Gluon Fragmentation into Heavy Quarkonium

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The dominant production mechanism for heavy-quark–antiquark bound states with large transverse momentum is fragmentation, the splitting of a high energy parton into a quarkonium state and other partons. We show that the fragmentation functions $D(z,\mu)$ describing these processes can be calculated using perturbative QCD. We calculate the fragmentation functions for a gluon to split into S-wave quarkonium states to leading order in the QCD coupling constant.

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Quantitative evidence for quantum chromodynamics (QCD) as the fundamental field theory describing the strong interactions has come primarily from high energy processes involving leptons and the electroweak gauge bosons which do not have strong interactions. The next simplest particles as far as the strong interactions are concerned are heavy quarkonia, the bound states of a heavy quark and antiquark. While not pointlike, the lowest states in the charmonium and bottomonium systems have typical radii that are significantly smaller than those of hadrons containing light quarks. They have simple internal structure, consisting primarily of a nonrelativistic quark and antiquark only. The charmonium and bottomonium systems exhibit a rich spectrum of orbital and angular excitations. They can therefore be very useful as probes of the strong interactions.

In most previous studies of the production of heavy quarkonia with large transverse momentum p_T, it was implicitly assumed that they are produced by *short distance* mechanisms, in which the heavy quark and antiquark are created with transverse separations of order $1/p_T$. At sufficiently large p_T, the dominant mechanism is actually *fragmentation*, the production of a parton with high transverse momentum which subsequently decays into the quarkonium state and other partons. The $Q\bar{Q}$ pair is created with a separation of order $1/m_Q$, where m_Q is the mass of the heavy quark Q. While it is sometimes of higher order in the QCD coupling constant α_s, the fragmentation mechanism dominates at sufficiently high p_T because the short distance mechanism is suppressed by powers of m_Q/p_T. The fragmentation of a parton into a quarkonium state is described by a fragmentation function $D(z,\mu)$, where z is the longitudinal momentum fraction of the quarkonium state and μ is a factorization scale. In this Letter, we point out that the fragmentation functions for the production of heavy quarkonium states can be calculated using perturbative QCD. We calculate the fragmentation functions for gluons to split into S-wave quarkonium states to leading order in α_s.

In a hadron collider, a charmonium state with large transverse momentum p_T can either be produced directly at large p_T or it can be produced indirectly by the decay of a B meson or a higher charmonium state with large p_T. In previous calculations of the rate for direct production of charmonium at large p_T [1], the dominant mechanisms were assumed to be those which arise at lowest order of perturbation theory. A typical Feynman diagram which contributes to the production of the $^1S_0$ charmonium state η_c at order α_s^3 is the diagram for $gg \to c\bar{c}g$ shown in Fig. 1. The order-α_s^4 radiative corrections to this process include the Feynman diagram for $gg \to c\bar{c}gg$ shown in Fig. 2. In most regions of phase space, the virtual gluons in Fig. 2 are off their mass shells by amounts of order p_T, and the contribution from this diagram is suppressed relative to the diagram in Fig. 1 by a power of the running coupling constant $\alpha_s(p_T)$. But there is a part of the phase space in which the virtual gluon attached to the $c\bar{c}$ pair in Fig. 2 is off shell by an amount of order m_c. The propagator of this virtual gluon enhances the cross section by a factor of p_T^2/m_c^2. At sufficiently large p_T, this overcomes the extra power of the coupling constant α_s. The enhancement is due to the fact that

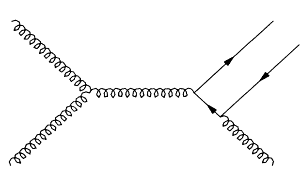

FIG. 1. A Feynman diagram for $gg \to c\bar{c}g$ that contributes to η_c production at order α_s^3.

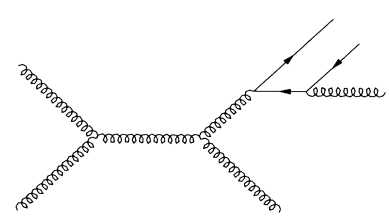

FIG. 2. A Feynman diagram for $gg \to c\bar{c}gg$ that contributes to η_c production at order α_s^4.

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the $c\bar{c}$ pair can be produced with transverse separation of order $1/m_c$ instead of $1/p_T$.

