## Comment on "Second-Order Dynamics in the Collective Temporal Evolution of Complex Systems"

In a recent Letter [1] Binder and Privman argue that it is possible to explain, using "second-order dynamics," some of the interesting behaviors recently discovered by two of us (Chaté and Manneville) [2,3] in classes of extended systems with local interactions and synchronous updating in high dimensions. Further, they attribute such behavior to the discreteness of time. The novel behavior discussed in Refs. [2,3] are nontrivial evolution patterns seen in the density of high-dimensional cellular automata and coupled map lattices, which typically repeat periodically or quasiperiodically in time. A characteristic feature of these densities is a superimposed noise, whose amplitude decreases with the system size, originating from the local chaos in the lattice. This noise was found to be indistinguishable from stochastic noise and to show scaling behavior with the lattice size [2,3(a)].

Binder and Privman determined the evolution of two 1D lattices of logistic maps of small sizes. By defining a density $m(t)$ for the maps as the average of site values, they compare the shapes of plots of $m(t+1)-2m(t)+m(t-1)$ against $m(t)-M$ and $m(t+1)$ against $m(t)$ with corresponding ones for the 5D $R^5_{5,9}$ automata. Based on the degree of *shape similarity* between such plots they imply the dynamics of the high-dimensional automata is ruled by the same second-order dynamics that they find to rule the coupled map lattice. However, the *non sequitur* of such an implication may be seen from two essential differences: (i) Their 1D examples are systems with too few sites to allow any conclusion. In fact, the behavior on which their conclusions are based is common in dynamical systems with few degrees of freedom. Moreover, they only investigated a very narrow size-dependent range of parameters. The behavior they report disappears in the limit of infinite size [4]. (ii) The most fundamental characteristic of the high-dimensional automata that both systems discussed by Binder and Privman miss is the *noisy structure.* The noisy structure can be clearly seen in their Fig. 1 (which reproduces results for rule $R^5_{5,9}$, discussed in Refs. [2] and [3]), while it is *totally absent* in Figs. 2 and 3, corresponding to their models. Actually, for coupled map lattices, this can be quantified by the largest Lyapunov exponent, which is clearly positive in the systems discussed in Ref. [2]. This shortcoming is independent of the size of the system.

In our opinion, the only similarity between the simulations performed by Binder and Privman and those discussed in Refs. [2] and [3] is the underlying quasiperiodicity in both the high-dimensional automata and the lattice of coupled maps. Quasiperiodicity is a common behavior of dynamical systems approximated by second-order difference or differential equations. One should not conclude on the sole basis of similar quasiperiodic behavior observed in different dynamical systems that the underlying mechanism responsible for them is the same.

Binder and Privman use the argument that since irrational frequencies can emerge from discrete-time dynamics, their second-order dynamics makes previous droplet-type arguments irrelevant as models of $R^5_{5,9}$. However, the appearance of irrational frequencies, the basic signature of quasiperiodicity, is by no means exclusively a consequence of discrete-time dynamics. Binder and Privman noticed that their discrete-time second-order dynamics generates quasiperiodic plots. Since the systems in Refs. [2,3] also display quasiperiodicity, they conclude a discrete-time second-order dynamics "to explain phenomenologically" the complex phenomena discussed in Refs. [2,3]. But discrete time is not needed: It is easily possible to generate plots similar to their Figs. 2 and 3 using, say, $A\sin(\omega_1 t)+B\sin(\omega_2 t)$, $\omega_1/\omega_2$ irrational, which is the solution of a continuous-time differential equation. That the dynamics is second order can hardly be a surprise since in its most fundamental manifestation quasiperiodicity involves *two* incommensurate frequencies.

In summary, after observing that some 1D discrete-time second-order dynamical models display quasiperiodic behavior, Binder and Privman proceed to draw conclusions about the behavior of collective states observed in the much more complex systems discussed in Refs. [2] and [3]. In our opinion such conclusions are certainly premature. Quasiperiodicity is a common behavior and not an exclusivity of the discrete-time second-order equations discussed in Ref. [1]. In order to conclude something about the complex behavior of the complicated and rich automata discussed in Refs. [2] and [3] and state that the models in Ref. [1] make "previous droplet-type arguments irrelevant," one should come up with a model dealing with at least some of the main characteristics of the subject in question, namely, the local noise, its origin, and its variation with system size.

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