# Morphology and Dynamics of Interfaces in Random Two-Dimensional Media

C. S. Nolle,[1] Belita Koiller,[1,2] Nicos Martys,[3] and Mark O. Robbins[1]
[1]*Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218*
[2]*Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro, CEP 22453, Rio de Janeiro, Brazil*
[3]*National Institute of Standards and Technology, 226/B348, Gaithersburg, Maryland 20899*
(Received 20 January 1993; revised manuscript received 21 July 1993)

We study the morphology and dynamics of an interface driven through a disordered two-dimensional medium by an applied force. At large length scales the interface is self-affine with roughness exponent $\alpha = 1/2$. The structure at small scales may be self-similar or self-affine, depending on the degree of disorder. Simulations of wetting invasion produce self-affine interfaces with $\alpha = 0.8$ and a power law distribution of local interface velocities. Numerical results are in excellent agreement with experiment. A technique that distinguishes between true self-affine scaling and a crossover is presented, and applied to the invasion model and a model for magnetic domain growth.

PACS numbers: 68.10.Gw, 05.40.+j, 47.55.Mh, 75.60.Ch

There has been tremendous recent interest in the effect of disorder on the morphology of moving interfaces [1]. Exact results have been obtained in two dimensions (2D) for the case of "annealed" disorder caused by time-varying thermal or statistical fluctuations [2,3]. This type of disorder occurs in deposition, aggregation, and many other growth processes. Except in special cases, the growing interface evolves into a self-affine fractal with roughness exponent $\alpha = 1/2$. Much less is known about the effect that time-independent or "quenched" disorder has on growing interfaces. Quenched disorder is present when a fluid invades a porous medium or a magnetic domain wall moves through a random magnet. The observation of anomalously large values of $\alpha$ (0.7 to 0.9) in experimental studies of 2D fluid invasion [4–7] has sparked a lively debate on the different effects of quenched and annealed disorder [3,7–12].

Several explanations for the anomalous roughness exponents have been advanced. Medina *et al.* [3] noted that annealed disorder could produce large values of $\alpha$ if it had power law spatial or temporal correlations. Zhang found that a power law distribution of local noise amplitudes could also increase $\alpha$ [9]. Neither group presented an explanation for the origin of such power law behavior or for the relevance of their models to systems with quenched disorder. However, experimental measurements of local interface velocities reveal a power law distribution that is consistent with Zhang's model [10].

Martys *et al.* [11] noted that power law scaling arises naturally at critical points. Their simulations of 2D fluid invasion showed that quenched disorder pinned interfaces until a critical driving force $f_c$ was exceeded. Studies of the critical behavior as the force increased to $f_c$ revealed two universality classes characterized by different interface morphologies at $f_c$. Invasion by a wetting fluid (weak disorder) produced self-affine interfaces with $\alpha = 0.8$, while invasion by a nonwetting fluid (strong disorder) produced self-similar interfaces ($\alpha = 1$) characteristic of percolation. Experiments also produce completely different patterns for wetting and nonwetting invasion [13,14]. Subsequent simulations of other models at $f < f_c$ show that percolative growth occurs whenever the effect of quenched disorder is strong [15–17]. Self-affine growth occurs in some models when the disorder is sufficiently weak [11,15]. In 3D, both simulations [15] and renormalization group calculations [18,19] yield larger roughness exponents for quenched disorder (2/3) than for annealed disorder (0.4).

Kessler *et al.* [12] have argued that the interface is always self-similar at $f_c$ in 2D and that the experimental values of $\alpha$ are an artifact. They presented results from a model which gave values of $\alpha$ near 0.75 due to a crossover from self-similar behavior at short scales to the annealed result, $\alpha = 1/2$, at large scales.

In this paper, we compare results obtained from simulations of wetting invasion to those from a model of magnetic domain wall motion that is known to produce self-similar percolating interfaces. Neither model has been studied previously at $f > f_c$. The two models give qualitatively different results for the interface morphology and dynamics. We show how the crossover from self-similar to self-affine scaling in the magnetic model is easily distinguished from self-affine scaling with an anomalously large roughness exponent. We then present results for the distribution of local interface velocities. Our results for wetting invasion are consistent with experiments [10]. Coupled with Zhang's work on anomalous exponents [9], they imply $\alpha = 0.8$ for wetting invasion, and self-similar structure for the magnetic model. These results agree with direct determinations of the structure.

