Radial Scale Length of Turbulent Fluctuations in the Main Core of TFTR Plasmas

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A new theory of microwave reflectometry is applied to turbulence measurements in the TFTR tokamak. Results indicate that in the main core of neutral beam heated plasmas in the supershot regime, the radial correlation length of density fluctuations is a weak decreasing function of heating power in the range 2–4 cm. Over the same interval of heating powers (0–14 MW), the level of density fluctuations is observed to steadily increase by more than an order of magnitude.

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Various observations [1–5] have indicated that a small-scale turbulence exists in tokamaks in the range of frequencies around $\omega_* = k_\theta c T_e/eBL_n$, where k_θ is the poloidal wave number and $L_n = |d\ln n/dr|^{-1}$ is the density scale length. While the initial observations [1,2] on small size tokamaks showed a maximum in the spectrum at wavelengths of the order of the ion Larmor radius (i.e., $k_\theta \rho_i \approx 1$), subsequent measurements [3–5] on larger devices showed spectra with monotonically decreasing amplitude with wave number, hence the conjecture that the scale length of turbulence in tokamaks is more sensitive to the size of the magnetic configuration than to the value of the ion Larmor radius. This is consistent with recent observations on TFTR [6] and JET [7] showing that the local plasma transport follows a Bohm-like scaling. The recent introduction of correlation reflectometry and beam emission spectroscopy on large tokamak experiments [8–10] has been motivated by the need for new localized measurements of long wavelength fluctuations.

In the method of correlation reflectometry [8], where the spectrum of density fluctuations is inferred from the characteristics of waves reflected from closely spaced cutoff layers, it was soon found that in the presence of a large level of density fluctuations the measured radial correlation length was clearly inconsistent with measurements obtained using standard microwave scattering techniques.

In this Letter, we present a theory of correlation reflectometry which accounts for all the major characteristics of waves reflected from strong fluctuations at the cutoff layer nearly 100% amplitude modulation in the reflected wave; a broadening in the angular spectrum of reflected waves when the coherent center frequency disappears; a corresponding dramatic broadening of the received frequency spectrum; and a corresponding reduction in the observed signal correlation length to values almost as small as the vacuum wavelength of the probe beam.

The theory presented in this Letter is used to infer the turbulence correlation length from data taken with a microwave reflectometer in the main core of TFTR plasmas in the supershot regime, and to investigate the dependence on heating power. The reflectometer system operated in the X mode and consisted of one fixed frequency (122 GHz) reference channel and one tunable frequency (111–123 GHz) channel. The use of scalar guides and Gaussian optics produced well collimated beams inside the plasma with a diameter of ≈ 5 cm.

In a previous study [11], it was shown that in the limit of small amplitude fluctuations (i.e., $\tilde{n}/n \leq 10^{-3}$ for the case of this Letter) the scattered field originates in a small region near the cutoff when the radial wavelength λ_r of the density fluctuation is larger than approximately 2 times the vacuum wavelength of the launched wave. For X-mode reflectometry on TFTR, this corresponds to the condition $k_r = 2\pi/\lambda_r < 12$ cm^{-1} which is well satisfied by density fluctuations existing in TFTR plasmas [5]. For $k_\theta \approx 0$, the scattered field corresponds to a phase modulation $\delta\phi$ of the reflected wave which, for long wavelength fluctuations, can be obtained from geometric optics. The importance of this result is that the geometric optics approximation is not limited to the case of small fluctuations. To find how this result is modified in the case of $k_\theta \neq 0$, we have solved numerically the wave equation in a 2D geometry (r,z) where a plane wave is launched perpendicularly to a plane stratified plasma equilibrium perturbed by the density fluctuation

$$\tilde{n} = an(r)\cos[k_r(r-r_c)]\cos(k_\theta z)\,, \tag{1}$$

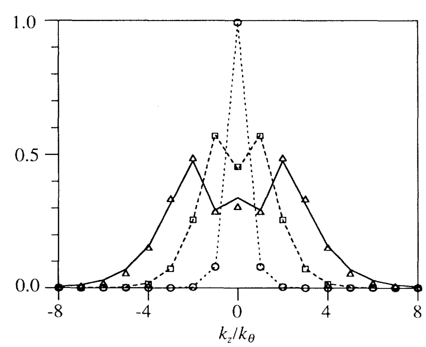

FIG. 1. Poloidal spectrum of reflected waves for the density perturbation of Eq. (1). Lines are from the numerical solution of the wave equation with $k_r = 2$ cm^{-1}, $k_\theta = 1.5$ cm^{-1}, and $a = 0.001$ (dotted line), $a = 0.01$ (dashed line), and $a = 0.02$ (solid line); symbols are from the geometric optics approximation.

