Interference of Two Electrons Entering a Superconductor

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The subgap conductivity of a normal-superconductor (NS) tunnel junction is thought to be due to tunneling of two electrons. There is a strong interference between these two electrons, originating from the spatial phase coherence in the normal metal at a mesoscopic length scale and the intrinsic coherence of the superconductor. We evaluated the interference effect on the transport through an NS tunnel junction. We propose the layouts to observe drastic Aharonov-Bohm and Josephson effects.

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Quantum phase coherence in solids manifests itself basically in two ways. First, there is an intrinsic coherence in the superconducting state. In superconductors, the phase is indeed a macroscopic variable. This can be observed in a variety of interference experiments, for example, with tunnel junctions [1]. Second, even in a normal metal electrons are coherent at a mesoscopic length scale. Interference between the electrons in a normal metal gives rise to a set of phenomena which constitutes the subject of a new branch of condensed matter physics [2,3].

At the normal-superconductor (NS) interface between a normal metal and a superconductor, these two sources of coherence may interplay. Very recent experiments [4] show how interesting such an interplay may be. In case of NS boundaries of a high transparency, the phenomenon of Andreev reflection [5] seems to be responsible for the peculiarities observed.

In the present paper we focus on the opposite case of a tunnel NS (N-I-S) interface. It is well known that an electron with energy less than Δ, Δ being the superconducting energy gap, cannot tunnel to the superconductor and therefore the transport through the junction is strongly suppressed at voltages below the gap [1]. On the other hand, two electrons can enter the superconductor converting into a Cooper pair since this process costs no energy. Such two-electron tunneling [6,7] determines the subgap conductivity of the junction.

It has been discussed recently [8] that the rate of this two-electron process is often determined by the interference of the electron waves on a space scale given by the coherence length, either in the normal or the superconducting metal. It was shown in [8] that interference plays an important role in junction geometries of present-day interest. The elastic mean free path in the electrodes on both sides of the tunnel junction is usually much smaller than the junction size due to boundary scattering on small length scales. As a result, two-electron tunneling in such geometries occurs between states of complex interference structure; hence the usual description in terms of plane electron waves fails. Instead, the diffusive motion should be considered of two electrons which are coherent over a distance given by $L_{T,\Delta} = \sqrt{D/\max\{eV,T\}}, \sqrt{D/\Delta}$, in the normal metal and the superconductor, respectively, where D is the diffusion constant [9]. This has drastic consequences for the subgap conductance, which can be enhanced by several orders of magnitude, compared to the value obtained by assuming ballistic electron motion in the electrodes. The enhancement of the subgap conductance is found again in the present paper.

The importance of interference effects motivates us to explore how agents which act on the phase will influence the subgap conductivity. Indeed we find the conditions under which a pronounced Aharonov-Bohm and Josephson effect can be observed. Since we consider the low-voltage subgap conductivity, we assume that $T, eV \ll \Delta$. Under these conditions the coherence length L_T is much larger than the one in the superconductor, L_Δ. Since the total subgap conductivity can be expressed as the sum of the interference contributions from the normal metal and the superconductor [8], we therefore concentrate on the interference in the normal metal.

The N-I-S interface is described by the Hamiltonian $\hat{H} = \hat{H}_N + \hat{H}_S + \hat{H}_T$, where $\hat{H}_N$ and $\hat{H}_S$ refer to the normal and the superconducting electrode, respectively. The tunnel Hamiltonian $\hat{H}_T$ is given by the usual form $\hat{H}_T = \sum_{\mathbf{k},\mathbf{p},\sigma} t_{\mathbf{kp}} \hat{a}^\dagger_{\mathbf{k},\sigma} \hat{b}_{\mathbf{p},\sigma} + t^*_{\mathbf{kp}} \hat{b}^\dagger_{\mathbf{p},\sigma} \hat{a}_{\mathbf{k},\sigma}$. Here, operators $\hat{a}_{\mathbf{k},\sigma}, \hat{b}_{\mathbf{p},\sigma}$ correspond to the normal and the superconducting electrode, and $t_{\mathbf{kp}}$ are the tunnel matrix elements which we take to be spin independent; the sum is taken over momenta $\mathbf{k}, \mathbf{p}$ and spin $\sigma = \uparrow, \downarrow$. Second order perturbation theory in $\hat{H}_T$ yields the lowest order contribution to the amplitude of two-electron tunneling. The $\hat{a}$ operators appearing in this amplitude remove two electrons from the normal metal electrode with energy ξ_k and $\xi_{k'}$. The amplitude thus consists of a sum over intermediate states in the superconductor

