Hall Effect of Vortices Parallel to CuO_2 Layers and the Origin of the Negative Hall Anomaly in $YBa_2Cu_3O_{7-\delta}$

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The Hall resistivity of vortices that lie parallel to the CuO_2 layers in $YBa_2Cu_3O_{7-\delta}$ is shown to be *negative*, in contrast with the Hall effect observed with field perpendicular to the layers (**H**‖**c**). The Magnus force component along the current is opposite in sign for vortices that lie parallel and perpendicular to the layers. In an oblique field, the opposing forces strongly influence vortex motion. This finding provides new insight into the negative Hall anomaly (observed with **H**‖**c**). We show that the anomaly arises from the negative sign of the interlayer segments produced by strong fluctuations about the direction of **H**.

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In the high-temperature superconductors, the layered structure of the lattice and the large anisotropy of the coherence lengths lead to strong variation of the resistivity [1,2] and torque [3] when the magnetic field is rotated. From the angular dependence of the flux-flow resistivity ρ_{xx} in $YBa_2Cu_3O_{7-\delta}$ (YBCO) [1], Kes *et al.* [4] proposed that there is a decoupling of the flux components perpendicular and parallel to the layers in the cuprates. However, it was recently pointed out that the angular dependence $\rho(\theta,B)\sim\rho(B\cos\theta)$ is a direct consequence of scaling in an anisotropic-mass model [5], and is not, in itself, sufficient proof of a breakdown of the Abrikosov lattice. Moreover, the angular variation of the torque in YBCO seems well described by the anisotropic-mass model [3]. It is an important issue whether the anisotropic-mass model is adequate for understanding vortex motion in "90 K" YBCO or if one needs to go beyond. From Hall measurements performed with field perpendicular and parallel to the CuO_2 layers, we have uncovered a sharp, *qualitative* difference in how the vortex core responds to current in the two cases. When the vortices are strictly parallel to the layers, their Hall effect is negative, in contrast with the well-studied Hall response of vortices perpendicular to the layers. The new results shed considerable light on the origin of the much-discussed negative Hall anomaly.

We recall that the flux-flow Hall effect (observed with field perpendicular to the layers) is holelike in YBCO, except in a narrow temperature range near T_c, where it displays a negative minimum versus field (the "negative Hall anomaly") [6–8]. Wang and Ting [9] have proposed that the change of sign relates to the ratio of the vortex core size to the quasiparticle mean free path. According to Ferrell [10], however, the negative Hall sign arises from the "upstream" force exerted by backflow of quasiparticles on the core. Other mechanisms have also been discussed [7,8,11]. None of these theories addresses the effect of tilting the field.

The measurements were performed on several fully oxygenated single crystals of $YBa_2Cu_3O_{7-\delta}$ that display sharp resistive transitions at $T_c=92$ K. In both experiments, the resistivity and Hall resistivity are recorded simultaneously as the field is swept from $+14$ to -14 T. The Hall voltage is measured by an ac nanovoltmeter with a current density **J**‖**x** of 80 A/cm^2. To attain a temperature stability of ± 10 mK, we used a bridge (Gen Rad 1615A) to amplify the capacitance sensor reading. In the first experiment, the field is fixed along **y**, so that the vortices lie parallel to the layers with their cores located preferentially between the CuO_2 layers (Fig. 1, inset). The x component of the vortex velocity $\mathbf{v}_L$ is sensed by the Hall contacts (solid ovals). The "out-of-plane" Hall resistivity ρ_{xz} derives from the antisymmetric part of this signal, viz., $\rho_{xz}=-B_y v_{Lx}/J$. As shown in Fig. 1, ρ_{xz} is negative at all temperatures that we can access with a field of 14 T. In the normal state, the out-of-plane Hall resistivity is negative, and almost temperature independent [12,13]. In Fig. 1, our measurements show that it is strictly linear in field (trace at 100 K). A few degrees below T_c, the flux-flow Hall resistivity displays a behavior dramatically different from the normal state. In moderate fields, it increases rapidly at a rate $d\rho_{xz}/dB$ that is 10–12 times larger than in the normal state. At high fields, ρ_{xz} saturates to values (2–3 $\mu\Omega$ cm) that are $\sim$5–10 times larger than the Hall resistivity ρ_{xy} observed with the field perpendicular to the layers.