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where a is a constant, $n(r)$ is the unperturbed plasma density, and r_c is the radial position of the cutoff. In vacuum, the reflected wave can be expanded in a series of plane waves with wave-vector components $k_z = lk_\theta$, where l is an integer. Taking an equilibrium profile similar to that of a typical TFTR discharge, we find the spectrum of amplitudes displayed in Fig. 1 for the cases of $k_0 = 25$ cm^{-1} (122 GHz), $k_r = 2$ cm^{-1}, and $k_\theta = 1.5$ cm^{-1}, and for $a = 0.001$, 0.01, and 0.02. In vacuum, we obtain a similar spectrum of plane waves assuming that the plasma density fluctuation modulates the phase of the reflected wave by the amount $\Delta\phi = \delta\phi\cos(k_\theta z)$. The amplitudes of the plane waves are then given by $|J_l(\delta\phi)|$, where J_l is the Bessel function of the first kind of order l. For the cases of Fig. 1, this agrees with the solution of the wave equation if we take for $\delta\phi$ the value given by geometric optics for the density perturbation of Eq. (1) with $k_\theta = 0$. Hence we conclude that, at least up to values of $a \le 0.02$ and $k_\theta \le 2$ cm^{-1}, the effect of density perturbations on the reflected wave can be approximated by a phase modulation given by geometric optics. Based on these findings, we present a theory of reflectometry in which the plasma is assumed to behave as a corrugated random phase screen of arbitrary amplitude. Such corrugations produce an angular distribution of waves in vacuum which are measured using directionally sensitive receivers. We may approximate the observable k_θ spectrum of the reflected waves in vacuum by the expression

$$E(k_\theta,\omega) \propto \exp(ik_r r)\int\int dz\,dt\,\exp[-i(k_\theta z - \omega t) + i\phi(z,t)], \quad k_r = \sqrt{k_0^2 - k_\theta^2}\,, \tag{2}$$

where $E(k_\theta,\omega)$ is a wave Fourier component,

$$\phi(z,t) = 2k_0\int_0^{r_c}\sqrt{\varepsilon(z,r,t)}\,dr \tag{3}$$

is the phase given by geometric optics, and ε is the plasma permittivity. For locally homogeneous stationary fluctuations the common power between two reflectometer channels with reflecting layers at r_1 and r_2 is

$$\langle E_1(k_\theta,\omega)E_2^*(k_\theta,\omega)\rangle = \int\int dz\,dt\,\exp[-i(k_\theta z - \omega t)]\langle e^{i\Delta\phi}\rangle\,, \tag{4}$$

where $\Delta\phi$ is the difference of the phases given by Eq. (3) for the two channels, the brackets $\langle\;\rangle$ represent ensemble averages, and $\langle e^{i\Delta\phi}\rangle$ is the characteristic function of $\Delta\phi$. The spectral coherence for waves reflected from two closely spaced cutoff layers is then given by

$$\gamma_E(\Delta r_c) = \frac{\langle \tilde{E}_1(k_\theta,\omega)\tilde{E}_2^*(k_\theta,\omega)\rangle}{\langle|\tilde{E}_1(k_\theta,\omega)|^2\rangle}\,, \tag{5}$$

where Δr_c is the separation of the cutoff layers and $\tilde{E}(k_\theta,\omega) = E(k_\theta,\omega) - \langle E(k_\theta,\omega)\rangle$ is the random component of the scattered field. The correlation length in reflectometry is then obtained from the measurement of the coherence for a range of Δr_c. For small fluctuation levels the characteristic function need only be expanded up to second order in $\Delta\phi$ so that knowledge of the probability density of ϕ is not required. However, to evaluate the common power for arbitrary fluctuation levels the characteristic function needs to be evaluated to higher order moments in $\Delta\phi$. For large amplitude fluctuations we must make the further reasonable assumption that the probability density of ϕ is approximately Gaussian

$$P(\phi) = \frac{1}{\sqrt{2\pi}\sigma_\phi}\exp(-\phi^2/2\sigma_\phi^2)\,, \tag{6}$$

where $\sigma_\phi = \langle\phi^2\rangle^{1/2}$. Under this assumption the common power and spectral coherence may be expressed in terms of powers of the normalized correlation $\gamma_\phi = \langle\phi_1\phi_2\rangle/\sigma_\phi^2$ as

