Model for the Evolution of River Networks

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We have developed a model, which includes the effects of erosion both from precipitation and from avalanching of soil on steep slopes, to simulate the formation and evolution of river networks. The avalanches provide a mechanism for competition in growth between neighboring river basins. The changing morphology follows many of the characteristics of evolution set forth by Glock. We find that during evolution the model maintains the statistical characteristics measured in natural river systems.

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Geological phenomena have provided the basis for numerous investigations into the driving mechanisms of spatial and temporal pattern formation in nature [1]. The rich morphology of natural river networks, in particular, has motivated numerous efforts to simulate their formation [2-6]. While the majority of these simulations, such as the random-walk models [4], have focused on reproducing their statistical properties [7], less emphasis has been placed on simulating the characteristic evolutionary cycle that geomorphologists have identified for these networks [8,9]. Given the interplay between the landscape features that dictate the river configurations and the erosional effects of the water which alter the landscape, one should not expect to capture the dynamical processes responsible for network formation through a model which addresses only the statistical characteristics of the networks without also modeling their evolution.

We present a lattice model for landscape erosion, concentrating on the temporal behavior of the landscapes and the rivers draining them. As distinct from previous models, we introduce avalanche events which lead to realistic hill-slope development and competition for survival among neighboring rivers [10]. The maintenance of statistical characteristics familiar to natural river systems is a central feature of the evolution. Our emphasis is on the time dependence of the model's statistics and its correspondence with the behavior of real river networks.

In our model a rectangular lattice of points simulates the terrain, and the height of the land, $h(x,y)$, is specified at each point. The lattice has periodic boundary conditions in the x direction. We begin the simulations with the landscape as a featureless incline, $h(x,y)=Iy$, where I defines the initial slope. Eroding water enters the simulation as precipitation landing at a random site on the lattice. The water flows across the landscape moving from the site it occupies to one of the four nearest neighbors according to the probability

$$P(\Delta h_i)=\begin{cases}\exp(E\Delta h_i)\left[\displaystyle\sum_{j=1,\Delta h_j\geq 0}^{j=4}\exp(E\Delta h_j)\right]^{-1}, & \Delta h_i\geq 0\,,\\ 0, \quad \Delta h_i<0\,, & \end{cases} \tag{1}$$

where Δh_i is the height of the site the water occupies minus the height of the neighbor i, and E is a free parameter in the model. Also, the water cannot return to a site it has immediately vacated unless that is the only direction with nonzero probability. This exponential dependence ensures that the water flows down any steep slopes it encounters but allows it to take a meandering path on more gentle terrain. The water continues to move to the nearest neighbors according to Eq. (1) until it reaches the bottom of the lattice, $y=0$.

After the water reaches $y=0$, each lattice point (x,y) visited loses D units of height to simulate the water's erosion: $h(x,y)\rightarrow h(x,y)-D$. Following this erosion, the values of Δh in the landscape are compared against a critical value M. If a site (x_h,y_h) has a height which is M units greater than any nearest neighbor, then it loses $\Delta h/4$ units to simulate the avalanching of soil: $h(x_h, y_h)\rightarrow h(x_h,y_h)-\Delta h/4$. The avalanching mass is not conserved and these units are simply eliminated as part of the eroded soil. (Preliminary results [11] on a more complicated model where the avalanched mass is conserved, through its redistribution onto the lower site, show no qualitative changes from the results reported here.) All sites are again checked against their nearest neighbors, and avalanching continues until all local slopes lie within the criterion set by M. The process then repeats with water again entering the simulation as precipitation at a new random lattice point.

The river network which drains the landscape can be defined at any time. In defining the rivers, a test drop of precipitation is placed at every lattice point. Each drop flows down the landscape, following the path of steepest descent, until it reaches $y=0$. After every drop has selected a path to $y=0$, all those points through which at least R drops have passed are considered part of a river.

