Effect of Quenching on the Kaon B Parameter

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We compute the kaon B parameter on an ensemble of lattices which include the effects of dynamical quark loops. Comparing with our previous quenched calculations, we find that dynamical fermions have negligible effect.

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As we seek to convert ever improving experimental data into a precise determination of the parameters of the standard model, one major source of uncertainty is the lack of knowledge of the hadronic matrix elements of the weak interaction effective Hamiltonian. One of the most relevant unknowns is the $\Delta S = 2$ matrix element responsible for kaon-antikaon oscillations, usually expressed in terms of the dimensionless ratio B_K. There have been a variety of attempts to calculate this quantity, ranging from chiral perturbation theory and the large N exapansion to QCD sum rules and lattice QCD. Of these, the lattice approach has the virtue of making the least number of assumptions, and is therefore likely eventually to emerge as the method of choice. As a practical matter, though, lattice calculations introduce their own systematic errors which can be sizable. These must be methodically addressed before we can fairly quote a lattice result.

There are three principal sources of systematic error in a lattice calculation: finite volume (V), finite lattice spacing (a), and quenching (N_f). As a matter of principle we would like to do a calculation with V about $(2F)^4$ so a hadron can be comfortably contained in the volume, with $a^{-1} \sim 4$ GeV so as to be in the perturbative scaling regime, and with $N_f = 2$ or 3 dynamical flavors of quarks. Since it is beyond our present capacity to do such a calculation, we have instead embarked on a program to investigate these errors one at a time. As a reference point we performed a calculation with a set of parameters which are admittedly inadequate in every respect: $16^3 \times 40$ at $\beta = 6.0$ ($a^{-1} \sim 2$ GeV), $N_f = 0$. We have subsequently studied the effects of improving either volume or the lattice spacing or the number of flavors. In a previous Letter [1] we reported the results of our finite volume comparison, arguing that the effect on B_K was indeed quite small, and that it could be understood in the context of (quenched) chiral perturbation theory [2]. We also noted that the coefficient of the chiral logarithm in B_K is the same in the quenched and the full theory, at least in the limit $m_s = m_d$ relevant to our calculation. Thus the quenched B_K has close to the correct functional dependence on m_K. This says nothing about the relative normalization of B_K in the quenched and full theories, but it does make it more plausible that B_K is less sensitive to the effects of dynamical fermions than other quantities. In the present Letter we take the necessary next step and report on the effects of dynamical fermions, using an ensemble of gauge configurations generated in unquenched QCD. A preliminary version of this calculation was reported in Ref. [3].

The quenched approximation to lattice QCD consists of the assertion that it is reasonable to replace the nonlocal fermion determinant ($\det[\not{D} + m]$) by a local gluon operator, usually by simply rescaling the gluon operator in the one-plaquette Wilson action. Certainly this is the leading order effect in perturbation theory, but there are in addition several pieces of evidence which argue that this approximation may be quite good even nonperturbatively. For one thing, several quenched calculations give results in agreement with the real world, which we assume is well described by unquenched QCD. Among these calculations is a recent determination of the spectrum of light hadrons, which obtained the correct mass ratios to within its statistical accuracy of about 6% [4]. Second, a recent comparison of quenched and unquenched calculations at finite temperature showed that the effects of dynamical fermions can be absorbed into a calculable shift in the effective coupling [5]. Such a shift is to be expected in the case of heavy quarks, but surprisingly it was found that the one-loop approximation to the effective gluon action correctly reproduces the observed Monte Carlo data (to 10% or 20%) even for quark

TABLE I. Configuration parameters.

		$\beta = 5.7$, $N_f = 2$		$\beta = 6$
	$m_{\text{sea}}a = 0.01$	$m_{\text{sea}}a = 0.015$	$m_{\text{sea}}a = 0.025$	Quenched
N_{config}	50	51	34	35
Plaquette	0.57743(5)	0.57728(5)	0.57710(7)	0.59368
β_{SD}	5.7953(7)	5.7950(7)	5.7943(8)	6

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TABLE II. Pseudoscalar masses.

		$\beta = 5.7$, $N_f = 2$		$\beta = 6$
M_{valence}	$m_{\text{sea}}a = 0.01$	$m_{\text{sea}}a = 0.015$	$m_{\text{sea}}a = 0.025$	Quenched
0.03	0.421(1)	0.419(2)	0.422(3)	0.412(3)
0.02	0.346(2)	0.343(3)	0.346(2)	0.339(3)
0.01	0.251(2)	0.246(3)	0.250(3)	0.244(3)

masses lighter than the strange quark. Most straightforwardly, there are direct comparisons of quenched and unquenched calculations with parameters tuned to give similar physical scales. One such study looked at meson decay constants [6], and again found that the effect of dynamical quarks could be absorbed into a redefinition of the gauge coupling. Here we present a similar study for the quantity B_K.

