Low-Energy Properties of Two-Dimensional Fermions with Long-Range Current-Current Interactions

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We calculate the one-particle Green function of 2D fermions interacting via a long-range transverse gauge field. Its asymptotic low-energy behavior is found within the eikonal expansion which consistently sums the infrared divergent terms given by "maximally crossed diagrams." Instead of power law corrections to Fermi liquid theory we observe a much stronger singularity which implies a more radical breakdown of Fermi liquid theory than the usual orthogonality catastrophe.

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Considerable interest has focused recently on the idea that many fermionic systems may possess "singular" effective interactions between the quasiparticles. In one development, Anderson has postulated a singular form for the fully renormalized interaction function between fermions in two dimensions [1]. However, the problem of singular interactions more commonly arises in a rather different way; one finds that some process or sum of diagrams leads to a singular form for the effective fermion-fermion interaction, and one would like to go beyond such an approximation in a nonperturbative way. It is important to realize that it is possible to do this, and to give a treatment of theories with singular interactions in an entirely consistent way [2]. It is also important to find examples of singular effective interactions for which the microscopic basis is quite unambiguous.

In the present paper we consider just such an example, in which the singular interaction is generated by the transverse gauge field. In the case of relativistic electromagnetic interactions in ordinary 3D metals this problem was first considered by Holstein, Norton, and Pincus [3] (see also [4]). In the framework of 3D QCD a similar discussion was given recently by Pethick and coworkers [5]. The general problem can be discussed in a standard model of spinless nonrelativistic fermions, at zero temperature, interacting via the Abelian gauge field $\mathbf{A}(\mathbf{r},t)$ which is described by the Hamiltonian written in the gauge $A_0=0$:

$$H=\int d\mathbf{r}\frac{1}{2m}\Psi^\dagger(-i\nabla-g\mathbf{A})^2\Psi-\mu\Psi^\dagger\Psi+\frac{1}{2}\int d\mathbf{r}\left(\frac{\partial\mathbf{A}}{\partial t}\right)^2+(\nabla\times\mathbf{A})^2. \quad (1)$$

Here we concentrate on the 2D counterpart of this problem which was first recognized in the context of gauge models of doped Mott insulators [6].

We shall be primarily interested in the effects of the transverse gauge field $\mathbf{A}_\perp(k)=[\mathbf{k}\times\mathbf{A}(k)]/k$, discarding the longitudinal one (it was recently shown to lead to less singular contributions [7]). As a general fact, in a metallic fermion state, with gapless charge excitations, transverse gauge fluctuations are described by the (retarded) propagator

$$D_{ij}(\omega,\mathbf{q})-\Pi_{ij}^{-1}(\omega,\mathbf{q})=\frac{\delta_{ij}-q_iq_j/q^2}{i\gamma\omega/q+\chi q^2}, \quad (2)$$

which is governed by RPA corrections of Fig. 1(a). The coefficients γ and χ can be approximately found from the one-loop fermion polarization. However, intending to proceed beyond perturbation theory, we will use the expression (2) with arbitrary coefficients.

All previous attempts to analyze the effects of the transverse gauge interaction relied on the calculation of the first self-energy correction shown in Fig. 1(b) which behaves as $\Sigma_{2D}(\epsilon)\approx -g^2(p_F/m\chi^{2/3}\gamma^{1/3})(i\epsilon)^{2/3}$ and $\Sigma_{3D}(\epsilon)\approx -g^2\epsilon\ln(\mu/\epsilon)$ in 2D and 3D, respectively [3-6], where in RPA $\gamma\sim g$ and χ depends weakly on g for g

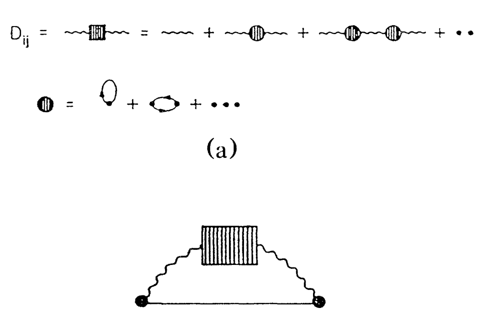

FIG. 1. (a) The RPA-screened transverse gauge field propagator; (b) the lowest-order contribution to the fermion self-energy.

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small. In fact, however, this contribution is only the first one in an infinite series of infrared divergent terms, which must all be dealt with on a consistent footing. In this paper we shall perform a summation of these terms by means of the eikonal expansion, which correctly accounts for the dominant processes of small angle scattering between the states close to the Fermi surface [2]. This will be done here using functional methods.