The sign of the Hall resistivity reflects that of the velocity component parallel to **J**. It is useful to think of the supercurrent imparting a "downstream" ("upstream") velocity component on the core when the Hall effect is observed to be positive (negative). Thus, our experiment implies that vortices parallel to the layers move upstream, whereas vortices perpendicular to the layers move downstream (in addition to their much larger velocity component in the direction of the Lorentz force). This has interesting implications for an oblique field.

Several groups [14] have proposed that, when the anisotropy is large, the structure of the vortex in an oblique field is comprised of vortex "pancakes" linked by segments that lie parallel to the layers (we will refer to the

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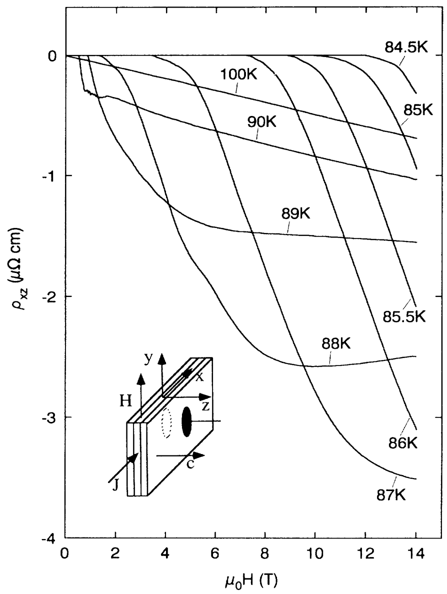

FIG. 1. The out-of-plane Hall resistivity ρ_{xz} versus field at fixed temperature in $YBa_2Cu_3O_{7-\delta}$ (**H** parallel to layers). The inset shows the field and current directions and the Hall contacts (ovals).

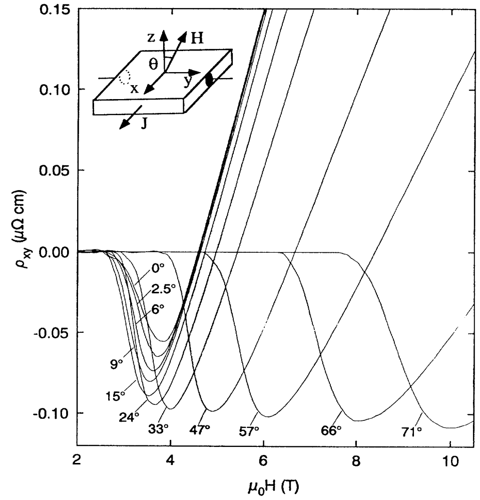

FIG. 2. The in-plane Hall resistivity ρ_{xy} versus field in twinned YBCO crystals for various angles θ at 84 K. The rapid deepening of the minimum as θ increases from 0° to 15° is associated with twin boundaries. At larger angles, the minimum continues to deepen while the onset field shifts to higher field values. The inset shows the experimental configuration.

latter as "interlayer" segments). We consider a vortex extending a distance L_z along **c**, with interlayer segments of total length L_y, moving as a unit, in response to a current $\mathbf{J} \| \mathbf{x}$. Since the x component of the force that acts on the interlayer segments and the pancakes are opposite in sign, we are presented with a novel situation for vortex motion. Let us call the Magnus force [15] on (unit length of) the interlayer segments $\mathbf{F}_{\|}$, and that acting on a single vortex pancake $\mathbf{f}_p$. The dominant motion along $\mathbf{J}\times\mathbf{H}$ is largely unaffected, while the motion along the current direction depends on the competition between these two opposing forces. Assuming that the velocity varies linearly with the bulk force [15], we write

$$v_{Lx} \sim \mathbf{x}\cdot[(\mathbf{f}_p L_z)/d + \mathbf{F}_{\|} L_y]\,, \qquad (1)$$

where d is the interlayer spacing. Obviously, tilting **H** close to the layers favors the interlayer segments and forces the vortex to drift upstream, thereby generating a negative Hall effect.