Since the two growth models have been described in detail in previous papers [11,15,16,20], we will restrict our discussion to a brief outline of the magnetic model, followed by a comparison to the fluid invasion model. The magnet consists of Ising spins, $s_i = \pm 1$, on a square lattice with unit lattice constant. The spins interact with a random field Ising model (RFIM) Hamiltonian:

$$\mathcal{H} = -J \sum_{<i,j>} s_i s_j + \sum_i (H + h_i) s_i \ , \tag{1}$$


0031-9007/93/71(13)/2074(4)$06.00

where $\langle i, j \rangle$ denotes a sum over nearest-neighbor pairs, $J > 0$ is the exchange interaction, $H > 0$ is an external magnetic field that favors the growth of up domains, and $h_i$ is the local random field at each site. The values of $h_i$ are chosen from a uniform distribution on the interval $[-\Delta, \Delta]$. When $\Delta/J$ is large, the spins are effectively decoupled and percolating clusters are formed [21]. We present results for this regime, $\Delta/J = 6$. Self-affine growth is not found for this model [16].

In our simulations, the domain wall begins as a flat line in the $x$ direction that spans the width $L$ of the system. Periodic boundary conditions are applied along $\hat{x}$ and the system height is unrestricted. All spins behind the interface are up $(+1)$, and spins in front of it are down $(-1)$. Growth of the up domain is started by applying a field $H > H_c$, where $H_c$ is the critical field needed to initiate steady motion of the interface. Only spins on the domain wall are allowed to flip [16]. Simulations are done at zero temperature, and spins flip when this lowers the energy of the system. For the results presented below, we assumed that the rate at which an unstable spin flips is proportional to the energy reduction. The quantities calculated do not depend explicitly on the definition of a time scale and other models for the dynamics gave equivalent results [17].

Many features of the fluid invasion model are analogous to those of the magnetic domain model. For example, pores filled by the invading (displaced) fluid correspond to up (down) spins. The driving force $f$ is an applied pressure rather than a magnetic field. In both models $f$ acts normal to the local orientation of the interface. Most other models of interface motion break spatial isotropy by imposing a preferred growth direction and many require the height of the interface along this direction to be single valued [1–3,7–9,12]. These restrictions can produce qualitative changes in the interface morphology. For example, they rule out self-similar fractal growth.

Disorder in the porous medium comes from variations in pore geometry. We construct a model 2D porous medium by placing solid disks of random radii ($0.05 < r < 0.49$) on the sites of a triangular lattice of unit lattice constant and width $L$ [11,20]. The invading fluid is injected from the bottom edge at an applied pressure $P$ and viscous effects are ignored. The interface consists of arcs connecting pairs of disks. Stable arcs must intersect both disks at the contact angle $\theta$ and have curvature $P/\gamma$, where $\gamma$ is the surface tension. If these conditions cannot be satisfied, the arc is unstable. At each time step all unstable arcs on the interface are advanced. The effective coupling between neighboring arcs, and thus the effective degree of disorder, are determined by $\theta$ [20]. Results for nonwetting invasion ($\theta > 50°$ in this model) are equivalent to those presented below for magnetic domains, and require more computational effort. We focus instead on a case of wetting invasion ($\theta = 25°$), where the interface is self-affine at $f_c$ [11].

The external interfaces produced by the magnetic and wetting invasion models are contrasted in Fig. 1. Representative snapshots of moving interfaces are shown for three values of the dimensionless driving force $(f - f_c)/f_c$ [22]. As $f$ decreases, the influence of quenched disorder grows, and the interfaces roughen. The limiting structure at $f_c$ is consistent with previous studies of the models at $f < f_c$ [11,15,16,20]. Magnetic domain boundaries develop the self-similar structure characteristic of percolation, while wetting invasion interfaces remain self-affine. Both models produce interfaces with overhangs—regions where the interface is a multivalued function of $x$. However, the number and scale of overhangs is much larger in the interfaces produced by the RFIM. Overhangs allow the interface to surround unfavorable regions rather than invade them, and are observed in experiments on both wetting and nonwetting invasion [4–7,13]. Without them the interface could be pinned by strong disorder at a single site.