$$A_{\mathbf{k}\uparrow\mathbf{k}'\downarrow} = \sum_{p,p'} t^*_{\mathbf{kp}} t^*_{\mathbf{k}'\mathbf{p}'} \left(\langle N | \hat{b}^\dagger_{\mathbf{p},\uparrow} \frac{1}{\xi_{k'} - \hat{H}_S} \hat{b}^\dagger_{\mathbf{p}',\downarrow} | N-2 \rangle - \langle N | \hat{b}^\dagger_{\mathbf{p}',\downarrow} \frac{1}{\xi_k - \hat{H}_S} \hat{b}^\dagger_{\mathbf{p},\uparrow} | N-2 \rangle \right) . \quad (1)$$

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The matrix elements between $|N\rangle$ and $|N-2\rangle$ connect states differing by two electrons. In coordinate representation they can be expressed in terms of the Fourier transform $\hat{F}^\dagger_{\uparrow\downarrow}(r_1', r_2'; \omega)$ of the usual anomalous Green's function $i\hat{F}^\dagger_{\uparrow\downarrow}(r_1', t; r_2', 0) = \langle N|T\hat{\psi}^\dagger_\uparrow(r_1', t)\hat{\psi}^\dagger_\downarrow(r_2', 0)|N-2\rangle$. This is the expectation value of the time-ordered product of two Heisenberg field operators, which can be expanded in terms of the operators $\hat{b}^\dagger_{\mathbf{p},\sigma}$. Since we assume that $T, eV \ll \Delta$, we consider only $\xi_k, \xi_{k'} \ll \Delta$ and find

$$A_{\mathbf{k}\uparrow\mathbf{k}'\downarrow} = 2\pi \int dr_1 dr_2 dr_1' dr_2' \psi_k(r_1)\psi_{k'}(r_2) \times t^*(r_1, r_1')t^*(r_2, r_2')\hat{F}^\dagger_{\uparrow\downarrow}(r_1', r_2'; \omega = 0) , \qquad (2)$$

where $\psi_k(r)$ denotes an eigenfunction of an electron in the normal metal; primed space arguments refer to the superconductor. In a disordered material, $\psi_k(r)$ and $\hat{F}^\dagger_{\uparrow\downarrow}(r_1', r_2'; \omega)$ are in general complicated functions depending on the realization of the disorder. Since we are interested only in the interference occurring in the normal metal, we perform an average of (2) over states in the superconductor. This may be done along the lines of [10]. Since tunneling occurs only between neighboring points, coordinates r and r' coincide; the product of two tunnel amplitudes will give the normal state conductance $g(r)$ of the tunnel interface per unit area such that the total conductance $G_T = \int d^2r\, g(r)$. The result reads

$$A_{\mathbf{k}\uparrow\mathbf{k}'\downarrow} = \frac{\hbar}{e^2}\frac{\pi}{\nu_N} \int d^2r\, g(r)\psi_k(r)\psi_{k'}(r)e^{i\phi(r)} , \qquad (3)$$

where $\phi(r)$ is the phase of the superconducting condensate and ν_N the density of states in the normal metal. The rate for two-electron tunneling as a function of the applied bias voltage V is obtained by applying Fermi's Golden Rule. To obtain the current we have to sum the tunnel rates in both directions. As a result we find

$$I(V) = \frac{\pi^2\hbar}{e^3\nu_N} \int d^2r_1 d^2r_2\, g(r_1)g(r_2) \exp i[\phi(r_1) - \phi(r_2)] \int d\omega [f(\omega/2 - eV) - f(\omega/2 + eV)][P^C_\omega(r_1, r_2) + P^C_{-\omega}(r_1, r_2)] . \qquad (4)$$