A more thorough analysis of the amplitude for $gg \to \eta_c gg$ reveals that the term that is enhanced by p_T^2/m_c^2 can be written in a factored form. The first factor is the amplitude for the production of a virtual gluon g^* with high p_T but low invariant mass q^2 via the process $gg \to gg^*$. In the limit $q^2 \ll p_T^2$, it reduces to the on-shell scattering amplitude for $gg \to gg$. The second factor is the propagator $1/q^2$ for the virtual gluon. The third and final factor is the amplitude for the process $g^* \to \eta_c g$, in which an off-shell gluon decays into an η_c and a gluon. The factoring of the amplitude allows this contribution to the differential cross section $d\sigma_{\eta_c}(p)$ for producing an η_c with 4-momentum p to be written in a factorized form:

$$d\sigma_{\eta_c}(p) \approx \int_0^1 dz\, d\widehat{\sigma}_g(p/z)\, D_{g\to\eta_c}(z) , \qquad (1)$$

where $d\widehat{\sigma}_g(p/z)$ is the differential cross section for producing a real gluon with 4-momentum p/z. All of the dependence on the transverse momentum p_T appears in the subprocess cross section $d\widehat{\sigma}_g$, while all the dependence on the quark mass m_c is in the function $D_{g\to\eta_c}(z)$. The variable z is the longitudinal momentum fraction of the η_c relative to the gluon. The physical interpretation of (1) is that an η_c with momentum p can be produced by first producing a gluon of larger momentum p/z which subsequently splits into an η_c carrying a fraction z of the gluon momentum. Such a process is called *fragmentation.*

The generalization of the leading order formula (1) to all orders in α_s is straightforward. At higher orders in α_s, the gluon that splits into a quarkonium state $\mathcal{O}$ can itself arise from the splitting of a higher energy parton into a collinear gluon. This splitting process gives rise to logarithms of p_T/m_Q. In order to maintain the factorization of the dependences on p_T and m_Q, it is necessary to introduce a factorization scale μ: $\ln(p_T/m_Q) = \ln(p_T/\mu) + \ln(\mu/m_Q)$. To all orders in α_s, the fragmentation contribution to the differential cross section for producing a quarkonium state $\mathcal{O}$ can be written in the factorized form

$$d\sigma_{\mathcal{O}}(p) = \sum_i \int_0^1 dz\, d\widehat{\sigma}_i(p/z,\mu)\, D_{i\to\mathcal{O}}(z,\mu) , \qquad (2)$$

where the sum is over all parton types i. The scale μ is arbitrary, but large logarithms of p_T/μ in the parton cross section $d\widehat{\sigma}_i$ can be avoided by choosing μ on the order of p_T. Large logarithms of μ/m_Q then necessarily appear in the fragmentation functions $D_{i\to\mathcal{O}}(z,\mu)$, but they can be summed up by solving the evolution equations [2]

$$\mu\frac{\partial}{\partial\mu}D_{i\to\mathcal{O}}(z,\mu) = \sum_j \int_z^1 \frac{dy}{y}\, P_{i\to j}(z/y,\mu)\, D_{j\to\mathcal{O}}(y,\mu) , \qquad (3)$$

where $P_{i\to j}(x,\mu)$ is the Altarelli-Parisi function for the splitting of the parton of type i into a parton of type j with longitudinal momentum fraction x. The boundary condition on this evolution equation is the fragmentation function $D_{i\to\mathcal{O}}(z,2m_Q)$ at the scale $2m_Q$. It can be calculated perturbatively as a series in $\alpha_s(2m_Q)$.

We proceed to calculate the fragmentation function $D_{g\to\eta_c}(z,2m_c)$ for a gluon to split into the ${}^1S_0$ charmonium state η_c to leading order in $\alpha_s(2m_c)$. A process (such as $gg \to gg$) that produces a real gluon of 4-momentum q has a matrix element of the form $\mathcal{M}_\alpha\epsilon^\alpha(q)$, where $\epsilon^\alpha(q)$ is the polarization 4-vector of the on-shell ($q^2=0$) gluon. In the corresponding fragmentation process (such as Fig. 2), a virtual gluon is produced with large energy $q_0 \gg m_c$ but small invariant mass $s=q^2$ of order m_c^2, and it subsequently fragments into an η_c and a real gluon. The fragmentation probability $\int_0^1 dz D(z,m_c)$ is the ratio of the rates for these two processes. In Feynman gauge, the fragmentation term in the matrix element for the η_c production has the form $\mathcal{M}_\alpha(-ig^{\alpha\beta}/q^2)\mathcal{A}_\beta$, where $\mathcal{M}_\alpha$ is the matrix element for the production of the virtual gluon and $\mathcal{A}_\beta$ is the amplitude for $g^* \to \eta_c g$. The fragmentation term is distinguished from the short distance terms in the matrix element by the presence of the small denominator q^2 of order m_c^2. The amplitude $\mathcal{A}_\beta$ can be written down using standard Feynman rules for quarkonium processes [3]. Multiplying $\mathcal{A}_\alpha$ by its complex conjugate and summing over final colors and spins, we get