To proceed with this eikonal expansion we first obtain a formal solution for the one-particle Green function $G(\mathbf{r},\mathbf{r}';t,t';\mathbf{A})$ in a given external field $\mathbf{A}(\mathbf{r},t)$:

$$\left[i\frac{\partial}{\partial t}-\frac{1}{2m}(-i\nabla-g\mathbf{A})^2+\mu\right]G(\mathbf{r},\mathbf{r}';t,t';\mathbf{A})=\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')\,. \tag{3}$$

This can be done in a standard way [8]; we Fourier transform in $\mathbf{r}-\mathbf{r}'$ and $t-t'$ and obtain $[\xi(p)=p^2/2m-\mu]$:

$$\left[\epsilon-\xi(p)+\frac{1}{m}\mathbf{p}\cdot(-i\nabla-g\mathbf{A})+i\frac{\partial}{\partial t}-\frac{1}{2m}(-i\nabla-g\mathbf{A})^2\right]G(\epsilon,\mathbf{p};\mathbf{A}(\mathbf{r},t))=1\,. \tag{4}$$

The solution of (4) can be represented in the integral form

$$G(\epsilon,\mathbf{p};\mathbf{A}(\mathbf{r},t))=i\int_0^\infty d\alpha\exp\{i\alpha[\epsilon-\xi(p)+i\delta_p]\}\exp\Psi(\epsilon,\mathbf{p};\mathbf{r},t;\alpha)\,, \tag{5}$$

where $\delta_p\equiv\delta\,\mathrm{sign}[\xi(p)]$, and $\delta=0^+$. Expanding $\Psi(\epsilon,\mathbf{p};\mathbf{r},t;\alpha)$ as a series in the coupling constant $\Psi=\sum_{n=1}^{\infty}g^n\Psi_n$ we have a set of equations for Ψ_n:

$$i\frac{\partial\Psi_n}{\partial\alpha}=\left[i\frac{\partial}{\partial t}+\mathbf{p}\cdot\frac{\nabla}{im}+\frac{1}{2m}\nabla^2\right]\Psi_n+\mathbf{A}\cdot\frac{\nabla}{im}\Psi_{n-1}+\delta_{n,1}\frac{1}{m}\mathbf{p}\cdot\mathbf{A}-\delta_{n,2}\frac{1}{2m}\mathbf{A}^2-\frac{1}{2m}\sum_{s=1}^{n-1}\nabla\Psi_s\cdot\nabla\Psi_{n-s}\,. \tag{6}$$

This expansion provides an efficient way to find recursively all higher order corrections to the leading eikonal approximation. The latter requires only the lowest order term Ψ_1 given by the expression

$$\Psi_1(\epsilon,\mathbf{p};\mathbf{r},t;\alpha)=\int d^2\mathbf{q}\,d\omega\exp[i(t\omega-\mathbf{r}\cdot\mathbf{q})]\mathbf{p}\cdot\mathbf{A}(\omega,\mathbf{q})\frac{1-\exp\{-i\alpha[\xi(\mathbf{p})-\xi(\mathbf{p}-\mathbf{q})+\omega]\}}{\xi(\mathbf{p})-\xi(\mathbf{p}-\mathbf{q})+\omega}\,. \tag{7}$$

Substituting (7) in (5) and averaging over the gauge fluctuations with the use of (2) we obtain the following formula [$n(p)$ is the Fermi distribution function]:

$$G(\epsilon,\mathbf{p})=i\int_0^\infty d\alpha\exp\{i\alpha[\epsilon-\xi(p)+i\delta_p]\}$$
$$\times\exp\left[ig^2\int d^2\mathbf{q}\,d\omega\,D_{ij}(\omega,\mathbf{q})p_ip_j[1-n(\mathbf{p}-\mathbf{q})]\frac{1-\exp\{-i\alpha[\xi(\mathbf{p})-\xi(\mathbf{p}-\mathbf{q})+\omega]\}}{[\xi(\mathbf{p})-\xi(\mathbf{p}-\mathbf{q})+\omega]^2}\right]. \tag{8}$$

Expanding (8) in g^2 one generates all crossed graphs, including the most important "maximally crossed" ones [2], which give the dominant singular contributions.

