Spin-Flavor Separation and Non-Fermi-Liquid Behavior in the Multichannel Kondo Problem: A Large-N Approach

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We present an analytical solution of a SU(N) × SU(M) generalization of the multichannel single-impurity Kondo model in the limit $N \to \infty$, $M \to \infty$, with $\gamma = M/N$ fixed. Non-Fermi-liquid behavior of the single electron Green function and of the local spin and flavor susceptibilities occurs in both regimes, $N \leq M$ and $N > M$, with leading critical exponents *identical* to those found in the conformal field theory solution for *all* N and M (with $M \geq 2$). We explain this remarkable agreement and connect it to "spin-flavor separation," the essential feature of the non-Fermi-liquid fixed point.

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In recent years, the multichannel Kondo model, first introduced by Nozières and Blandin [1], has been the focus of intense activity. This model involves a single local moment of spin S_I coupled through antiferromagnetic exchange to M identical conduction bands. It is now established [2,3] that in the "overcompensated" regime, $2S_I < M$, the screening of the local moment by the conduction electrons drives the metal into a non-Fermi-liquid critical state at $T = 0$, fully validating Nozières and Blandin's original arguments.

The renewed interest in this model is, in part, due to a number of suggestions for realizations of two-channel overcompensated behavior in explicit experimental contexts, most notably: (i) the quadrupolar Kondo effect [4] in heavy fermion alloys [5]; (ii) non-Fermi-liquid scattering rates in narrow copper point contacts [6]; and (iii) "marginal" Fermi liquid normal state properties [7] of the high-T_c materials [4,8]. Moreover, it has been argued that the two-channel Kondo model provides a link to exotic superconductivity [3,4,8]. Related non-Fermi-liquid behavior and enhanced pairing correlations have been recently found in the mixed-valence regime at low temperatures in certain extended Anderson single-impurity models motivated by the electronic structure of the high-T_c materials [9] .

From a theoretical point of view, remarkable progress has been made in understanding the universal properties of the multichannel Kondo and other impurity models through the use of conformal field theory techniques [3]. This approach provides a clear picture of the separation of spin, channel, and charge excitations at non-Fermi-liquid fixed points and describes the subtle recombination of these degrees of freedom required in the Fermi liquid case. In spite of its elegance and power this method only classifies the *possible* critical behaviors without providing a constructive route to the solution of a *particular* model.

In this Letter we formulate a controlled calculation method which can, in principle, incorporate both the complications of real materials *and* the conceptual insights gained from conformal field theory. Apart from the obvious practical application to dilute impurity systems, this question is important in studying the possibility for non-Fermi-liquid behavior in lattice systems, for which the conformal field theory techniques are not immediately applicable. An explicit route for addressing the latter problem involves constructing mean-field theories for a lattice in infinite spatial dimensions by solving a certain single-impurity model embedded in a self-consistent medium [10].

We concentrate on the SU(N) × SU(M) generalization of the multichannel Kondo problem, where N and M are the degeneracies in the spin and flavor quantum numbers, respectively. We use a functional integral approach based on the "slave-boson" representation [11] which explicitly separates the (local) spin and flavor excitations. The limit $N, M \to \infty$ with $\gamma = M/N$ fixed leads to a closed set of coupled self-consistent integral equations which can be solved analytically in the asymptotic low-frequency, zero temperature limit. These are identical to the "Noncrossing Approximation" (NCA) equations of perturbation theory [11]. Although these equations break down for sufficiently low energies in the Fermi liquid (single channel) case [12], it is the unique feature of our large N, M treatment of the multichannel Kondo problem that the NCA becomes *exact*. In fact, we show that the single electron Green function and the local spin and flavor susceptibilities display non-Fermi-liquid behavior with leading critical exponents *identical* to those found in the conformal field theory solution [3] for *all* $N, M \geq 2$. We explain this apparently surprising result by demonstrating that the fluctuations provide *no* corrections to the leading exponents obtained from NCA *to all orders in* $1/N, 1/M$ [13]. We regard this as an explicit realization of Anderson's "spin-charge separation" idea [14] at the non-Fermi-liquid fixed point of the multichannel Kondo problem.

