Comment on "Attractive Forces between Electrons in (2+1)-Dimensional QED"

It has recently been claimed by Girotti *et al.* [1,2] that electron-electron bound states are possible in QED_3. They have furthermore used their model to perform extensive numerical calculations in order to calculate transition temperatures in high-T_c superconductors. The point to be made in this Comment consists of the observation that the binding term of Girotti *et al.* is an improper reduction of the Aharonov-Bohm (AB) [3] potential $[(l+\alpha_{\rm in}/2)^2-l^2]/r^2$ to simply $l\alpha_{\rm in}/r^2$ at large r. In fact footnote [4] of Ref. [1] makes it clear that its authors are aware of at least one work which includes the additional $\alpha_{\rm in}^2$ repulsive term [4]. However, they imply that their approach is defensible because it is "based solely on relativistic quantum field theory."

The model of Ref. [1] in fact consists of a relativistic perturbation theory calculation which is then inserted into a Schrödinger equation. Since the final result thus requires a Galilean limit, it is perfectly sensible to test the model of Ref. [1] by taking its $c\to\infty$ limit together $\theta_{\rm in}\to-\infty$ with $\theta_{\rm in}/e^2$ finite. While this eliminates the mass $|\theta_{\rm in}|$ particle from the physical spectrum, it has the advantage of yielding a theory which is exactly soluble as well as fully relativistic (with respect to the Galilei group of space-time transformations) and is thus able to provide an unambiguous test of the proposed model in the indicated limit. It is to be emphasized that such a procedure is fully consistent with the analysis of Refs. [1] and [2] which claim validity for $e^2/m\ll 1$. In fact the vertex corrections which are shown there to be of order e^2/m in Ref. [1] are well known to be totally absent in the Galilean limit.

Actually, this problem has been solved some years ago [5]. It was shown at that time that the Hamiltonian of the field theory is

$$H=-\frac{1}{2m}\int d^2x\,\psi^\dagger\left[\nabla_i+ig^2\bar{\nabla}_i\int\mathcal{D}(x-x')\times\rho(x')dx'\right]^2\psi\,,$$

where $\bar{\nabla}_i=\epsilon_{ij}\nabla_j$, $g^2=-e^2/\theta_{\rm in}$, and $\mathcal{D}(x)$ is the negative of the inverse Laplacian. The operator ρ is the charge density $\psi^\dagger\psi$ where $\psi(x)$ satisfies the equal time commutation relation

$$[\psi(x),\psi^\dagger(x')]=\delta(x-x')\,.$$

For simplicity the spin has been ignored, a feature which can, however, readily be restored [6].

The eigenvalue equation $H|N\rangle=E|N\rangle$ can be solved in the two-particle ($N=2$) sector. Then one writes

$$|2\rangle=\int d^2x_1d^2x_2\,\psi^\dagger(x_1)\psi^\dagger(x_2)f(x_1,x_2)|0\rangle$$

and finds in the center-of-mass frame that $f(x_1,x_2)$ reduces to $f_l(r)e^{il\phi}$ (where r and ϕ are the polar coordinates of $\mathbf{x}_1-\mathbf{x}_2$). Significantly, $f_l(r)$ satisfies the equation

$$\left[\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}+mE-(l+g^2/2\pi)^2/r^2\right]f_l(r)=0\,,$$

which has a strictly repulsive $1/r^2$ effective potential. With the identification $g^2/\pi=\alpha_{\rm in}$ this reproduces the Schrödinger equation of Ref. [1] in the $\theta_{\rm in}\to-\infty$ limit except for the omission in the latter of the repulsive $(g^2/2\pi r)^2$ term. It is to be emphasized that the present approach fully realizes the ideal of Ref. [1] of a purely field theoretical formulation. This is only partially achieved in Ref. [1] where it is necessary to make a crucial junction between a perturbative field theory calculation and the Schrödinger equation. The result obtained here establishes what would have been anticipated merely on the grounds of gauge invariance—namely, that the quadratic term cannot be neglected. Thus it is not possible to obtain in a consistent way the long range attractive potential claimed in Refs. [1] and [2] and the numerical results presented there cannot be expected to be relevant to the high-T_c problem.

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[4] The absence of this term in Ref. [1] can be traced to their incorrect calculation of the matrix element of γ_i between initial and final electron states. It is known from the AB solution of Ref. [3] that a plane wave is modified by a factor of $e^{-i\alpha_{\rm in}\phi/2}$. Inclusion of this long range correction (analogous to terms well known in Coulombic wave functions) restores the full AB potential and thereby eliminates the attractive part of the potential found in Refs. [1] and [2].
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