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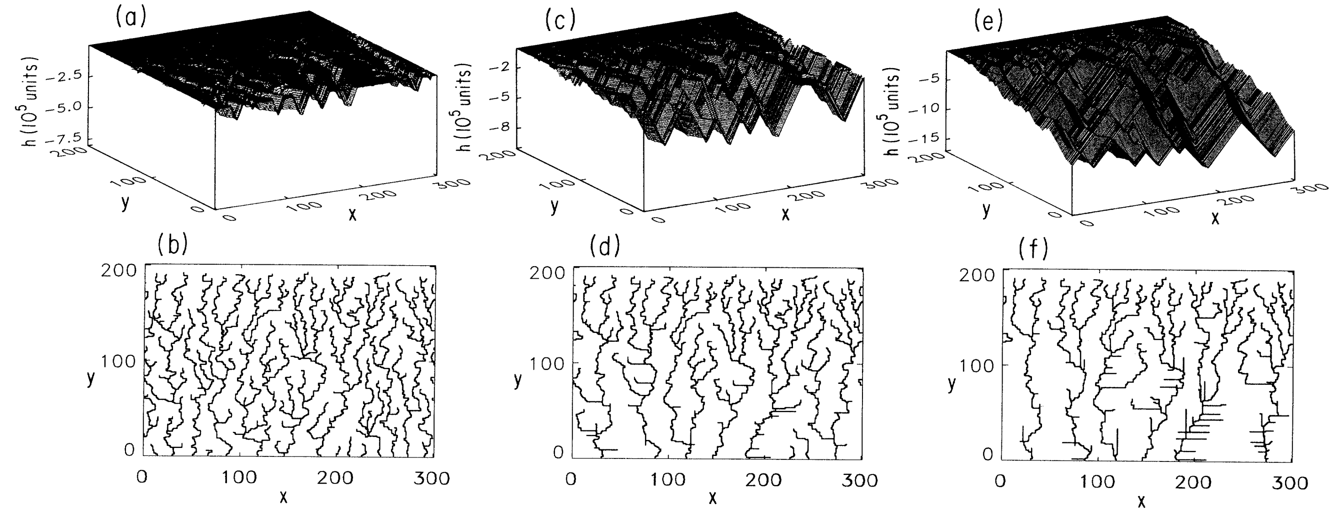

FIG. 1. The landscape (upper panel) and river networks (lower panel) at three times in a simulation: (a) and (b) after 1×10^5 iterations, (c) and (d) after 3×10^5 iterations, and (e) and (f) after 7×10^5 iterations. The simulation consisted of a 300×200 site system with model parameters: $D=10$, $E=\frac{1}{20}$, $M=20000$, $I=1.0$, and $R=100$.

Typically, the model parameters used in our simulations had $EI<1$ so that the early paths for the water flow were not heavily biased by the initial slope, resulting in meandering paths. Also, we chose $D>I$ so that water which encountered the path of previous precipitation would be biased toward joining that path and $M\gg D$ so that the eroding effects of a typical precipitation event were kept small. We examined the river patterns formed for different R (between 25 and 150) and found that, besides the obvious variation in drainage density (the ratio of total length of rivers to drainage basin area) and a weak effect on the apparent evolution rate, its value did not influence the statistical characteristic of the river networks.

Figures 1(a), 1(c), and 1(e) show the development of a 300×200 site landscape formed from this model at three points in its evolution. Figures 1(b), 1(d), and 1(f) show, for the choice $R=100$, the river networks corresponding to Figs. 1(a), 1(c), and 1(e), respectively. The evolution contains two distinct time scales. During the first few iterations water typically lands on an uneroded site and takes a meandering path down the initial slope before encountering the eroded path from a previous unit of water. Once it has encountered this path, Eq. (1) biases the water to follow it. This early aggregation process forms shallow incisions resembling the branched networks of the random-walk models [4]. With this pattern intact, further evolution follows a largely deterministic process on a much longer time scale. Advantageously positioned branches deepen progressively due to the enhanced collection of precipitation and expand their areas of influence at the expense of other networks through the avalanching process. This competition between neighbors provides the dynamics through which successful branches eliminate or subordinate others and produces network evolution.