$$\gamma_E(\Delta r_c) = \sum_{n=1}^{\infty}\frac{\sigma_\phi^{2n}}{n!}\Phi_n(k_\theta,\omega;\Delta r_c)\Big/\sum_{n=1}^{\infty}\frac{\sigma_\phi^{2n}}{n!}\Phi_n(k_\theta,\omega;0)\,, \tag{7}$$

where

$$\Phi_n = \int\int dz\,dt\,\exp[-i(k_\theta z - \omega t)]\gamma_\phi^n(z,t;\Delta r_c)\,. \tag{8}$$

Note that as σ_ϕ increases higher order terms in γ_ϕ begin to contribute to the observed spectrum and measured coherence. As we will show, assuming that γ_ϕ decreases monotonically with increasing Δr_c, the radial correlation length and correlation time of the received signals will decrease as the plasma fluctuation level increases. From Eq. (8), the angular spread of the reflected waves should also increase as higher order terms in γ_ϕ begin to dominate the scattered spectrum, as is also shown by the numerical results of Fig. 1.

In order to evaluate Φ_n to all orders we assume an approximate Gaussian form for γ_ϕ as

$$\gamma_\phi = \exp(-r^2/2\sigma_r^2)\exp[-(z - vt)^2/2\sigma_\theta^2]\exp(-t^2/2\sigma_t^2)\,, \tag{9}$$

where v is the apparent poloidal velocity of the fluctuations and σ_r, σ_θ, and σ_t are the radial correlation length, poloidal correlation length, and correlation time, respectively. Inserting Eq. (9) into Eqs. (8) and (7) we obtain

$$\gamma_E = \frac{\sum_{n=1}^{\infty}(\sigma_\phi^{2n}/nn!)\exp(-nr^2/2\sigma_r^2)\exp(-k_\theta^2\sigma_\theta^2/2n)\exp[-(\omega - k_\theta v)^2\sigma_t^2/2n]}{\sum_{n=1}^{\infty}(\sigma_\phi^{2n}/nn!)\exp(-k_\theta^2\sigma_\theta^2/2n)\exp[-(\omega - k_\theta v)^2\sigma_t^2/2n]}\,. \tag{10}$$

The spectrum of plane waves for any one reflectometer channel is proportional to the denominator of Eq. (10). Note that for $k_\theta \ne 0$ the peak in the frequency spectrum occurs for $\omega \approx vk_\theta$, independent of the fluctuation amplitude. If

k_θ is known from the alignment of the receiver and transmitter to the plasma reflecting layer, then the plasma rotation may be inferred from the Doppler shift in the peak of the frequency spectrum. This is consistent with the notion that the fluctuations at the plasma cutoff act as a rotating diffraction grating where the nonspecular reflection experiences a Doppler shift determined only by the scattering angle and the plasma velocity. Restricting the measurement to $k_\theta \approx 0$ and $\omega \approx 0$ we obtain

$$\gamma_E \approx \frac{\mathrm{Ei}(\sigma_\phi^2 \gamma_r) - \xi - \ln(\sigma_\phi^2 \gamma_r)}{\mathrm{Ei}(\sigma_\phi^2) - \xi - \ln(\sigma_\phi^2)} , \qquad (11)$$

where Ei is the exponential integral, $\xi = 0.577$ is the Euler number, and $\gamma_r = \exp(-r^2/2\sigma_r^2)$ is the radial component of γ_ϕ. In the limit of $\sigma_\phi \ll 1$ we arrive back at the linear or Born approximation where $\gamma_E \approx \gamma_r$, whereas for $\sigma_\phi^2 \gamma_r > 2.5$ we obtain from the asymptotic form of Ei

$$\gamma_E \approx \frac{1}{\gamma_r} \exp[-\sigma_\phi^2(1-\gamma_r)] . \qquad (12)$$

An important result is that for sufficiently large σ_ϕ the value of the signal correlation may be much smaller than the corresponding value of γ_r. Expanding γ_r to second order it is readily shown that $\gamma_E \approx \exp(-r^2/2\sigma_E^2)$ where $\sigma_E = \sigma_r/\sigma_\phi$ so that the correlation length of the measured signal is a factor of σ_ϕ smaller than the correlation length of the phase. It may also be shown numerically from Eq. (10) that the k_θ and ω spectrum of reflected waves broadens by a factor of σ_ϕ over the corresponding spectral width of the fluctuations. This broadening is also shown in the numerical results of Fig. 1.