$$\sum \mathcal{A}_\alpha\mathcal{A}_\beta^* = \frac{8\pi}{3}\alpha_s^2\frac{|R(0)|^2}{m_c}\frac{1}{(s-4m_c^2)^2}[-(s-4m_c^2)^2 g_{\alpha\beta} + 2(s+4m_c^2)(p_\alpha q_\beta + q_\alpha p_\beta) - 4s p_\alpha p_\beta - 16m_c^2 q_\alpha q_\beta] , \qquad (4)$$

where p is the 4-momentum of the η_c. Terms proportional to q_α or q_β are gauge artifacts and can be dropped. In an appropriate axial gauge, q_α and q_β are of order m_c^2/q_0 when contracted with the numerator of the propagator of the virtual gluon. In covariant gauges, the q_α and q_β terms are not suppressed but are canceled by other diagrams. In the $p_\alpha p_\beta$ term, we can set $p = zq + p_\perp$ up to corrections of order m_c^2/q_0, where z is the longitudinal momentum fraction and $p_\perp$ is the transverse part of the 4-vector p. In a frame where $q=(q_0,0,0,q_3)$, $z=(p_0+p_3)/(q_0+q_3)$ and $p_\perp = (0,p_1,p_2,0)$. After averaging over the directions of the transverse momentum, $p_\perp^\alpha p_\perp^\beta$ can be replaced by $-g^{\alpha\beta}\vec{p}_\perp^{\,2}/2$, up to terms that are suppressed in axial gauge. The terms in (4) that contribute to fragmentation then reduce to

$$\sum \mathcal{A}_\alpha\mathcal{A}_\beta^* = \frac{8\pi}{3}\alpha_s^2\frac{|R(0)|^2}{m_c}\frac{1}{(s-4m_c^2)^2}\left[(s-4m_c^2)^2 - 2(1-z)(zs-4m_c^2)s\right](-g_{\alpha\beta}) . \qquad (5)$$

We have used the conservation of the q_0-q_3 component of the 4-momentum in the form $s=(\vec{p}_\perp^{\,2}+4m_c^2)/z+\vec{p}_\perp^{\,2}/(1-z)$.

At this point, it is easy to calculate the rate for production of $\eta_c g$ in the limit $q_0^2 \gg q^2 \sim m_c^2$ and divide it by the rate for production of an on-shell gluon. The resulting fragmentation probability is

$$\int_0^1 dz\, D_{g\to\eta_c}(z) = \frac{\alpha_s^2}{6\pi}\frac{|R(0)|^2}{m_c}\int_{4m_c^2}^{\infty} ds \int_{4m_c^2/s}^{1} dz\, \frac{s^2+16m_c^4-2z(s+4m_c^2)s+2z^2s^2}{s^2(s-4m_c^2)^2}\,, \tag{6}$$

where $R(0)$ is the nonrelativistic radial wave function at the origin for the S-wave bound state. We have increased the upper end point of the integration over s to infinity, because the resulting error is of order m_c^2/q_0^2, which we have been consistently neglecting. Interchanging orders of integration, we can read off the fragmentation function:

$$D_{g\to\eta_c}(z,2m_c) = \frac{1}{24\pi}\alpha_s(2m_c)^2\frac{|R(0)|^2}{m_c^3}\left[3z-2z^2+2(1-z)\ln(1-z)\right]. \tag{7}$$

We have set the scale in the fragmentation function and in the running coupling constant to $\mu = 2m_c$, which is the minimum value of the invariant mass $\sqrt{s}$ of the fragmenting gluon. The value of the S-state wave function at the origin $R(0)$ is determined from the ψ electronic width to be $|R(0)|^2 = (0.8\ \text{GeV})^3$. We use the value $\alpha_s(2m_c) = 0.26$ for the strong coupling constant. The largest uncertainty comes from the value of the quark mass, which we take to be $m_c = 1.5$ GeV.