Our starting point is a path integral treatment of the generalized infinite U Anderson model Hamiltonian,

$$H = \sum_{k,\sigma,\alpha} \epsilon_k c^\dagger_{k,\sigma,\alpha} c_{k,\sigma,\alpha} + \epsilon_f \sum_\sigma f^\dagger_\sigma f_\sigma + (V/\sqrt{N_s}) \sum_{k,\sigma,\alpha} [f^\dagger_\sigma b_{\bar{\alpha}} c_{k,\sigma,\alpha} + \text{H.c.}], \quad (1)$$

where the fermion, $f^\dagger_\sigma$, creates a local spin excitation and the boson, $b_{\bar{\alpha}}$, transforms according to the conjugate representation of SU(M) and annihilates the flavor quantum number of the "vacuum" state produced by destroying a conduction electron; N_s is the number of lattice sites. The "completeness" of the local states at the impurity site, represented by the constraint, $\sum_\sigma f^\dagger_\sigma f_\sigma + \sum_{\bar{\alpha}} b^\dagger_{\bar{\alpha}} b_{\bar{\alpha}} = 1$, is implemented in the usual way [11], by introducing a fictitious field λ coupling to the constrained charges, which is taken to ∞ at the end of the calculation. In the limit $\epsilon_f < 0, V/|\epsilon_f| \ll 1$ (1) leads to the SU(N) $\times$ SU(M) Coqblin-Schrieffer model, with exchange coupling $\mathcal{J} = V^2/|\epsilon_f|$.

The most compact way of presenting our arguments is in terms of the impurity contribution to the partition function Z_{imp} obtained after performing a Gaussian integral over the conduction electron fields. (This leads to an overall multiplicative factor Z_c of the free-electron gas partition function to the full partition function, $Z = Z_c Z_{\text{imp}}$.) By completing squares, the hybridization term is eliminated in favor of a spin-flavor interaction contribution, $S_{\text{int}} = -(\tilde{V}^2/N) \sum_{\sigma,\bar{\alpha}} \int d\tau \int d\tau' f^\dagger_\sigma(\tau) f_\sigma(\tau') G^0(\tau - \tau') b^\dagger_{\bar{\alpha}}(\tau') b_{\bar{\alpha}}(\tau)$, where $G^0(\tau - \tau') = -\sum_k (\partial/\partial\tau + \epsilon_k)^{-1}/N_s$ is the noninteracting conduction electron Green function at the impurity site; and, as usual, in order to obtain a nontrivial large N, M limit, we have defined a rescaled hybridization matrix element, $\tilde{V} = \sqrt{N} V$, which should be considered of order unity at the end of the calculation. The next step is to introduce two composite fields, nonlocal in imaginary time, $\Phi_f(\tau, \tau')$ and $\Phi_b(\tau, \tau')$ [with $\Phi^\dagger_{f,b}(\tau, \tau') = \Phi_{f,b}(\tau', \tau)$], which decouple the interaction term S_{int}, enabling us to write, $Z_{\text{imp}} = \int [\mathcal{D}f][\mathcal{D}b][\mathcal{D}\Phi_f][\mathcal{D}\Phi_b] \exp(-\tilde{S})$. The collective field contribution to the effective action, $\tilde{S} = \tilde{S}_{\text{free}} + \tilde{S}_\Phi$, is given by

$$\tilde{S}_\Phi = -\tilde{V}^2 \int_0^\beta d\tau \int_0^\beta d\tau' G^0(\tau - \tau') \Big(N \Phi_f(\tau', \tau) \Phi_b(\tau, \tau') - \sum_\sigma f^\dagger_\sigma(\tau) \Phi_b(\tau, \tau') f_\sigma(\tau') - \sum_{\bar{\alpha}} b^\dagger_{\bar{\alpha}}(\tau') \Phi_f(\tau', \tau) b_{\bar{\alpha}}(\tau) \Big). \quad (2)$$

