Comment on "Experimental Measurements of the Roughness of Brittle Cracks"

In a recent Letter, Måløy *et al.* [1] reported an elegant experimental study of the roughness exponent ζ for fractured surfaces of six different brittle materials. This exponent describes the scaling of roughness w, defined as the width of the profile, with the length L of a one-dimensional cut through the surface, $w \propto L^{\zeta}$ [2]. Måløy *et al.* argue that they find a value of $\zeta = 0.87 \pm 0.07$ which is universal for all brittle materials, and conjecture that this universal value may also apply for ductile fracture. In this Comment (i) we point out that there are strong indications that these values are probably not universal and depend on material properties, and (ii) we propose a method to test this issue.

A large body of data on fracture surfaces dating back to the pioneering paper by Mandelbrot, Passoja, and Paullay [3] can be compared with that of Måløy *et al.* Much of these data have been analyzed in terms of the local fractal dimension D of the self-affine surface, fractal behavior being expected for either brittle or ductile fracture when regarded as growth or defect percolation processes, respectively. The value of ζ is equal to the codimension given as $3-D$ if D is determined by box counting, by Fourier analysis, or by the slit island method [3]. Alternatively it is equal to $1/(D_d - 1)$, where D_d is a divider dimension evaluated from the scaling of the profile length with the ruler size [4]. The data available in the literature at mesoscopic level for D give ζ that ranges between 0.7 and 1 for a wide variety of materials and modes of fracture [5]. Moreover, within this range many studies find correlations between ζ and mechanical properties [3,5], an observation that contradicts the very notion of universality, and therefore casts doubt on this hypothesis.

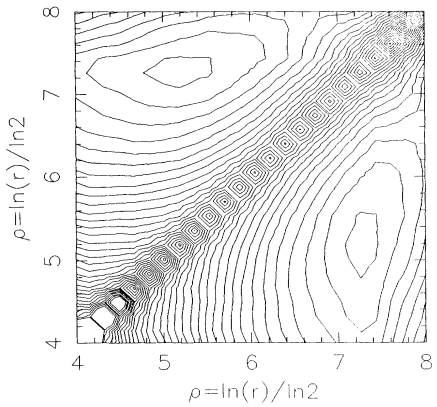

FIG. 1. Contour plot of $M(\rho,\rho')$ for the STM image of the tungsten fracture surface (recording step is 20 nm).

In addition, it is not uncommon to find that materials exhibit different values of ζ on different scales [3,6]. Data of scanning tunneling microscopy (STM) reported for MgO, Si, and Cu [5] suggest that ζ is 0.6 ± 0.1 on a nanometer scale, which is beyond the error margin of mesoscopic studies. We have analyzed STM images of brittle fracture surfaces of tungsten single crystals and of highly oriented pyrolitic graphite taken with sampling steps from 0.1 to 20 nm and found $\zeta = 0.40 \pm 0.15$ from Fourier analysis and from the pair correlation function. The same value was derived from direct evaluation of w as a function of L.

We suggest that a test of self-affinity, and better characterization, could be obtained by generalizing a recently proposed correlation matrix [7],

$$M(\rho,\rho') = \frac{\langle [h(\mathbf{r}_1)-h(\mathbf{r}_2)]^2 [h(\mathbf{r}_1)-h(\mathbf{r}_3)]^2 \rangle_{|\mathbf{r}_1-\mathbf{r}_2|=2^{\rho};|\mathbf{r}_1-\mathbf{r}_3|=2^{\rho'}}}{\langle [h(\mathbf{r}_1)-h(\mathbf{r}_2)]^2 \rangle_{|\mathbf{r}_1-\mathbf{r}_2|=2^{\rho}} \langle [h(\mathbf{r}_1)-h(\mathbf{r}_3)]^2 \rangle_{|\mathbf{r}_1-\mathbf{r}_3|=2^{\rho'}}} . \quad (1)$$

That analysis provides a quantitative description of structural features beyond the fractal dimension. For a self-affine surface, M should depend only on the relative separation, $|\rho-\rho'|$. The contour plot of $M(\rho,\rho')$ for the tungsten surface shown in Fig. 1 exhibits such behavior apart from end effects. We have found that subsets of the structure are uncorrelated for $|\rho-\rho'| \approx 4$, while for the graphite surface $|\rho-\rho'| \approx 3$.

To conclude, our and previous works show different roughness exponents for different materials and different length scales, as well as correlations between ζ and material properties. This, combined with the limited scale measurements of microscopic and mesoscopic surfaces (usually less than two decades), makes difficult any claim for universality. We suggest that more rigorous tests of self-affinity be carried out to support the self-similarity and to discriminate between different structures with close values of ζ. This is important because ζ alone is not sufficient to describe the surface geometry, which may explain the failure of numerous attempts to find a universal relation between D and the fracture toughness [5,6].

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