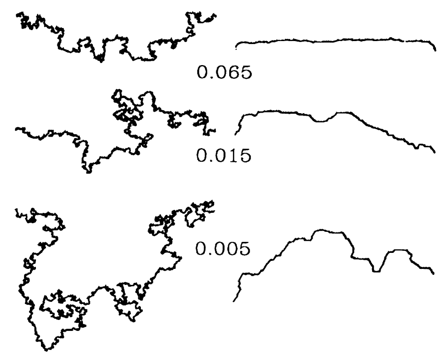


FIG. 1. Interfaces generated with $L = 1000$ at the indicated values of $(f - f_c)/f_c$ for the RFIM (left) and the fluid invasion model (right). The invading domain is below the interface.

The roughness of an interface is generally characterized by calculating the rms fluctuation in height $w(l)$ over intervals of width $l$ [1]. For a self-affine fractal $w(l) \sim l^{\alpha}$, and the value of $\alpha (< 1)$ can be obtained from the slope of a log-log plot. Since $w$ grows less rapidly than $l$, self-affine curves can be represented as single-valued functions at sufficiently large scales [1]. For this reason, the roughness exponent is often determined from the rms fluctuation, $w_{sv}(l)$, of the single-valued interface formed by the highest point at each $x$ [4–7]. We now show that apparent values of $\alpha$ obtained from $w$ and $w_{sv}$ can differ dramatically from each other, and from the true asymptotic value that is found at large $l$.

Figure 2(a) contrasts RFIM results for $w$ and $w_{sv}$ at several values of $f$. Deviations between the two curves are directly related to overhangs. Overhangs produce a nonzero value of $w(0)$ because there is a spread in height at the same $x$. They lead to very rapid rises with $l$ in $w_{sv}$, because the single-valued interface makes large vertical jumps at the edge of each overhang.

We find that the size of the largest overhangs is di-

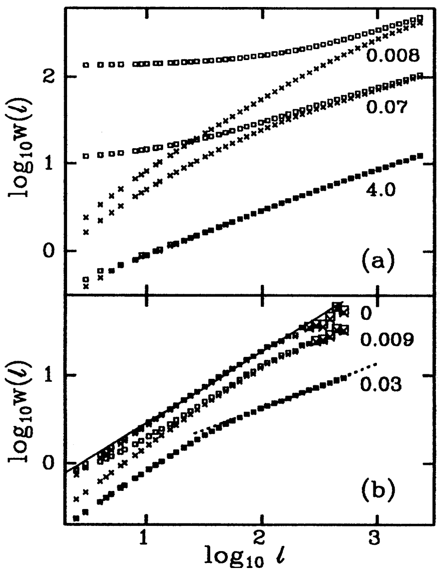


FIG. 2. Values of $w$ (squares) and $w_{sv}$ (crosses) at the indicated values of $(f - f_c)/f_c$ in (a) the RFIM with $L = 8000$ and (b) the fluid invasion model with $L = 1000$. Results were averaged over 10 to 100 independent interfaces and statistical uncertainties are no greater than the symbol size.

rectly related to a correlation length $\xi \propto (f - f_c)^{-\nu}$, where $\nu = 4/3$ is consistent with percolation [17]. Results for different $f$ collapse onto a single universal curve when $w$ and $l$ are normalized by $\xi$. At lengths less than $\xi$, the interface and invaded regions are self-similar fractals whose dimensions are also consistent with those of percolation. For $l > \xi$, results for $w$ and $w_{sv}$ converge, and an asymptotic slope of $\alpha = 1/2$ is obtained [Fig. 2(a)]. This is consistent with the roughness exponent for models with annealed disorder.

Although the structure at $l < \xi$ is not self-affine, one may obtain apparent roughness exponents at these scales from the slopes of the curves in Fig. 2(a). Reasonably good fits may be obtained over a decade or more, but the results from $w$ and $w_{sv}$ are completely different. The apparent roughness exponent obtained from $w$ is nearly zero, while that obtained from $w_{sv}$ is above 0.8. Neither is close to the asymptotic value of 1/2 that is obtained at large $l$ after the two curves join.

Kessler *et al.* had argued that the crossover from self-similar percolation patterns to self-affine scaling could produce anomalously large roughness exponents [12]. Our results show that this is only true if one considers single-valued interfaces. The results for the full interface are actually depressed below 1/2. Comparing $w$ and $w_{sv}$ provides a useful test of whether there is a crossover or true self-affine scaling.

