslides occurred resulting in sharp changes of the profile over a short interval and widely varying length scales, and (c) the material accumulated at the bottom part due to the landslides was carried away by the flowing water. Apparently all three processes have to take place in order to produce fractal profiles.

Although the relationship of the processes occurring in our experiments to the mechanisms leading to the formation of real mountains with self-affine profiles should be the subject of further study, we think that our findings are suggestive about the emergence of fractal surfaces in geomorphology. We have made several observations which are likely to be relevant from the point of fractal roughening of mountainous regions: (i) A mixture of materials (sand and earthy soil) resulted in more realistic profiles implying the importance of inhomogeneities in the erosion processes. (ii) If the conditions were such that no landslides occurred, no fractally rough surfaces developed in spite of the appearance of river networklike patterns on the surface. (iii) The materials we used led to relatively small characteristic slopes; however, this did not result in a change as concerning the roughness exponent, representing an evidence in favor of the universal nature of the observed behavior. Finally, the landslide mechanism is likely to play an essential role in the fractal roughening of ridges only on length scales below a few miles, and it naturally does not determine the roughness on scales 10–100 miles, where different processes become dominant.

The authors are grateful to E. Illy and A. Mindszenthy for useful discussions. We thank J. Pfeifer for her help in determining the grain size distribution. The present research was supported by the Hungarian Research Foundation Grant No. T4439.

[1] B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, San Francisco, 1982).
[2] D. Norton and S. Sorenson, Pure Appl. Geophys. **131**, 77 (1989); L. E. Gilbert, Pure Appl. Geophys. **131**, 241 (1989); M. Matsushita and S. Ouchi, Physica (Amsterdam) **38D**, 247 (1989); S. Ouchi and M. Matsushita, Geomorphology **5**, 15 (1992); C. G. Chase, Geomorphology **5**, 39 (1992); B. L. Cox and J. S. W. Wang, Fractals **1**, 86 (1993).
[3] G. Dietler and Y.-C. Zhang, Physica (Amsterdam) **191A**, 213 (1992).
[4] See, e.g., *Dynamics of Fractal Surfaces*, edited by F. Family and T. Vicsek (World Scientific, Singapore, 1991); *Surface Disordering*, edited by R. Jullien, J. Kertész, P. Meakin, and D. Wolf (Nova Science, New York, 1992); T. Vicsek, *Fractal Growth Phenomena* (World Scientific, Singapore, 1992).
[5] S. Kramer and M. Marder, Phys. Rev. Lett. **68**, 205 (1992); H. Takayasu and H. Inaoka, Phys. Rev. Lett. **68**, 966 (1992); A. Rinaldo, I. Rodriguez-Iturbe, R. Rigon, E. Ijjasz-Vasquez, and R. L. Bras, Phys. Rev. Lett. **70**, 822 (1993).
[6] E. G. Purdy, in *Reefs in Time and Space*, edited by L. S. Leporte (Society of Economy, Paleontology and Mineralogy, Tulsa, 1974).
[7] R. Wittmann, T. Kautzky, A. Hübler, and E. Lüscher, Naturwissenschaften **78**, 23 (1991).
[8] G. d'Affara, *Il Caldore* (Communita Mountana Centro Caldore, Cortina, 1990).

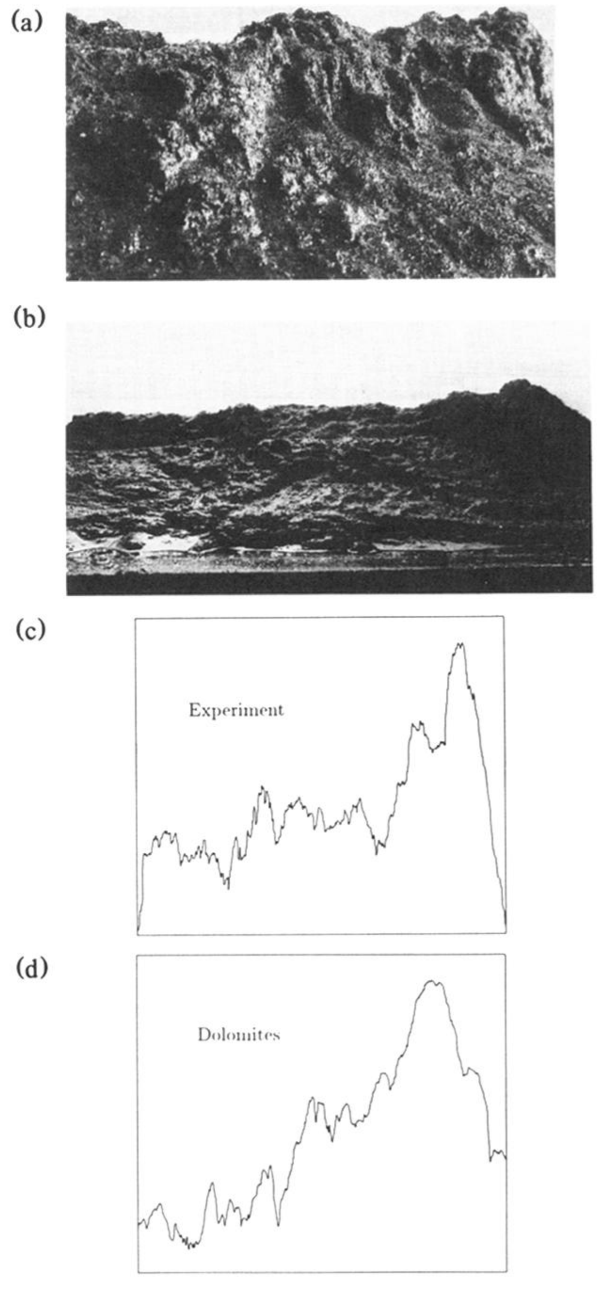

FIG. 1. (a),(b) Two representative morphologies obtained by spraying the model ridges of length 65 cm for 20 and 10 min, respectively. In (c) we display the digitized and rescaled (reduced in the horizontal direction by a factor of 4) image obtained from the profile of the picture in (b). (d) For comparison the digitized profile of one of the mountains in the Dolomites is also shown.