The large N, M calculation proceeds as usual by carrying out the Gaussian integral over the local fermions and bosons, f_σ and $b_{\bar{\alpha}}$, to produce an effective action for the composite fields, $\Phi_{f,b}$; this is then evaluated by a saddle point integration [15]. (Hereafter we drop the bar on the channel index, $\bar{\alpha}$.)

In the $N, M = \infty$ limit with $\gamma = M/N$ fixed the saddle point approximation becomes exact and leads to time translationally invariant solutions, $\Phi_{f,b}(\tau - \tau')$. After Fourier transforming in terms of fermionic and bosonic imaginary frequencies, ω_n, ν_n, the saddle point equations can be written as $\Phi_f(i\omega_n) = [i\omega_n - \epsilon_f - \Sigma_f(i\omega_n)]^{-1}$ and $\Phi_b(i\nu_n) = [i\nu_n - \Pi_b(i\nu_n)]^{-1}$. This defines the self-energies, Σ_f and Π_b, which, on the real frequency axis ($i\omega_n = \omega + i0^+, i\nu_n = \omega + i0^+$), satisfy the self-consistent equations,

$$\Sigma_f(\omega) = \frac{\gamma\tilde{\Gamma}}{\pi} \int d\epsilon f(\epsilon) \Phi_b(\epsilon + \omega), \quad (3)$$

$$\Pi_b(\omega) = \frac{\tilde{\Gamma}}{\pi} \int d\epsilon f(\epsilon) \Phi_f(\epsilon + \omega). \quad (4)$$

Here $\tilde{\Gamma} = \pi\rho\tilde{V}^2$ is the bare hybridization width (which includes a factor of N introduced by the rescaling), and ρ is the conduction electron density of states, assumed constant in the energy range of interest. Also note that the $\lambda \to \infty$ limit has already been taken. In principle, the functions Σ_f and Π_b elsewhere in the complex plane can be obtained by the appropriate analytic continuation.

In the low-frequency, low-temperature limit, the above self-consistent equations can be easily solved either by direct substitution or by reducing them to differential equations as done for the single channel case [11,12]. The solutions for Φ_f and Φ_b can be written as power series in the reduced frequency variable, $\Theta = \{[(1 + \gamma)/\gamma](E_0 - \omega)/T_0\}^{1/(1+\gamma)}$, where E_0 is the ground state energy and $T_0 = D(\gamma\tilde{\Gamma}/\pi D)^\gamma \exp(\pi\epsilon_f/\tilde{\Gamma})$ is the Kondo scale. Near $\omega = E_0$, the $T \to 0$ spin fermion and flavor boson spectral functions, $A_{f,b}(\omega) = \text{Im}\Phi_{f,b}(\omega - i0^+)/\pi = A^{(+)}_{f,b}(\omega)\theta(\omega - E_0)$, and the corresponding spectral functions for occupied states, $A^{(-)}_{f,b}(\omega) = \lim_{T\to 0}[\text{Im}\Phi_{f,b}(\omega - i0^+)/\pi] \exp(\beta[E_0 - \omega])$, take the forms $A^{(\pm)}_f(\omega) \sim |\Theta|^{-\gamma}$ and $A^{(\pm)}_b(\omega) \sim |\Theta|^{-1}$.