Here f is the Fermi distribution for electrons in the normal metal. Equation (4) is the central result of our paper, which clearly shows the interplay between phase coherence in a superconductor and a normal metal, connected by a single junction [11]. The intrinsic coherence of the superconductor is reflected by the appearance of the phase difference $\phi(r_1) - \phi(r_2)$. In the normal metal, the two incoming electrons undergo many elastic scattering events in the junction region before they tunnel through the N-I-S interface, leading to interference on a length scale given by L_T [8]. These interference effects have been taken into account by averaging the rate in the standard way [3] over possible scattering events. The result (4) therefore contains the sum of two Cooperon contributions $P^C_\omega(r, r')$, which obey the equation [12]

$$\{-\hbar D[\nabla - i2\pi A(r)/\Phi_0]^2 - i\omega\}P^C_\omega(r, r') = \delta(r - r') , \qquad (5)$$

where A is the vector potential and $\Phi_0 = hc/2e$ the flux quantum. From this equation it is clear that the result does not only depend on properties of the junction (via G_T), but also on its surroundings over a distance L_T, due to the interference occurring on this length scale [9].

As a simple example we calculate first the subgap conductance corresponding to a layout where a semi-infinite normal wire of thickness $d \ll L_T$ is connected to a superconducting electrode by a tunnel junction. In this case, the only contribution from the spatial integrations in (4) originates from the tunnel junction at $r_1 = r_2 = 0$. The solution of (5) for a wire yields $P^C_\omega(0,0) = 1/d\sqrt{-i\omega\hbar D}$. It leads to

$$G_{\text{wire}} = 8\pi^{5/2}(1 - 2\sqrt{2})\zeta(-\tfrac{1}{2})R_{\text{cor}}G_T^2 \approx 53.8 R_{\text{cor}}G_T^2 , \qquad (6)$$

where $R_{\text{cor}} = L_T/e^2\nu_N Dd$ is the resistance of the wire per correlation length L_T. The explicit dependence on L_T arises after integrating the Cooperon together with the Fermi factors over ω in Eq. (4). This dependence causes an enhancement of the subgap conductance due to interference in the normal wire, as discussed in [8]. Qualitatively, one may interpret this enhancement as induced superconductivity in the normal wire over the length L_T.

Interference effects in a mesoscopic system threaded by a magnetic flux lead to the Aharonov-Bohm effect: the total resistance depends periodically on the applied flux [13]. A similar effect can be observed with the layout depicted in Fig. 1(a), where a small loop with circumference L is inserted into a wire at distance l of the junction. The resistance of the loop is denoted by R_L; R_l is the resistance of the piece of wire between loop and junction. The loop is threaded by a magnetic flux Φ. The conductance (4) at zero temperature ($R_L, R_l \ll R_{\text{cor}}$) reads

$$G_{\text{loop}} = 4\pi^2 G_T^2 \left(R_l + \frac{R_L}{\sin^2 \pi\Phi/\Phi_0} \right) . \qquad (7)$$

