Surface Waves in Vertically Vibrated Granular Materials

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Noncohesive granular materials driven by a vertically vibrated horizontal surface become unstable when the acceleration exceeds g. Then there is internal convective transport of materials balanced by continuous surface avalanches leading to an inclined surface. We describe studies above the onset to flow where new surface wave motions occur. The surface waves travel upward against gravity on the free surface. They occur only above minimum accelerations and dimensionless energies, and only with large vibration amplitude.

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Granular materials have both solidlike and fluidlike behavior: under weak shear, they deform plastically; under high shear, they flow like a fluid. Granular materials exhibit other unusual behavior such as size segregation [1], density waves [2], convective transport [3], and anomalous sound propagation [4].

Recently there has been great interest in the physics of noncohesive granular materials lying on a vertically vibrating surface [3,5–9] which typically vibrate with sinusoidal oscillations, $z = a\sin(\omega t)$. A relevant dimensionless parameter is the reduced dimensionless acceleration amplitude, $\Gamma = a\omega^2/g$ (g is the acceleration of gravity, a the physical amplitude of the vibrating floor, and ω the angular frequency of the vibration). As Γ increases above a critical value Γ_c, the free surface of the granular material becomes unstable. In many circumstances, a single heap forms, making an angle θ_d with the horizontal (a dynamic angle of repose). Continuous surface avalanches result which are sustained by a slower internal convective transport of material towards the top of the heap. Evesque and Rajchenbach [5] have shown that the transition to convection depends only on Γ. However, the processes involved in convective flows are only partially understood. At the very basic level it is important to determine if there are other relevant dimensionless parameters, such as a typical dimensionless kinetic energy, $E \equiv (a\omega)^2/gd = \Gamma a/d$ where d is the grain diameter. It is also relevant to pursue the effects of grain shape and the dynamics of the states which occur above the onset of convection.

In this Letter, we describe experiments on an annular layer of vertically vibrated sand. A new kind of surface wave motion is found [9] which occurs in granular materials which can sustain a relatively high repose angle in the presence of vibration, typically materials with nonspherical grains. This state is distinct from that reported by Douady *et al.* [6]. The waves move upward against gravity on the free surface of the granular material. Both Γ and E are relevant parameters for these waves.

The experiments are carried out using a mechanical shaker driven by a dc motor, as sketched in Fig. 1. The shaker is mounted on a bearing-guided precision steel bar in order to ensure the truest possible vertical motion. We use annular cells with narrow gaps. This geometry eliminates the influence of one pair of lateral boundaries, giving a periodic boundary condition in the azimuthal direction. The cell is made of two concentric cylinders of Plexiglas. The diameter of the inner cylinder is about 10 cm, and the outer diameter is varied to give radial gap sizes from 0.3 cm to 1.0 cm. The grain diameters d range between 0.02 cm and 0.2 cm. The gap is filled with granular materials to a depth of 3 to 5 cm which corresponds to a large number of particle diameters, in contrast to the experiments of Douady *et al.* [6]. Several materials are used, including metal spheres and ellipsoids, rough and smooth sand, glass spheres, and grass seed. Vibration frequencies range over $0 < \omega/2\pi \leq 30$ Hz; most of the experiments are carried out at vibration amplitudes between 0.03 cm and 0.6 cm. The vertical displacement of the cell and its vibration frequency are measured using an optical technique to be described elsewhere. The patterns formed in the gap are sampled by a video camera, and we use several techniques to visualize

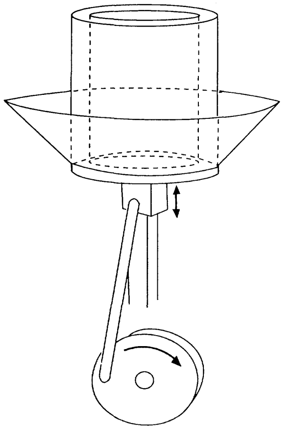

FIG. 1. Sketch of the shaker apparatus.

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the convective and wave motion. In the first, we mount a conical mirror outside the walls of the shaker; a distant video camera with a telephoto lens and fast shutter provide images in which the vertical vibrational motion is effectively removed. Additionally, when $\omega/2\pi$ is related rationally to the video acquisition frequency $\omega_v/2\pi = 30$ Hz, a charged coupled device camera with a fast shutter provide particularly useful images. If $\omega/\omega_v = p/q$, where p and q are integers, the sequence of images consisting of every qth frame shows the slow evolution of the flow without the shaking motion, regardless of the position of the camera.