The behavior of the Green function in the vicinity of the Fermi energy can be simply found at $p=p_F$:

$$G(\epsilon,p_F)=i\int_0^\infty d\alpha\exp[i\alpha(\epsilon+i\delta_p)-i^{1/3}g^2\alpha^{1/3}]$$
$$=-i\frac{\pi}{\epsilon}\frac{d^2}{dz^2}\mathrm{Hi}(z)\,, \tag{9}$$

where $\epsilon>0$, $\tilde g^2=g^2p_F/m\chi^{2/3}\gamma^{1/3}$ and $z=\exp(-2i\pi/3)\tilde g^2/\epsilon^{1/3}$. The special function $\mathrm{Hi}(z)$ is defined in terms of Airy functions $\mathrm{Ai}(Z)$, $\mathrm{Bi}(z)$, as follows:

$$\mathrm{Hi}(z)=\tfrac{2}{3}\,\mathrm{Bi}(z)+\int_0^z[\mathrm{Ai}(t)\mathrm{Bi}(z)-\mathrm{Ai}(z)\mathrm{Bi}(t)]dt\,. \tag{10}$$

Now at $\epsilon\gg\tilde g^6$ we reproduce the lowest order result of perturbation theory

$$G(\epsilon,p_F)=\frac{-i\pi}{\epsilon}c_1\left[1+c_2\frac{i^{-4/3}\tilde g^2}{\epsilon^{1/3}}+\cdots\right], \tag{11}$$

where $c_1=2/[\sqrt{3}\Gamma(\frac{1}{3})\Gamma(\frac{2}{3})]$, and $c_2=\Gamma(\frac{1}{3})/3^{2/3}$. However, at $\epsilon\ll\tilde g^6$ we find the asymptotic behavior

$$G(\epsilon,p_F)\sim-\pi^{-1/2}\tilde g^{3/2}\epsilon^{-5/4}\exp\left[-\frac{2}{3}\frac{\tilde g^3}{\epsilon^{1/2}}\right]. \tag{12}$$

Notice that the result (12) is essentially nonperturbative and nonexpandable in a power series in g^2. In fact, it implies a more drastic breakdown of the Fermi liquid theory (FLT) for the model (1) than Anderson's orthogonality catastrophe [1] or any kind of Luttinger liquid behavior. In particular, the wave function renormalization arising from (11) is $Z_{p_F}(\epsilon)\sim\exp(-\tilde g^3/\epsilon^{1/2})$, shown in Fig. 2. Moreover, the one-particle distribution function $n(p)$ remains analytic near p_F, with a finite slope; $n(p)=\frac{1}{2}-\xi(p)/\tilde g^6$, for $\xi(p)\ll\tilde g^6$.

A better understanding of $G_{p_F}(\epsilon)$ is obtained by looking at its physical consequences. The perturbative high-energy limit has the dimensionless expansion parameter $\tilde g^2\epsilon^{-1/3}$ (or, equivalently, $\epsilon/\tilde g^6$). At order $\tilde g^{2n}$, the leading contributions to this expansion are coming from max-

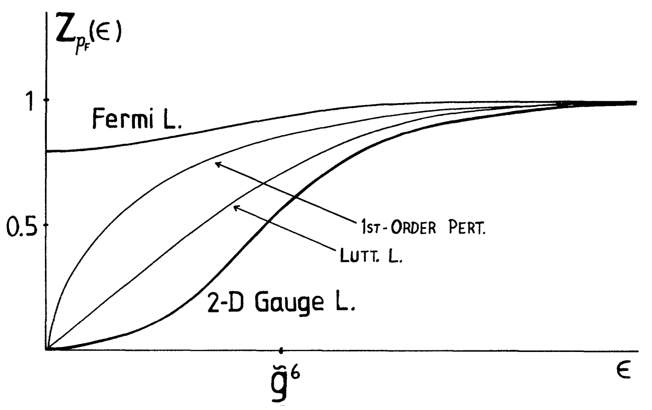

FIG. 2. A graph of $Z_{p_F}(\epsilon)$, the one-particle wave function renormalization. We also show, for comparison, the behavior of a Fermi liquid, a Luttinger liquid, and the extrapolation of the first order perturbative result (11) to zero energy.

imally crossed graphs [2] (with n screened gauge propagators). This expansion is around $z=0$ in (9); the Airy functions are analytic at $z=0$. We note that the energy scale $\tilde{g}^6$ coming from the dimensionless parameter derives ultimately from the peculiar pole at $q\sim(i\omega)^{1/3}$ in $D_{ij}(q,\omega)$; this "anomalous skin effect" type pole is also responsible for the $\frac{1}{3}$ power of α in (9).

The opposite low-energy (or $|z|\gg 1$) limit is nonanalytic in $\tilde{g}^2\epsilon^{-1/3}$ (recall that Airy functions $\sim\exp[\frac{2}{3}z^{3/2}]$, in the wedge of interest, when $|z|\to\infty$), and in fact (12) can be recovered directly from (9) via saddle-point integration. Thus $\epsilon=0$ is a singular point of $G_{p_F}(\epsilon)$, so that even though the eikonal expansion is a sum of crossed diagrams (each of order $\tilde{g}^{2n}$), the low-energy limit is not analytic in $\tilde{g}^2$; we have a genuinely divergent series.