In order to relate the measured signal correlation to the plasma fluctuations we introduce the approximation for the perturbed phase $\tilde{\phi} = k_0 \int_0^{r_c} \tilde{\varepsilon}/(\varepsilon_0)^{1/2} dr$ which yields numerical results very close to those of Eq. (3). Also, since the measurement is localized to a narrow region around the cutoff, we use the linear approximation $\varepsilon_0 \approx (d\varepsilon_0/dr)_{r=r_c}(r-r_c)$. From this we derive the result

$$\Gamma_\phi(k_r) \approx 2\pi \frac{k_0^2}{k_r |d\varepsilon_0/dr|_{r=r_c}} [C^2(\alpha) + S^2(\alpha)] \Gamma_\varepsilon(k_r) , \qquad (13)$$

where $\Gamma_\phi(k_r)$ and $\Gamma_\varepsilon(k_r)$ are the radial Fourier transforms of the phase and permittivity correlations, $C(\alpha)$ and $S(\alpha)$ are the Fresnel integrals, and $\alpha = (2k_r r_c/\pi)^{1/2}$. From this we may determine the correlation length of density fluctuations from reflectometer measurements.

The experimental results presented in this Letter have been obtained in neutral beam heated TFTR discharges in the supershot regime [12]. The main plasma parameters were minor radius 0.8 m, major radius 2.45 m, toroidal field 4.2–4.8 T, plasma current 1.2 MA, central electron density $(4\text{–}6)\times 10^{19}$ m^{-3}, central electron temperature 5–8 keV, central ion temperature 5–20 keV, and neutral beam power 2–14 MW.

Figure 2 shows the frequency spectrum of the reflected waves during the Ohmic and the neutral beam (NB) phase of a TFTR discharge with $P_{\mathrm{NB}} = 14$ MW. These spectra were obtained from the common power between two closely spaced reflectometer channels at $r/a \approx 0.3$. The reflecting layers were separated by a distance of the order of 3 mm. Consequently, in both cases the spectral coherence (also shown in Fig. 2) is close to unity over the entire spectral range. Note that the spike in the frequency spectrum at $\omega \approx 0$ during the Ohmic phase, which represents the unscattered component of the reflected wave, has totally disappeared in the NB phase of the discharge. This is evidence for a transition from weak to strong scattering. As suggested by the preceding theory, this transition should be accompanied by a broadening of the frequency spectrum as shown in Fig. 2, together with a narrowing of the signal correlation length as shown in Fig. 3, which displays the signal coherence at $\omega \approx 0$ as a function of the reflecting layers separation. Within the accuracy of the measurement, the data can be fitted to a Gaussian. From these data it appears that during NB heating the spectral width of the signal increases by more than an order of magnitude over the Ohmic phase together with a corresponding decrease in the measured coherence length as predicted by our random phase screen model. By scanning the relative angle between transmitter and receiver we have determined that the Ohmic k_θ spectrum is in the range $k_\theta \le 1$ cm^{-1}, which is of the order of the instrumental k resolution of the receiver. The observed frequencies in the Ohmic phase also correspond to the drift wave range of frequencies for $k_\theta \le 1$ In contrast, the NB spectrum is well outside the range of drift frequencies, since during the heating phase the electron temperature increases by less than a factor of 2, leading to no more than a factor of 2 increase in the drift wave frequency. The observed large spectral width can be explained in part by the Doppler broadening caused by the plasma toroidal rotation which is induced by NB injection. This broadening can be estimated from the measured plasma velocity if the poloidal magnetic field profile

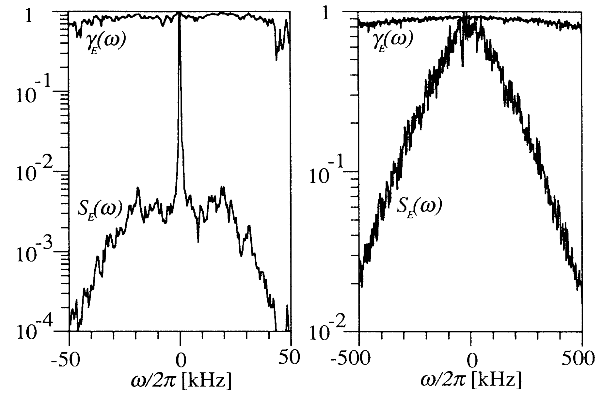

FIG. 2. Frequency power spectra of reflected waves during Ohmic (left) and NB injection (right) with $P = 14$ MW. $S_E(\omega)$ is the power spectrum; $\gamma_E(\omega)$ is the spectral coherence for a channel separation of $\approx 2\pi/k_0$.