The onset of fluidization and flow is independent of the material used and is found to occur at $\Gamma_c = 1.17\pm0.05$, in essential agreement with other shaker experiments [5–7]. Below Γ_c, the granular particles respond as solid materials, i.e., they keep their original positions relative to the shaker, as long as the angle of a given heap is not too steep. Just above Γ_c, steady convective motion occurs in the form of two counterrotating rolls leading to a single symmetric heap. The azimuthal position of the heap drifts with time as long as a is not too big, which implies that asymmetries in the experiment are small. We will refer to the angular position of the heap as ϕ_h. There is a corresponding valley which occurs at $\phi_h + \pi$.

As Γ increases further above Γ_c, a new surface wave motion is observed for granular materials which have relatively large θ_d, a condition which is met by rough sand or long thin grass seed. In the remainder of this work, we will focus on materials which show the surface waves. (Materials like grass seed are also interesting because they manifestly contain an orientational degree of freedom which can play an important role in flows [10]. In addition, the grains tend to align with the flow, so that streamlines are easily visualized.)

The qualitative features of the surface wave are as follows: (a) Initially, one small bulge appears on the free surface near the lowest point of the heap, $\phi_h + \pi$ (where the two different avalanches meet). The bulge splits into two parts which travel upward on each side of the heap faster than the convective velocity of the particles near the surface, but under the avalanche layers. Figure 2 shows a time series of the generation and initial outward propagation of these pulses for long thin grass seed. Figure 3 shows one side of the heap highlighting the upward motion of a pulse. Arrows in the figures indicate the positions of the pulses. As the pulse travels up the slope, it spreads out, increasing its length and decreasing its height, and eventually disappears near the top of the heap. During the upward climb of the surface wave, there is still a continuous downward avalanche on the top of the wavy surface layer. (The surface wave motion is observed for a rectangular cell also, but the surface waves are better defined in the annular cell.) (b) The onset of surface waves coincides with the formation of shear bands within the convective region. Shear bands are localized regions of intense shearing in the granular materials which separate domains of granular materials. For grass seed, the orientational order of the grains is poorly defined within a shear band, and the density is low compared to the rest of the sample. During the free flight of each cycle, the shear bands dilate and the granular particles from the avalanche layers diffuse into the bands. Each pulse

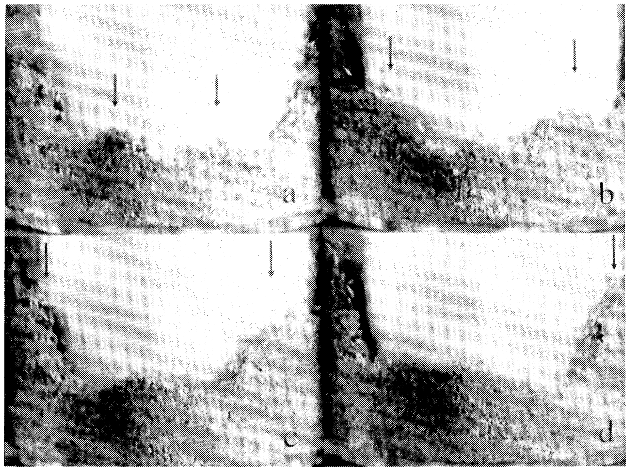

FIG. 2. Images showing a side view near the origin of the waves at the lowest point of the heap. A pulse forms and splits into two parts as it begins to travel up the slope. These images and those in Figs. 3 and 4 were obtained using video rates of 15 Hz. Times are $t = 0$ (a), $t = 0.4$ sec (b), $t = 0.8$ sec (c), and $t = 1.13$ sec (d). Here, $\Gamma = 3.17$, $\omega/2\pi = 15$ Hz, and $a = 0.35$ cm. The material for this and Fig. 3 is grass seed.

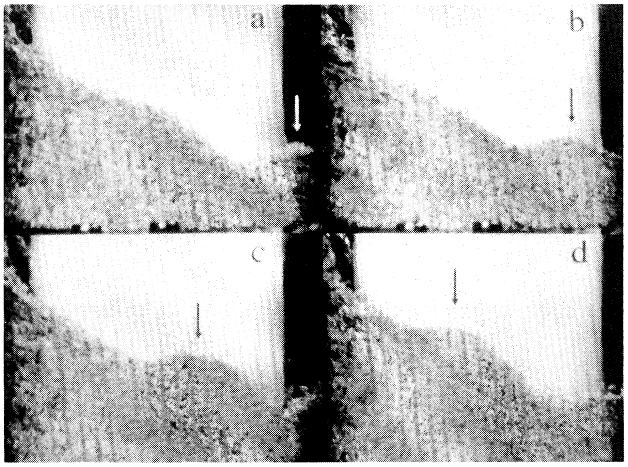

FIG. 3. Side view of the surface waves traveling upward along the surface of the heap. Here, $\Gamma = 3.17$, $\omega/2\pi = 15$ Hz, and $a = 0.35$ cm. Times are $t = 0$ (a) (note the pulse forming at the far right), $t = 0.4$ sec (b), $t = 0.8$ sec (c), and $t = 1.26$ sec (d).

on the surface is created simultaneously with one shear band near the bottom of the heap. As the pulse travels upward, it becomes disconnected from its shear band which is convected inwards and eventually disappears.