The fluid invasion model produces very different results. Pinned interfaces at $f_c$ have only a few small overhangs, and $w$ and $w_{sv}$ coincide over nearly the entire range of $l$. Both are consistent with $\alpha = 0.8$. Overhangs become more important when $f$ is increased slightly above $f_c$, as illustrated in Fig. 2(b) for $(f - f_c)/f_c = 0.009$. These overhangs are temporary stages of the growth process that occur as the interface surrounds unfavorable regions. They cause a difference between $w$ and $w_{sv}$ at small scales, but at large scales both $w$ and $w_{sv}$ are consistent with $\alpha = 0.8$. As $f$ increases further, overhangs disappear and $w$ and $w_{sv}$ coincide over the entire range of $l$. There is a clear crossover in both curves from $\alpha = 0.8$ at small scales to the annealed disorder result, $\alpha = 1/2$, at large scales. The crossover length decreases as $f$ increases and corresponds to a correlation length $\xi \sim (f - f_c)^{-1.3}$ [17]. A similar crossover has been observed in experiments on wetting invasion [5].

The dynamic exponents of the two growth models are also very different. Results for the power law relating the mean velocity $v$ to $f - f_c$, the power law noise in the instantaneous velocity, and the power law describing the total width of the interface will be presented in a subsequent paper [17]. Here we focus on a particularly interesting example, the exponent describing the distribution of growth rates. This exponent has been measured in wetting invasion experiments [10] and is related to Zhang's model for anomalous roughness exponents [9].

Zhang considered a model of annealed noise with a power law distribution of noise amplitudes and overdamped dynamics [9]. Horvath *et al.* [10] noted that this would lead to a power law distribution in the rate of advance of local sections of the interface. They determined the experimental distribution from the displacement between single-valued interfaces, $z(x)$, at times separated by a small $\Delta t$. The effective noise amplitude and deviation from the mean velocity are both proportional to

$$\eta(x,t) = \frac{z(x,t+\Delta t) - z(x,t) - \Delta z}{\Delta z}\,, \qquad (2)$$

where $\Delta z = v\Delta t$ is the mean displacement [23]. Horvath *et al.* found a power law distribution $\rho(\eta) \sim 1/\eta^{1+\mu}$ with $\mu = 2.7(2)$. This corresponds to a value of $\alpha$ near 0.8 in Zhang's model, which is close to the value measured directly from the interface.

Results for $\rho(\eta)$ from our two growth models are compared in Fig. 3. In both cases we decreased $\Delta z$ until the distribution became constant, and then averaged over many pairs of interfaces. If $\Delta z$ was too large, fluctuations in the local velocity averaged out and the distributions were cut off. The distributions were also cut off by the correlation length $\xi$. At shorter scales, the distributions were well fit by power laws.

The exponents describing the velocity distributions for the two models are very different: $\mu = 2.7(2)$ for wetting invasion and $\mu = 0.7(1)$ for the RFIM. The result for wetting invasion is consistent with experiments [10]. The RFIM distribution has a much longer tail because of overhangs. The single-valued interface at a given $x$

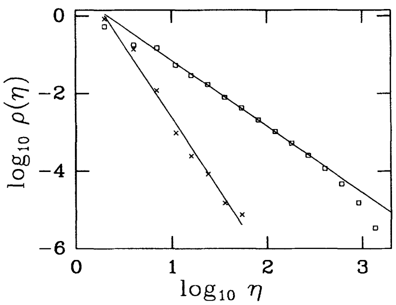


FIG. 3. Distribution of local interface velocities for the RFIM with $L = 3000$, $(f - f_c)/f_c = 0.017$ (squares) and the fluid invasion model with $L = 1000$, $(f - f_c)/f_c = 0.009$ (crosses). Statistical errors are smaller than the symbols.

jumps forward as soon as an overhang appears above it. Zhang's calculations yielded $\alpha \approx 0.8$ for $\mu = 2.7$ and self-similar growth ($\alpha = 1$) for $\mu < 2$. These results are completely consistent with direct analysis of the interface morphology in both cases.