This evolution contains many elements in common with the prevailing theory of natural river network evolution proposed by Glock [8]. Glock's theory, which has received support from numerous field observations [12] and laboratory experiments [13], divides the evolution of the morphology into an early period of "extension" followed by a later period of "integration." During extension the river system experiences "elongation" by headward growth of main branches and an "elaboration" with the addition of tributaries off of the main branches. After maximum extension has been reached, integration begins with the "abstraction" or elimination of small internal branches, as the main rivers increase their drainage area. A further property of integration, which Glock suggests may not be easily identified by inspecting present day river systems, is the aggressive adjustment of the path of the main branch in an effort to minimize its route to the sea. Another major characteristic of evolution that Glock identifies is stream capture, or "piracy," which occurs when an aggressively growing basin encroaches on the path of a stream from a neighboring basin and diverts that stream into its own network [9]. As we will now show, we are able to identify all of these features in our simulations.

As Figs. 1(a), 1(c), and 1(e) illustrate, the important feature in the evolution of the landscape is the broadening of V-shaped valleys and their headward invasion into the terrain, reminiscent of Glock's extension period. As the valleys grow, the hill slopes maintain a constant slope equal to $3M/4$ dictated by the avalanche erosion, and exhibit parallel retreat. This parallel retreat of the valley sides mimics the effects of erosion due to rainwash observed in the field [14]. (The river patterns correspond-

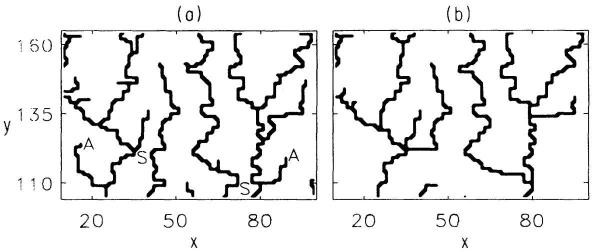

FIG. 2. Details from Figs. 1(b) and 1(d) showing examples of the abstraction of streams and of stream capture, two important characteristics of real river network evolution. In (a) "A" labels streams that will experience abstraction and "S" marks where stream capture will occur.

ing to these landscapes do not share the headward growth that the valley heads experience. This failure is an artifact of the rule defining the rivers in terms of R. Because no lattice point can be farther than R sites from a river, the networks by necessity appear at full extension at very early times.)

As the simulations progress, the rivers also follow the patterns of natural evolution. The main branches experience a slow migration to increasingly direct routes toward $y=0$. Meanwhile, abstraction takes place as internal tributaries lose their identity to the growing valleys of the main branches. Evidence of abstraction appears in Figs. 2(a) and 2(b), which show details from Figs. 1(b) and 1(d), respectively. The rivers marked with "A" in Fig. 2(a) experience abstraction and no longer exist in Fig. 2(b). Stream capture also plays an important role. In Fig. 2(a) the points marked with "S" are places where stream capture occurs and a river becomes part of a competing network by the time of Fig. 2(b).

Eventually, as the valleys of the main branches become large compared to R, tributaries from runoff down the valley sides appear. The formation of such second generation tributaries has been observed in real landscapes where conditions allow rapid incision of the main valley into the terrain [13]. Thus, new evolution occurs as the valley sides provide a new environment for the development of rivers. However, because $M \gg I$, the characteristics of these second generation networks differ greatly from those of the first.

We have compared several statistical relations among features of the model with the known statistical properties of natural systems. Hack's law [15,16] relates the area A of a drainage basin (the area of land which collects precipitation that contributes to the network) to the length of a principal river in the basin, L_p. From a series of field measurements, Hack concluded that

$$L_p \sim A^{\alpha}, \tag{2}$$

where α is a constant. Because the definition of a river in our model requires that at least R units flow through a river site, every basin formed on the model's landscapes must have at least an area R. Consequently, to account for this offset we modify Hack's law as