The surface waves in the grass seed and the rough sands are always reproducible and well defined. For the smooth sand and monodisperse glass spheres in the range of a/d and Γ considered here, we find that surface waves sometimes occur but that they are transient and soon disappear. Not all faceted or nonspherical materials show the surface waves, rather, only those with a maximum dynamic angle of repose $\theta_{\mathrm{dm}} \gtrsim 30°$.

Unlike the onset of convection, the onset value of Γ for the surface waves, Γ_w, depends on E or a/d. In Fig. 4(a), we show Γ_w vs a at the onset of the waves for three different sizes of rough sand. There is a minimum and apparently common value of Γ_0 ($\Gamma \simeq 1.6$) below which the surface waves never form. If the same data for the onset of the waves are presented as E_w vs a/d, part (b) of Fig. 4, they collapse reasonably well onto the single curve of

$$E \equiv \Gamma(a/d) = b_1 + b_2(a/d), \tag{1}$$

with $b_1 = 2.55 \pm 0.25$ and $b_2 = 1.59 \pm 0.05$. The solid line in the figure is this fit. Note that this relation implies minimum values for both E and Γ, namely, b_1 and b_2, respectively, below which the waves will not occur [11]. A key point of this figure is that it demonstrates the importance of an additional parameter besides Γ.

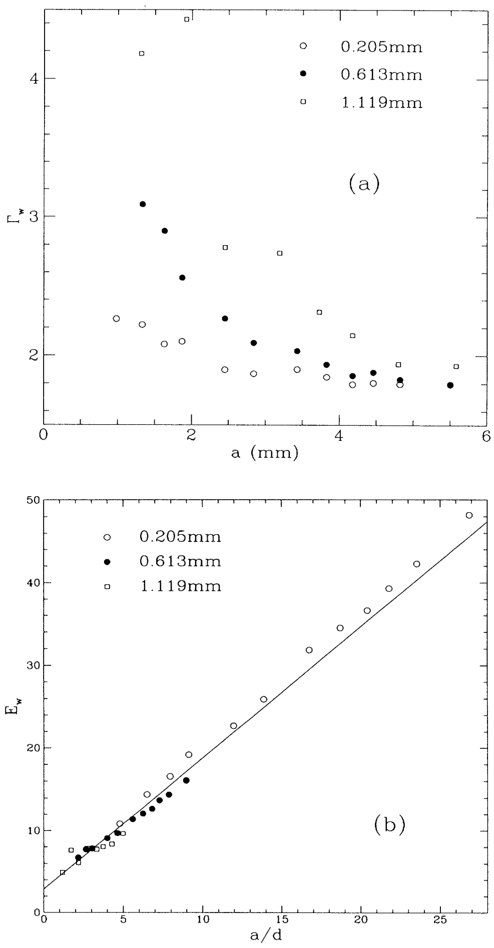

FIG. 4. Data for the onset of the surface waves for three different sizes (indicated in the figures) of rough sand. In (a) we show data for Γ_w vs a, and in (b), we show the same data for E_w vs a/d. The solid line in (b) is a fitted curve from Eq. (1).

Waves associated with subharmonic bifurcations and $\theta_d = 0$ have been previously reposted by Douady *et al.* [6]. However, the waves reported here are fundamentally different because (a) they are not related to a subharmonic bifurcation, (b) they occur when $\theta_d > 0$, and (c) they occur when the height of the layer is large. Specifically the waves reported here can occur below the subharmonic bifurcation of Douady *et al.* Also, if we gradually reduce the height of the layer, the traveling waves vanish and we recover the standing waves of Douady *et al.*

There is currently no model which can describe the complex processes which occur in this system. However, some insight into these observations may be provided by simple arguments and by considering the 1D motion of a single particle driven by an oscillating surface. For instance, the propagation of the waves up the surface may be explained as follows. Because of the continuous supply of granular material by avalanches on the uphill side of a wave pulse, the higher edge grows upward. At the lower edge of a pulse, the slope is steeper, so the avalanche rate is higher. The combination of these two effects moves the pulse up the slope. The surface wave is, therefore, not a material wave but is more like a solitary wave. The fact that the pulse broadens as it goes upward is due to the fact the leading edge travels faster than the trailing edge.