By expressing the resulting frequency dependence as $|E_0 - \omega|^{2\Delta_{f,b}-1}$ we can identify the scaling dimensions of the spin and flavor fields, $\Delta_f = [2(1 + \gamma)]^{-1}$ and $\Delta_b = \gamma\Delta_f$. It then follows that the spin, channel, and physical fermion fields, all of which are bilinears of f and b, have scaling dimensions, $\Delta_{\text{sp}} = 2\Delta_f = 1/(1 + \gamma)$, $\Delta_{\text{ch}} = 2\Delta_b = \gamma/(1 + \gamma)$, and $\Delta_F = \Delta_f + \Delta_b = 1/2$, respectively. The leading frequency dependence of the corresponding correlation function, $|E_0 - \omega|^{2\Delta_{\text{sp,ch,F}}-1}$, is indeed what we observe. Below we summarize some of our explicit results.

Single-electron Green function.—The local electron Green function, $\mathcal{G}_{\sigma\alpha}$, can be calculated as a convolution

of the spin and flavor propagators, leading to a local spectral function, $\rho_{\sigma\alpha}(\omega,0) = \mathrm{Im}\,\mathcal{G}_{\sigma\alpha}(\omega - i0^+)/\pi$, of the form

$$\rho_{\sigma\alpha}(\omega,0) \approx [\pi/(1+\gamma)^2 N\tilde{\Gamma}] \times [1+\theta(\omega)f_+(\tilde{\omega})+\theta(-\omega)f_-(\tilde{\omega})], \quad (5)$$

with $f_\pm(\tilde{\omega}) = (a_\pm|\tilde{\omega}|^{\Delta_{sp}} + b_\pm|\tilde{\omega}|^{\Delta_{ch}})$, $a_- = -[4\gamma/(2+\gamma)\pi]\sin(\pi\Delta_{ch})B(2\Delta_{sp},\Delta_{ch})$, $a_+ = -\cos(\pi\Delta_{ch})a_-$, $b_+ = -[4W_{ch}/(1+2\gamma)\pi]\sin(\pi\Delta_{ch})B(2\Delta_{ch},\Delta_{sp})$, and $b_- = \cos(\pi\Delta_{ch})b_+$. Here, $W_{ch} = \pi T_0/\tilde{\Gamma}$ is the weight of channel fluctuations in the ground state, $\tilde{\omega} = [(1+\gamma)/\gamma](\omega/T_0)$, and $B(x,y)$ is the beta function. In the overscreened case, $M \geq N$, both the leading and next-to-leading frequency dependence are the same as that obtained by Affleck and Ludwig from conformal field theory [3]. Also, note the explicit breaking of particle-hole symmetry displayed by the different signs above for positive and negative frequencies, consistent with the nonsymmetric form of (1).

Resistivity.—From the single-particle Green function we can obtain the resistivity from the standard transport theory formula, $\rho(T) \sim [\int d\epsilon(\frac{-\partial f}{\partial\epsilon})\tau(\epsilon,T)]^{-1}$, in terms of the scattering rate, $\tau_{\sigma\alpha}(\omega,T)^{-1} = -2\,\mathrm{Im}t^{(1)}_{\sigma\alpha}(\omega+i0^+,T) = [2\tilde{\Gamma}\rho_{\sigma\alpha}(\omega,T)]/(\rho N)$, where we have used the relation between the conduction electron t matrix and the single-electron Green function, $t^{(1)}_{\sigma\alpha}(\omega,T) = V^2\mathcal{G}_{\sigma\alpha}(\omega,T)$. The resulting leading behavior, $\rho(T)/\rho(0) \sim [1 - c(T/T_0)^{\min(\Delta_{sp},\Delta_{ch})} + \cdots]$ is in agreement with that obtained by Affleck and Ludwig [3]. In particular, for the special case of $N = M$, $\Delta_{sp} = \Delta_{ch} = 1/2$, we obtain a $\sqrt{T}$ correction as does the conformal approach [3]. Estimating the coefficient c requires a knowledge of the temperature dependence of $\rho_{\sigma\alpha}$ which is beyond the scope of the present paper. It is gratifying that for $N = 2$ the magnitude of the spin contribution to the rate, $\{(\pi\rho)/[2\tau_{\sigma\alpha}(0,0)]\} = 3\pi^2/[4(2+M)^2]$ (which is obtained after removing 1/4 of the total rate due to potential scattering), agrees with the results of Affleck and Ludwig [3] to within 8% for all $M \geq 2$.