$$L_p \sim (A-R)^{\alpha}. \tag{3}$$

We identify basins of all Strahler order [7] on the terrain and define L_p within each basin as the longest river measured from a river head to the basin's mouth. The inset of Fig. 3 shows an example of the data for L_p versus A. Because of the deviations from a power law at low L_p, we consider only the asymptotic behavior ($L_p > 10^2$) in evaluating α. Figure 3 shows the value of α at different points of evolution. As the landscape evolves, the value of α quickly drops from $\alpha=0.67$ (which is the value found in the directed-walk model on a square lattice [17]) to a fixed point near $\alpha=0.6$, the value originally measured by Hack. At a more developed stage, α rapidly decreases to a second fixed point at $\alpha \approx 0.47$. Finally, at very late stages the quality of data deteriorates and Hack's law no longer holds for our simulations.

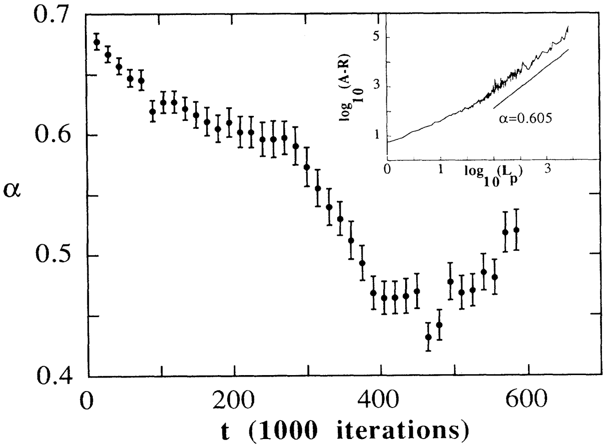

FIG. 3. The time development of α, the scaling exponent in Hack's law, for the model's networks. The exponent shows fixed points near $\alpha=0.6$ and $\alpha=0.47$, the same values measured by Mueller [18] for small and large natural drainage basins, respectively. The inset shows an example of the scaling of the principle river length with basin area after 1.8×10^5 iterations. The simulation was on 1000×1500 sites with model parameters: $D=10$, $E=\frac{1}{20}$, $M=2000$, $I=1.0$, and $R=70$.

An interesting correspondence exists between the behavior of α in the model and variations observed in nature. Mueller [18], analyzing several thousand sets of data, found that Hack's relation with $\alpha=0.6$ holds well for basins of area less than 8000 square miles. However, the exponent experiences a rapid change to $\alpha=0.5$ for basins between 8000 and 100000 square miles, and again changes rapidly to $\alpha=0.466$ for larger basins. Thus, the early and late fixed points of α in the evolving networks of our model match the exponents for small and large natural basins, respectively. The connection between basin size and age has been the subject of debate [19], and one should take care in drawing conclusions based on the substitution of size for age. However, following a headward growth description, one certainly can associate large basins with the highly developed stages of evolution. In such an association, the change in α in the networks of our model and the variation of α measured in the field have a natural correspondence.

We have also analyzed the model for comparison with other statistical properties of fluvial morphology, such as Horton's laws [20] and the stream length distribution [21], and find good agreement with natural systems during the model's evolution. A more complete discussion of these results will be published elsewhere [11]. The maintenance of Hack's law and these other statistical laws throughout a realistic evolutionary cycle, which has been the focus of our study, seems a necessary feature of any model which aspires to capture the important geomorphic mechanisms for the production of river patterns.

Our introduction of an avalanche scheme, which follows from the simple idea that the landscape cannot support steep slopes, leads naturally to a competition in growth among neighboring drainage basins. From this dynamical process we both recover the realistic evolutionary cycle for river networks and maintain the statistical laws characteristic of natural systems. Our ability to track the temporal behavior of these statistical features with our model provides a method for viewing trends in network morphology, as the correspondence between our results and Mueller's measurements illustrates. Clearly, modifications to the model can extend its description of natural systems. For example, a more physical criterion for channel initiation [6] could replace the model's reliance on random-walk paths. Also, allowing the resedimentation of eroded soil [11] should provide insight into the role of mass conservation in the model.

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