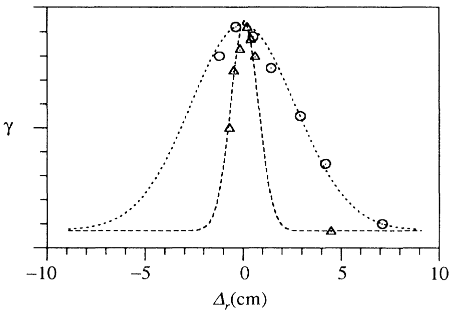

FIG. 3. Signal coherence (γ_E) at $\omega \approx 0$ as a function of reflecting layers separation for the two cases of Fig. 2 (circles are for Ohmic; triangles are for NB). Lines are Gaussian best fits.

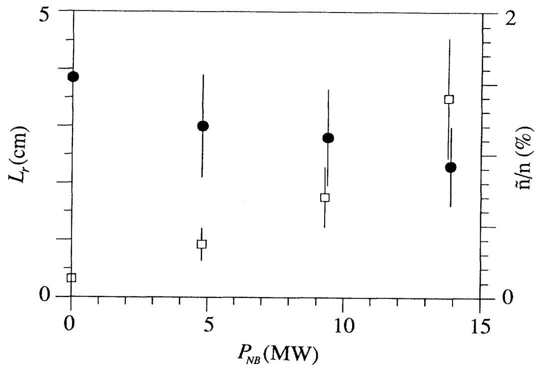

FIG. 4. Density radial scale length (circles) and density fluctuation level (squares) as a function of NB power at $r/a \approx 0.3$.

is known, which is rarely the case. Alternatively, the propagation velocity and hence the Doppler broadening may be directly obtained from the peak of the measured frequency spectra for $k_\theta \neq 0$. We conclude that although the Doppler effect is important, it cannot account for the observed frequency broadening of the signal. From our random phase screen model, we know that the spectrum of reflected waves must also be broadened by an increased level of turbulence, which for Fig. 2 corresponds to an increase of σ_ϕ from ≈ 0.4 to ≈ 3.6 rad. In fact, while the Ohmic value of σ_ϕ is small enough to guarantee that only the first term in the series of Eq. (10) is important so that $\gamma_E \approx \gamma_r$, the value of σ_ϕ during NB injection is much larger than unity and therefore higher order terms contribute to the measured spectral coherence. For large σ_ϕ we must apply Eqs. (11) and (13) to the measured coherence of reflected waves (Fig. 3) in order to derive the density correlation. The results can again be fitted by a Gaussian with a half-width (L_r) of the $1/e$ points which is a decreasing function of beam power, from ≈ 4 cm in Ohmic to ≈ 2 cm at 14 MW (Fig. 4). From the value of L_r we obtain the spectral width $\Delta k_r \approx 2/L_r$ corresponding to $\Delta k_r \rho_i \approx 0.1$–$0.3$. In the same range of heating powers the level of density fluctuation $\tilde{n}/n$ increases with heating power from 1×10^{-3} to approximately 1.5×10^{-2} as shown in Fig. 4. This steady increase of the measured density fluctuation with beam power is inconsistent with the mixing length criterion $\tilde{n}/n \approx L_r/L_n$ since the measured value of L_n is independent of heating power (≈ 50 cm). Finally, if we assume from simple random walk arguments that $\chi \approx L_r^2/t_c$ (with t_c the correlation time) and we take $t_c \propto 1/\omega_*$ and $k_\theta \approx \Delta k_r$, since during the heating phase ion and electron temperatures at $r/a \approx 0.3$ increase by less than a factor of 2, the trend shown by the data of Fig. 4 is consistent with the observation that in the supershot regime the local thermal conductivity of both ions and electrons is only a weak function of beam power [13].

In conclusion, a new theory of microwave reflectometry has been applied to data taken in the main core of neutral beam heated plasmas of the TFTR tokamak in the supershot regime. The results indicate that the radial correlation length of density fluctuations is a weak decreasing function of beam power. Over the same range of heating powers, the level of density fluctuations is observed to steadily increase with beam power. This is inconsistent with mixing length estimates given the weak variation of the density scale length with beam power in these plasmas.

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