Consider next the 1D motion of a single particle driven by an oscillating surface. We envision that this picture provides a heuristic model of the oscillatory component of the center of mass motion of a large collection of grains. Even though individual granular interactions are characterized by nonzero restitution coefficients, there is very rapid dissipation of kinetic energy by multiple collisions between neighbor grains, particularly during the compression phase of each cycle. Therefore, the effective restitution coefficient between a large collection of grains and the oscillating floor is much smaller (more inelastic) than that of one granular particle and the oscillating floor.

The dissipation per unit length in the azimuthal direction depends on the angular position. This is related to the fact that the time averaged density is greatest at ϕ_h and least at $\phi_h + \pi$ where the valley occurs. Thus, the collision rates are highest at ϕ_h and lowest at $\phi_h + \pi$, so the effective restitution coefficient between the granular mass and the floor is minimum at the center of the heap and maximum in the valley. This effect is manifested in the experiments by the nature of the gap which forms during part of each cycle between the bottom of the sand and the shaker. This gap occurs for smaller Γ at $\phi_h + \pi$ than for ϕ_h. In turn, this leads to an upward bulge, as in Fig. 2, near the valley and a weakening of the material there.

When the dynamic angle of repose is small, both the inhomogeneity in the effective restitution coefficient and the avalanche rate on the surface are small. In this case, there may not be enough driving force to generate the pulses, explaining why only materials with high θ_{dm} show the traveling surface waves.

The surface waves observed here are a novel phenomena, and it is interesting to compare them to surface waves in other kinds of materials. For solids, the disappearance of the stress at the free surface causes Rayleigh waves [12]. For liquids, there are gravity and capillary waves [12], including capillary waves driven by vertical vibration [13]. None of these bears a strong similarity to the surface waves of the granular materials under vibration. Since the surface flow avalanche is moving in the opposite direction from the internal convective transport of the granular material, this situation is somewhat similar to stratified shear flow [14]. In these flows, a closed long rectangular tube is filled with two immiscible fluids of different densities and then inclined from the horizontal to produce counterpropagating accelerating flows with large shear at the interface between the two fluids. Instabilities develop resulting in a wavy interface, but not pulses as seen in the shaker experiments.

The waves seen here are a phenomenon which appears to be unique to granular dynamics. A brief summary of their properties includes the following: (a) the waves occur only for materials with at least a dynamic angle of repose of 30° or more; (b) the onset of these waves is a function not only of Γ but also of E or a/d, as given by Eq. (1); and (c) some qualitative features can be understood in terms of simple models such as a bouncing inelastic ball. A detailed understanding of their origins is a challenging problem.

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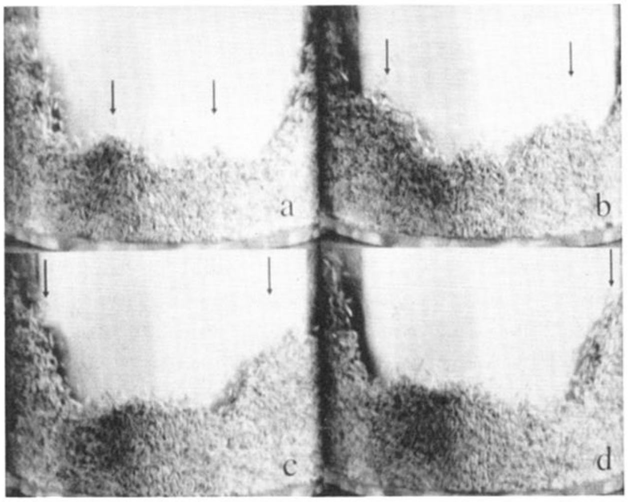

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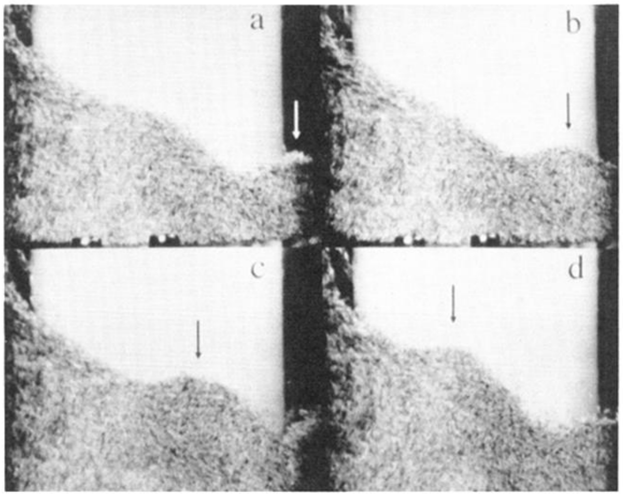

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