Local spin and channel flavor dynamical susceptibilities.—The linear response to external fields coupling to the impurity spin and channel quantum numbers can be easily calculated from the bubble diagrams for the spin and channel excitations, f and b, respectively. The leading contributions to the absorptive part of the local spin and flavor susceptibilities (per spin or channel degree of freedom), $\tilde{\chi}''_{sp} = \mathrm{Im}\chi_{sp}/N$ and $\tilde{\chi}''_{ch} = \mathrm{Im}\chi_{ch}/M$, are given by

$$\tilde{\chi}''_{sp,ch}(\omega,0) \sim (C_{sp,ch}/T_0)\mathrm{sgn}(\omega)|\tilde{\omega}|^{(\Delta_{sp,ch}-\Delta_{ch,sp})}, \quad (6)$$

with $C_{sp} = \gamma\Delta^2_{sp}\sin(\pi\Delta_{sp})B(\Delta_{sp},\Delta_{sp})$ and $C_{ch} = W^2_{ch}\Delta^2_{sp}\sin(\pi\Delta_{ch})B(\Delta_{ch},\Delta_{ch})$. The next corrections vanish as $|\tilde{\omega}|^{(2\Delta_{sp,ch}-\Delta_{ch,sp})}$. Note that for $N = M$ (which includes the important case $N = M = 2$) both susceptibilities reduce to the general form $\tilde{\chi}''(\omega,0) \approx A\,\mathrm{sgn}(\omega)[1 - B\sqrt{|\tilde{\omega}|/T_0} + \cdots]$. This leading step function behavior in χ'' and the associated logarithmic dependence in the real part, $\chi'(\omega,T) \sim -\ln(\max\{\omega,T\}/T_0)$, provides a possible connection with the marginal Fermi liquid phenomenology of the high-T_c oxides [7], as first noted in Ref. [4(b)]. For $N > M > 1$ the real part of the spin susceptibility is constant ($\sim 1/T_0$) and the non-Fermi-liquid behavior is dominated by the $|\tilde{\omega}|^{-(\Delta_{sp}-\Delta_{ch})}$ divergence of the channel susceptibility (see Fig. 1). In the opposite limit, $N < M$, the flavor susceptibility is constant ($\sim W^2_{ch}/T_0$) and a $|\tilde{\omega}|^{-(\Delta_{ch}-\Delta_{sp})}$ divergence occurs in the spin susceptibility. For $N > M$ the system displays two-parameter universality: the channel fluctuations start dominating below a new energy scale, $T_{ch} \sim T_0 W_{ch}^{[1/(\Delta_{sp}-\Delta_{ch})]}$, with $T_{ch} << T_0$ for $N > M$ in the Kondo limit ($W_{ch} << 1$). We note that T_{ch} evolves into the "pathology" temperature below which non-Fermi-liquid behavior ensues in the NCA treatment of the one-channel Kondo model [11,12].

1/N fluctuations.—The functional integral formulation outlined above gives a natural framework for estimating the effects of fluctuations, and allows us to explain the remarkable fact that the saddle point exponents remain unchanged to all orders in $1/N$. The arguments are very much in the spirit of the argument usually made to justify the fact that *perturbation theory* gives the exact exponents characterizing the low-energy behavior in conventional Fermi liquids, and suggest the possibility of extending Nozières' phenomenological discussion to non-Fermi-liquid situations. More precisely, all $1/N$ fluctuations can be incorporated into the renormalization of interaction vertices in all diagrams (f and b self-energies and the single-particle Green function as well as all susceptibilities). We have checked explicitly to order $1/N^2$ that these vertex renormalizations modify