Given the initial fragmentation function (7), $D_{g\to\eta_c}(z,\mu)$ at larger values of μ is determined by solving the evolution equation (3) with (7) as a boundary condition. To illustrate the effects of evolution, we evolve the fragmentation function using only the $P_{g\to g}$ term in (3). The z dependence of $D_{g\to\eta_c}(z,\mu)$ at the energy scales $\mu = 2m_c$ and $\mu = 20m_c$ is illustrated in Fig. 3. The evolution causes the fragmentation function to decrease at large z and to diverge at $z = 0$. A physical cross section like (2) is still well behaved, because phase space limitations place an upper bound on the transverse momentum p_T/z of the parton which translates into a lower bound on z. The evolution of the fragmentation function can significantly increase the total production rate because of the large enhancement at small z.

The fragmentation function for a gluon into J/ψ can be calculated to leading order in α_s from the Feynman diagrams for $g^* \to \psi gg$. The square of the amplitude $\sum \mathcal{A}_\alpha \mathcal{A}_\beta^*$ for this process can be extracted from a calculation of the matrix element for $e^+e^- \to \psi gg$ [4]. The resulting fragmentation function is proportional to $\alpha_s(2m_c)^3|R(0)|^2/m_c^3$. The z dependence is given by a two-dimensional integral that must be evaluated numerically [5]. In Fig. 4, we show the fragmentation function at the initial scale $2m_c$ and after evolution to the scale $\mu = 20m_c$ using the $P_{g\to g}$ term in the evolution formula (3).

An order of magnitude estimate of the gluon fragmentation contribution to quarkonium production in any high transverse momentum process can be obtained by multiplying the cross section for producing gluons with $p_T > 2m_Q$ by the initial fragmentation probability $\int_0^1 dz D(z,2m_Q)$. For the η_c and ψ, these probabilities are

$$\int_0^1 dz D_{g\to\eta_c}(z,2m_c) = \frac{1}{72\pi}\alpha_s(2m_c)^2\frac{|R(0)|^2}{m_c^3}\,, \tag{8}$$

$$\int_0^1 dz D_{g\to\psi}(z,2m_c) = (1.2\times10^{-3})\,\alpha_s(2m_c)^3\frac{|R(0)|^2}{m_c^3}\,. \tag{9}$$

Their numerical values are 4.5×10^{-5} and 3.2×10^{-6}. The initial fragmentation probability for the splitting of a gluon into the $^3S_1$ radial excitation ψ' is also given by (9), except that $R(0)$ is replaced by the radial wave function at the origin for the ψ'. The initial fragmentation probability for the $^3S_1$ bottomonium state Υ is also given by (9), except that m_c is replaced by m_b and $R(0)$ is the

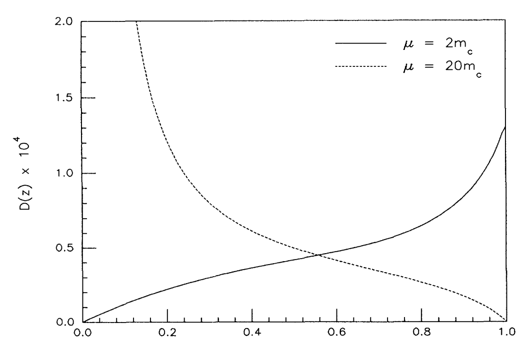

FIG. 3. The fragmentation function $D_{g\to\eta_c}(z,\mu)$ as a function of z for $\mu = 2m_c$ (solid line) and $\mu = 20m_c$ (dotted line).

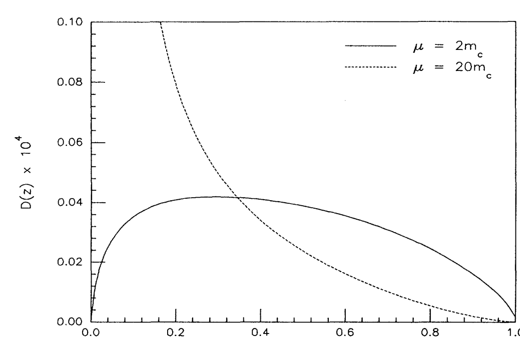

FIG. 4. The fragmentation function $D_{g\to\psi}(z,\mu)$ as a function of z for $\mu = 2m_c$ (solid line) and $\mu = 20m_c$ (dotted line).

appropriate wave function at the origin.