Any physical quantity depending on the one-particle Green function will thus show quite a different behavior as $\epsilon\to 0$ from the perturbative result in (11). For example, the de Haas–van Alphen (dHvA) oscillatory magnetization can be calculated using the same eikonal technique. We have verified explicitly that a factor $\exp[-\pi/6(\tilde{g}^6/\omega_c)r]$ appears in the rth harmonic of the dHvA amplitude (ω_c is the cyclotron frequency); this is analogous to the "Dingle factor" which would appear if impurity scattering were to smooth out the Fermi surface over an energy $\sim\tilde{g}^6$. Here, however, the smoothing in $n(p)$ arises from the interaction itself, as described above.

One can also use the eikonal technique to calculate the entire partition function and various multipoint Green functions summing up the most infrared divergent contributions. It is particularly important to establish that the various response functions are properly related by Ward identities. Gauge invariance can be verified by the observation that within the leading eikonal approximation the irreducible three-point vertex function $\Lambda^\mu(p,q)$ obeys the Ward identity $[q_\mu=(\omega,\mathbf{q})]$:

$$q_\mu\Lambda^\mu(p,q)=G^{-1}(p)-G^{-1}(p+q)\,, \tag{13}$$

where $G(p)$ is taken from (8). This is also a check on the relation between the wave-function renormalization $Z_p(\epsilon)$ and the temporal component of $\Lambda^\mu(p,q)$, i.e., that $Z_p(\epsilon)=\Lambda^0(p,q)|_{q\to 0,\omega/q\to 0}$. This relation is an important check of self-consistency [2].

A closely related check on the application of the eikonal technique is the calculation of the two-particle Green function which describes correlations in particle-particle as well as particle-hole channels. A useful and efficient way to derive this is via Schwinger's "bilinear shift operator" [9]:

$$G_2(x_1,x_2;x_3,x_4;\mathbf{A}(x))=\exp\left(\frac{i}{2}\int_{x_1}^{x_3}dx\int_{x_2}^{x_4}dx'\frac{\delta}{\delta A_i(x)}D_{ij}(x-x')\frac{\delta}{\delta A_j(x')}\right)$$
$$\times[G(x_1,x_3;\mathbf{A}(x))G(x_2,x_4;\mathbf{A}(x))-(x_3\leftrightarrow x_4)]\,. \tag{14}$$

In the context of the eikonal expansion this equation plays the same role for problems involving singular interactions, as the Bethe-Salpeter equation. The averaging of (14) over the gauge fluctuations governed by (2) then generates insertions of the propagator $D_{ij}(x-x')$ joining two fermion lines in all possible ways, including the all-important vertex corrections [2], which again preserves the relevant Ward identities.

One may also calculate correlation functions starting from (14). As one might expect from the Ward identities the singularity in G does not necessarily appear in the response functions; in fact we have found that the static compressibility is finite and regular at low momenta. A full characterization of this system must therefore await a study of all the correlation functions.

The above results indicate a breakdown of FLT in the model (1). Moreover, the low-energy behavior found within the eikonal approximation appears to be quite different from either the "orthogonality catastrophe" [10] [which involves exponentiation of logarithmic divergences to give $Z(\epsilon)\sim\epsilon^\eta$] or the usual phase instabilities (such as pairing or charge density wave). The initial deviation from FLT is captured by the lowest IR-divergent diagram [Fig. 1(b)], but at $\epsilon\sim\tilde{g}^6$ this power-law behavior turns into the exponential asymptotics of Eq. (12), as shown in Fig. 2. In view of this observation we conclude that those physical systems which can be adequately described by this gauge model should not be treated in the Fermi liquid framework. Physical systems which are believed to be described by the 2D gauge model include the $\nu=\frac{1}{2}$ fractional quantum Hall effect [11]; it has also been argued that the normal state of high-temperature superconductors may be describable in these terms.

Another interesting point which arises from this calculation is that the fermions in this gauge model appear to be beyond the reach of the bosonization methods recently developed by Haldane [12] for higher-dimensional Fermi liquids; this is because the spectrum is no longer linear in the vicinity of the Fermi surface. A similar conclusion was reached recently, by applying Haldane's methods directly [13]. It would be interesting to see whether some generalization of these methods could be applied to this kind of problem (or indeed to other kinds of singular interaction).

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