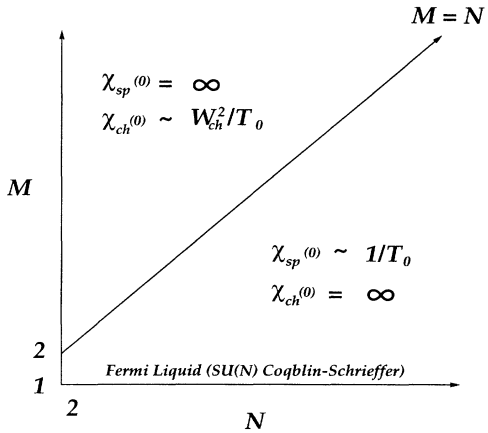

FIG. 1. Phase diagram in the N, M plane. The NCA is strictly controlled in the large M, N limit for $M \geq 2$, but gives the exact critical exponents *for all* $N, M \geq 2$. The universality class is the same for all lines of fixed slope $\gamma = M/N$.

the amplitudes but only give subleading singular contributions. [For example, the subleading corrections to Σ_f and Σ_b behave as, $\mathrm{Im}\delta\Sigma_f(\omega) \sim |\omega - E_0|^{3\Delta_{\mathrm{ch}}}$ and $\mathrm{Im}\delta\Sigma_b(\omega) \sim |\omega - E_0|^{3\Delta_{\mathrm{sp}}}$, respectively.] In fact, the saddle point gives the *exact* low-energy singularities to all orders in $1/N$, as can be seen by considering arbitrary order diagrams in perturbation theory (around the saddle point), propagators which carry spectral functions which diverge as $|\omega - E_0|^{\Delta_{f,b}-1}$. As an illustration, consider a generic diagram contributing to Π_b which contains L loops (thus L independent energy integrations), L fermion propagators, and $L-1$ boson propagators. The most singular contribution behaves as $|\omega - E_0|^{\zeta_b(L)}$ where $\zeta_b(L) = L + L(\Delta_f - 1) + (L-1)(\Delta_b - 1) = 1 - \Delta_b$ since unitarity of the scattering amplitude requires $\Delta_f + \Delta_b = 1$. This is indeed the behavior found at the saddle point. Similar arguments apply to Σ_f, the one particle Green function, and the spin and channel susceptibilities. It is natural to speculate that in systems with spin-charge separated, non-Fermi-liquid ground states, the correct low-energy behavior can be obtained on the basis of an appropriate (self-consistent) perturbation theory involving the "separated" spin and charge degrees of freedom. In analogy with conventional Fermi liquids, such perturbative arguments should be valid even when no obvious small parameters are available, provided no phase transition occurs to a Fermi liquid state through the "binding" of spin and charge.

Crossover effects.—In the presence of spin or channel symmetry breaking fields, H_{sp} or H_{ch}, we find crossovers to Fermi liquid behavior below respective scales, $T_{\mathrm{sp}} \sim H_{\mathrm{sp}}^{1+1/\gamma}, T_{\mathrm{ch}} \sim H_{\mathrm{ch}}^{1+\gamma}$. The corresponding crossover exponents, $\phi_{\mathrm{sp}} = 1 + 1/\gamma, \phi_{\mathrm{ch}} = 1 + \gamma$, are precisely those obtained from conformal field theory [3]. However, the Fermi liquid behavior below $T_{\mathrm{sp(ch)}}$ is outside the scope of the NCA. This can be traced back to the fact that the Kondo screening characteristic of the Fermi liquid fixed point involves the formation of a singlet bound state between a conduction electron and the local spin excitation, f_σ [16]; the residue of the bound state pole plays the role of the slave-boson mean-field amplitude in the conventional large-N approach to the Kondo problem. In the multichannel problem this Fermi liquid saddle point becomes possible only in the presence of a channel symmetry breaking field.

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