In hadron colliders, short distance processes dominate the direct production of charmonium at small p_T, because fragmentation processes are suppressed by powers of $\alpha_s(2m_c)$. At sufficiently large p_T, fragmentation processes must dominate, because the short distance processes are suppressed by powers of m_c^2/p_T^2. We can make a quantitative estimate of the p_T at which the crossover occurs by comparing the differential cross section for the short distance process with the differential cross section for the gluon scattering process $gg \to gg$ multiplied by the appropriate fragmentation probability. For simplicity, we consider 90° scattering in the gg center of mass frame. In the limit $p_T \gg m_c$, the differential cross section for $gg \to g\eta_c$ is [1] $d\sigma/dt = 81\pi\alpha_s^3|R(0)|^2/512m_c p_T^6$. The differential cross section for $gg \to gg$ is $d\sigma/dt = 243\pi\alpha_s^2/128p_T^4$. To allow for fragmentation of either of the two outgoing gluons, we multiply by twice the initial fragmentation probability (8). The resulting differential cross section exceeds that for the short distance process at $p_T \approx \sqrt{3\pi/\alpha_s}\ m_c \approx 6m_c$. The differential cross section for $gg \to g\psi$ is [1] $d\sigma/dt = 5\pi\alpha_s^3|R(0)|^2 m_c/64p_T^8$ and for this case we estimate the crossover point to be $p_T \approx \sqrt{4.2/\alpha_s} m_c \approx 4m_c$. Thus fragmentation should dominate over short distance production at values of p_T that are being measured in present collider experiments.

The importance of fragmentation for charmonium production at large transverse momentum can also be seen from the surprisingly large rate [6] for $Z^0 \to \psi c\bar{c}$, which is 2 orders of magnitude larger than that for $Z^0 \to \psi gg$. The explanation for this is that $Z^0 \to \psi gg$ is a short distance process, while $Z^0 \to \psi c\bar{c}$ includes a fragmentation contribution enhanced by $(M_Z/m_c)^2$ [7]. The enhanced contribution arises from the decay $Z^0 \to c\bar{c}$, followed by the splitting $c \to \psi c$ or $\bar{c} \to \psi\bar{c}$. With this insight, the lengthy calculation presented in Ref. [6] can be reduced to the calculation of the fragmentation function $D_{c\to\psi}(z, M_Z)$.

The probability for a virtual gluon to decay into a ψ was calculated by Hagiwara, Martin, and Stirling [8] and used to study J/ψ production from gluon jets at the CERN e^+e^- collider LEP. They did not calculate the fragmentation function $D_{g\to\psi}(z,\mu)$ and thus were unable to sum up large logarithms of M_Z/m_c. Furthermore, their expression for the fragmentation probability is missing a factor of $1/16\pi^2$, which explains the surprisingly large rate that they found for this ψ-production mechanism.

A complete calculation of the fragmentation contribution to ψ production at high transverse momentum must include the production of the P-wave charmonium states χ_{cJ}, followed by their radiative decays into ψ. In calculating the fragmentation functions for gluons to split into P-wave charmonium states, there are two distinct contributions that must be included at leading order in α_s. The P-wave state can arise either from the production of a collinear $c\bar{c}$ pair in a color-singlet P-wave state, or from the production of a collinear $c\bar{c}$ pair in a color-octet S-wave state [9]. Calculations of the P-wave fragmentation functions will be presented elsewhere [5].

We have shown in this Letter that the dominant production mechanism for heavy quarkonium at high transverse momentum is fragmentation, the production of a high energy parton followed by its splitting into the quarkonium state. We have also shown that the fragmentation functions that describe this process can be calculated using perturbative QCD. We calculated the fragmentation functions $D(z,\mu)$ for gluons to split into S-wave quarkonium states to leading order in α_s. For ψ production in hadron colliders, we estimated the crossover value of p_T, where fragmentation begins to dominate over the conventional short distance production mechanisms, to be only about 6 GeV. All previous calculations of quarkonium production at large transverse momentum must therefore be reexamined, taking into account the fragmentation mechanism.

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Note added.—For direct ψ production in $p\bar{p}$ colliders, the crossover value of p_T at which the fragmentation mechanism dominates over the conventional short distance mechanism has been calculated explicitly and found to be around 7 GeV at Fermilab Tevatron energies [10]. This value is in good agreement with the naive estimate given above.

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