The results presented here clearly imply that the anomalous roughness exponent observed in wetting invasion experiments is not an artifact resulting from a crossover between percolation and self-affine growth. Such a crossover is easily detected by comparing the scaling of $w$ and $w_{sv}$. The actual origin of the anomalous exponent appears to be proximity to a critical point and the associated power law behavior. At any finite velocity, power law scaling is cut off at the correlation length. The structure and dynamics cross over to those for annealed noise at larger scales. Viscous effects may also introduce cutoffs in fluid invasion experiments [6].

Power law scaling shows up directly in the distribution of local interface velocities. While the independently determined values of $\mu$ and $\alpha$ are consistent with experiment [10] and Zhang's work, the factors determining these exponents remain unknown. There has been recent progress in developing scaling relations between critical exponents near $f_c$ [11,18,19] and this line of research holds continued promise.

We thank M. Cieplak, F. Family, D. S. Fisher, O. Narayan, T. Natterman, and P.-z. Wong for useful discussions. Support from NSF Grant No. DMR-9110004 is gratefully acknowledged.

[1] See, for example, *Dynamics of Fractal Surfaces*, edited by F. Family and T. Vicsek (World Scientific, Singapore, 1991).
[2] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).
[3] E. Medina, T. Hwa, M. Kardar, and Y.-C. Zhang, Phys. Rev. A **39**, 3053 (1989).
[4] M. A. Rubio, C. Edwards, A. Dougherty, and J. P. Gollub, Phys. Rev. Lett. **63**, 1685 (1989); **65**, 1339 (1990).
[5] V. K. Horváth, F. Family, and T. Vicsek, Phys. Rev. Lett. **65**, 1388 (1990); J. Phys. A **24**, L25 (1991).
[6] S. He, G. L. M. K. S. Kahanda, and P.-z. Wong, Phys. Rev. Lett. **69**, 3731 (1992).
[7] S. V. Buldyrev, A.-L. Babarasi, F. Caserta, S. Havlin, H. E. Stanley, and T. Vicsek, Phys. Rev. A **45**, R8313 (1992).
[8] L.-H. Tang and H. Leschorn, Phys. Rev. A **45**, R8309 (1992).
[9] Y.-C. Zhang, J. Phys. (Paris) **51**, 2129 (1990).
[10] V. K. Horváth, F. Family, and T. Vicsek, Phys. Rev. Lett. **67**, 3207 (1991).
[11] N. Martys, M. Cieplak, and M. O. Robbins, Phys. Rev. Lett. **66**, 1058 (1991); Phys. Rev. B **44**, 12 294 (1991).
[12] D. A. Kessler, H. Levine, and Y. Tu, Phys. Rev. A **43**, 4551 (1991).
[13] R. Lenormand, J. Phys. Condens. Matter **2**, SA79 (1990), and references therein.
[14] J. P. Stokes, D. A. Weitz, J. P. Gollub, A. Dougherty, M. O. Robbins, P. M. Chaikin, and H. M. Lindsay, Phys. Rev. Lett. **57**, 1718 (1986).
[15] H. Ji and M. O. Robbins, Phys. Rev. B **46**, 14 519 (1992).
[16] H. Ji and M. O. Robbins, Phys. Rev. A **44**, 2538 (1991); B. Koiller, H. Ji, and M. O. Robbins, Phys. Rev. B **46**, 5258 (1992).
[17] C. S. Nolle, B. Koiller, N. Martys, and M. O. Robbins (to be published).
[18] T. Natterman, S. Stepanow, L.-H. Tang, and H. Leschorn, J. Phys. II (France) **2**, 1483 (1992).
[19] O. Narayan and D. S. Fisher (to be published).
[20] M. Cieplak and M. O. Robbins, Phys. Rev. Lett. **60**, 2042 (1988); Phys. Rev. B **41**, 11 508 (1990).
[21] R. Lenormand and S. Bories, C.R. Acad. Sci. Ser. B **291**, 279 (1980); R. Chandler, J. Koplik, K. Lerman and J. F. Willemsen, J. Fluid Mech. **119**, 249 (1982).
[22] Values of $f_c$ were determined by finding the mean force needed to span a square system of size $L$ and extrapolating the result to infinite system size [11,16]. The fractional uncertainty is less than $10^{-3}$.
[23] Since $\eta$ does not depend on the definition of time scale, different growth algorithms give the same $\mu$.