The dynamical configurations used in this calculation were generated on the Columbia lattice parallel processor, and are a subset of those used by the Columbia group in their spectrum calculation [7]. The lattices are of size $16^3 \times 32$, and were generated using the hybrid molecular dynamics algorithm with a gauge coupling of $\beta = 5.7$ and $N_f = 2$ flavors of staggered fermions. Three ensembles of lattices were used, with quark masses $m_{\text{sea}}a = 0.01$, 0.015, and 0.025. In the present study, lattices were saved every 250 units of molecular dynamics time and transferred to the Crays at the National Energy Research Supercomputer Center for propagator calculations and analysis. The numbers of configurations are listed in Table I. Most every quantity we examined was relatively insensitive to the value of the dynamical sea quark mass. There is discernible but small dependence of the plaquette on the sea quark mass. Alternatively, one can define an effective coupling by using the simplest Schwinger-Dyson equation which relates the expectation value of the plaquette to various 1×2 and L-shaped loops [8]. Measuring these loops, we solved for the effective β_{SD} which is listed in Table I. In Table II we list the mass of the lightest pseudoscalar on each of the ensembles for valence quark masses of $m_q a = 0.01$, 0.02, and 0.03. Again the lack of dependence on the sea quark mass is apparent. We take this insensitivity to m_{sea} as an indication that these masses are already small compared to the QCD scale. In physical terms the lightest mass ($m_{\text{sea}}a = 0.01$) corresponds to about $\frac{1}{2}m_{\text{strange}}$.

The parameters of the dynamical quark lattices are well matched with our previous quenched calculation [1] of B_K at $\beta = 6.0$ on a $16^3 \times 40$ lattice. In particular, the lattice spacing a is similar, though its precise value depends on the observable chosen. If we take the nucleon mass to set the scale, we find $a^{-1} = 2.0$ GeV for both the dynamical [7] and the quenched [9] ensembles, while from f_π we would get $a^{-1} = 2.1$ GeV and $a^{-1} = 1.7$ GeV, respectively. In either case we conclude that lattice spacing effects and finite volume effects should be essentially the same for the two ensembles, and we can make a head-on comparison of the effects of quenching.

The calculation of B_K was performed in the same way as in our previous calculation [1]. We used quark wall propagators in Landau gauge, and imposed periodic boundary conditions in space and Dirichlet boundaries in the time direction. To make the comparison as direct as possible, we replicated the $16^3 \times 32$ dynamical lattices in the time direction, enlarging them to $16^3 \times 40$ before computing propagators.

The results for B_K are shown in Table III. The lack of dependence on m_{sea} is evident. Extrapolating linearly to the chiral limit, $m_K^2 = 0$, we extract the lowest order parameter in the chiral Lagrangian, $B_K(0)$. To obtain B_K at the physical point (m_K) we need to know the lattice spacing a. Taking $a^{-1} = 2.1$ GeV and $a^{-1} = 1.9$ GeV for the dynamical and quenched configurations, respectively, we obtain results quoted in Table III. Using the same choice of scale, we collect the results in Fig. 1, including two more points from quenched simulations with nondegenerate quarks ($m_{\text{val}} = 0.01 + 0.02$ and $m_{\text{val}} = 0.02 + 0.03$). We conclude that the effect of quenching on our calculation of B_K cannot be large. In particular it must be less than a few percent, and is consistent with zero given our statistical errors. This gives us confidence that B_K can be extracted reliably in the quenched approximation.

It should be stressed that the numbers quoted here are for the bare *lattice* value of B_K. To convert these results

TABLE III. Results for B_K. Statistical errors are computed with a single-elimination jackknife.

		$\beta = 5.7$, $N_f = 2$		$\beta = 6$
M_{valence}	$m_{\text{sea}}a = 0.01$	$m_{\text{sea}}a = 0.015$	$m_{\text{sea}}a = 0.025$	Quenched
0.03	0.818(09)	0.811(10)	0.803(11)	0.777(13)
0.02	0.771(11)	0.769(13)	0.761(13)	0.749(16)
0.01	0.658(18)	0.694(21)	0.697(22)	0.697(29)
$B_K(0)$	0.61(2)	0.65(3)	0.65(3)	0.67(3)
$B_K(m_K)$	0.67(2)	0.70(2)	0.70(2)	0.71(2)

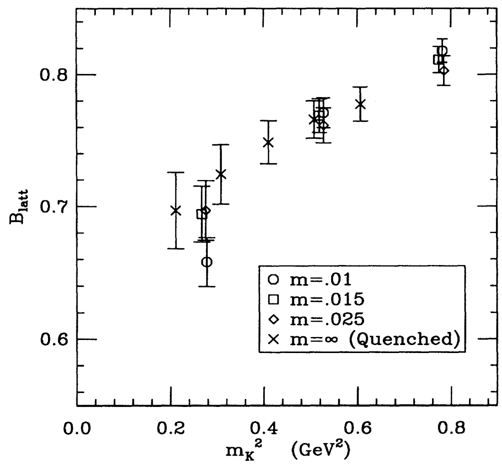

FIG. 1. B_K versus square of kaon mass for quenched and dynamical simulations.

to the conventionally quoted $\hat{B}_K$, there remains the issue of taking the continuum limit. The perturbatively calculable matching factors and coefficient functions are under control, but in addition one finds significant power law corrections from the finite lattice spacing. We are presently studying these issues, using a finer lattice and improved operators.

This calculation has also been performed by another group, using essentially the same set of parameters [10]. Where we can compare, our results are essentially the same. The results are also consistent with our recent calculation using dynamical Wilson fermions [11], although with Wilson fermions the errors were too large to make a truly meaningful test.

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