This result is plotted in Fig. 2 (upper curve). It shows the usual $h/2e$ periodicity [13] related to the Cooperon. Moreover it diverges each time when $\Phi = n\Phi_0$ for integer n. At finite temperature the result (6) restricts the maximal conductance at zero flux by the value $53.8 R_{\text{cor}}G_T^2$. But even at zero temperature, when R_{cor} diverges, (7) will still be finite, due to the penetration of magnetic flux in the wires that constitute the loop. This penetration leads to a shift in the Cooperon energy $\omega \to \omega + i\alpha^2(\hbar D/4L^2)(2\pi\Phi/\Phi_0)^2$ where $\alpha = S_{\text{wire}}/S_{\text{loop}}$ is the ratio of the area of the wire and the loop. As a result we obtain instead of (7)

$$G_{\text{loop}} = \frac{8\pi^2 G_T^2 R_L[1+2(l/L)\sin^2 \pi\Phi/\Phi_0]^2}{(\pi\alpha\Phi/2\Phi_0)(\cos 2\pi\Phi/\Phi_0 + 1) + (2\sin^2 \pi\Phi/\Phi_0)[1+2(l/L)\sin^2 \pi\Phi/\Phi_0]} . \tag{8}$$

Thus the divergencies are removed as can be seen in Fig. 2 (lower curve), where the result is plotted, taking $l/L = 0.5$ and $\alpha = 0.1$. We note that the flux dependence presented here is caused by the fact that the dominant transport mechanism under subgap conditions consists of a phase-coherent transfer of *two* electrons.

We now turn to a different geometry [Fig. 1(b)], where instead of a ring a fork is attached to the wire, such that we have two tunnel junctions to the superconductor at different positions r_1 and r_2. The subgap conductance will be determined not only by the flux threading the closed area between the fork and the superconductor, but also by the magnetic field distribution in the superconductor. Let us consider the curves C_N and C_S connecting the junctions 1 and 2 in the normal metal and the superconductor, respectively. The effect will be governed by the phase $\theta = \phi(r_1) - \phi(r_2) + (2e/\hbar c)\int_{C_N} \mathbf{A}\cdot d\mathbf{x}$. In order to obtain a gauge-invariant expression for θ we use the relation $\boldsymbol{\nabla}\phi - (2e/\hbar c)\mathbf{A} = \mathbf{p}_s/\hbar$, where $\mathbf{p}_s$ is the momentum of the superconducting condensate. In this way we arrive at $\theta = 2\pi\Phi/\Phi_0 + \int_{C_S}(\mathbf{p}_s/\hbar)\cdot d\mathbf{x}$, with Φ the flux penetrating the closed loop formed by $C_N + C_S$. This result does not depend on the choice of C_S. It reflects the dependence of θ on the penetration of the magnetic field as well as on the vortex positions in the superconductor. The effect can be used to monitor these positions. Introducing the conductances G_1, G_2 of the junctions we obtain

$$G_{\text{fork}} = 53.8 R_{\text{cor}}\left[G_1^2 + G_2^2 + 2G_1G_2\cos\theta\right] \tag{9}$$

when L_T is larger than the size of the fork. The conductance G_{fork} thus combines the phase coherence in the normal and the superconducting metal.

If we consider tunneling to a superconductor of finite size, the transport will be influenced by charging effects, if the total capacitance C of the superconducting island is small enough, such that the charging energy $E_c = e^2/2C$ is of the order of Δ [7,14]. We will restrict ourselves to the case $\Delta > E_c$. In this case the transport to the superconductor will be due to two-electron processes. We will present a simple relation between the rate for these processes and the current which would flow in the absence of charging effects. The transfer of two electrons to the superconducting island increases its electrostatic energy by an amount E_{2e}. This energy can be changed with the help of an additional potential V_g applied to the island; we assume that $E_{2e} \ll E_c, \Delta$. The first electron entering the superconductor as a quasiparticle then increases the electrostatic energy of the island by an amount E_c. The energy of the intermediate states in the superconductor will therefore be shifted by this amount, leading to a different amplitude $\tilde{A}_{\mathbf{k}\uparrow\mathbf{k}'\downarrow} = [F(E_c)/F(0)]A_{\mathbf{k}\uparrow\mathbf{k}'\downarrow}$, with

$$F(E) = \frac{4\Delta}{\sqrt{\Delta^2 - E^2}} \arctan\sqrt{\frac{\Delta + E}{\Delta - E}} . \tag{10}$$

The rate is then given by $2e\tilde{\Gamma} = [F(E_c)/F(0)]^2 I(E_{2e}/2e)$.

Finally we investigate a Josephson-like effect, which occurs in the geometry depicted in Fig. 1(c). In this layout

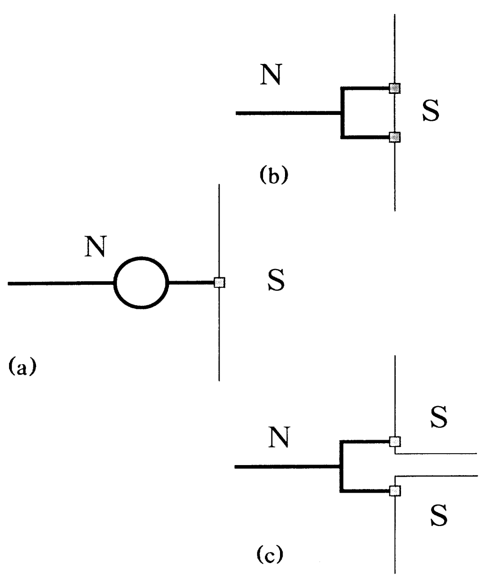

FIG. 1. Three geometries discussed in the text: (a) one-dimensional normal wire containing a loop connected to a superconducting electrode by a single junction; (b) fork geometry connected to the superconductor by two junctions; (c) fork geometry connected to two different superconductors.

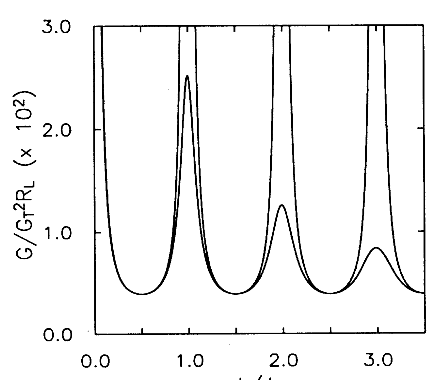

FIG. 2. Subgap conductance at zero temperature for the geometry of Fig. 1(a), as a function of flux. Curves correspond to Eqs. (7) (upper curve) and (8) (lower curve) in the text.

a normal fork is connected to two different superconductors, to which a small voltage difference eV_S is applied. The phase difference between the two superconductors and hence between the junctions at the extensions of the fork will increase linearly with time: $\theta = eV_S t/\hbar$. Substituting this phase difference into Eq. (9) we obtain

$$G_J = 53.8R_{\rm cor}\left[G_1^2 + G_2^2 + 2G_1G_2\cos(eV_S t/\hbar)\right] . \quad (11)$$

This conductance is the ordinary three-terminal conductance of the fork. It could be measured by determining the current through the normal wire under subgap conditions ($eV \ll kT, \Delta$). The effect of V_s on the transport voltage V may be neglected when V_s is choosen small enough. One sees that the conductance oscillates with a frequency $\omega_J = eV_S/\hbar$; the modulation is of the order of the conductance itself.

In conclusion we studied the effect of interference on the subgap conductivity of an N-I-S tunnel junction. Transport is determined by the transfer of electrons in pairs from the normal metal to the superconductor. At low temperatures, interference between the two electrons occurs in the normal metal over a longer length scale than in the superconductor. Therefore, the subgap conductance is determined not only by properties of the tunnel interface, but also by the layout on the normal side near the interface over a distance L_T. These novel interference effects can be made visible by influencing the electron phase, e.g., with the help of the Aharonov-Bohm effect or the Josephson effect. We discuss these effects for various layouts of practical interest, and present results for the subgap conductance.

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