In the other limit, when **H** is perpendicular to the layers, an interesting question is whether Eq. (1) affects vortex motion. We ask if the negative Hall anomaly, in fact, reflects the competition between the interlayer segments and pancakes. If so, the anomaly should be sensitive to tilts of the field away from the c axis. We describe this test next. (In this geometry, **H** lies in the y-z plane at an angle θ with the c axis, as drawn in the inset of Fig. 2. The Hall contacts are aligned along **y**, so that they sense only the *in-plane* component of the electric field, i.e., the in-plane Hall resistivity $\rho_{xy} = B_z v_{Lx}/J$, where B_z is the z component of the B field.) As shown in Fig. 2, the profile of ρ_{xy} vs field is sensitive to slight deviations of θ from zero. At $\theta=0$, the profile is similar to previously published data [7,8], displaying a broad negative minimum near 3.7 T. However, if the field is tilted slightly, the Hall profile changes rapidly. The threshold field also decreases with increasing θ (from 3 T at $\theta=0$ to 2.6 T at 15°). At larger angles, the minimum continues to deepen, while the threshold field shifts rapidly to higher fields. These changes with θ are clarified considerably if we replot the data against $H_z = H\cos\theta$ rather than H (this compensates for the reduction in the flux density normal to the layers). As shown in Fig. 3 (main panel), all the curves merge into one curve when B_z exceeds 4.1 T. However, for $B_z < 4.1$ T, there remains strong θ dependence after the flux-density compensation (inset).

To analyze the remaining angular dependence, we plot the value at the minimum $\rho_{xy}^{\min}$ against θ (Fig. 4). At small angles, $\rho_{xy}^{\min}$ displays a sharp cusp (broken lines). From measurements on both twinned and untwinned crystals, we have determined that this cusp is associated with the presence of twin boundaries (no trace of it is seen in the untwinned crystals [16]). Twin boundaries present in the former act as strong pinning barriers to vortex motion when **H** is parallel to **c**, but they rapidly become ineffectual when θ deviates from zero. The cusp is the Hall analog of the resistivity cusp previously studied by Kwok and co-workers [17]. Our results demonstrate that depinning from the walls is accompanied by a change in ρ_{xy} towards *larger* negative values. In effect, removing the walls as barriers increases the upstream component of the freed vortices. The second prominent feature in Fig. 4 is the slower decrease of ρ_{xy} at angles

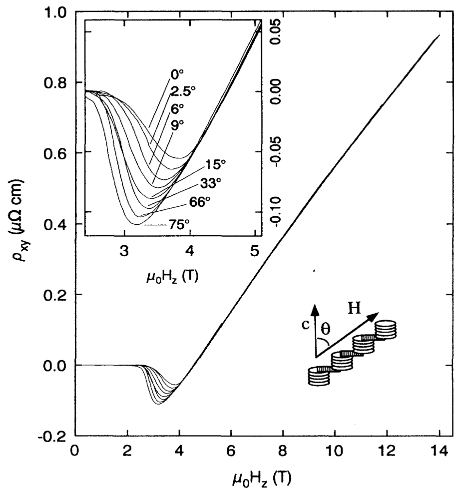

FIG. 3. (Main panel) A replot of the in-plane Hall resistivity ρ_{xy} versus the *normal* field component H_z at fixed angle θ. For values of H_z above 4.1 T, the curves merge, indicating that ρ_{xy} depends on H_z only. Below 4.1 T, ρ_{xy} is strongly affected by changes in θ (top inset). The lower inset shows the proposed vortex structure in an oblique field (stacks of vortex pancakes connected by interlayer segments).

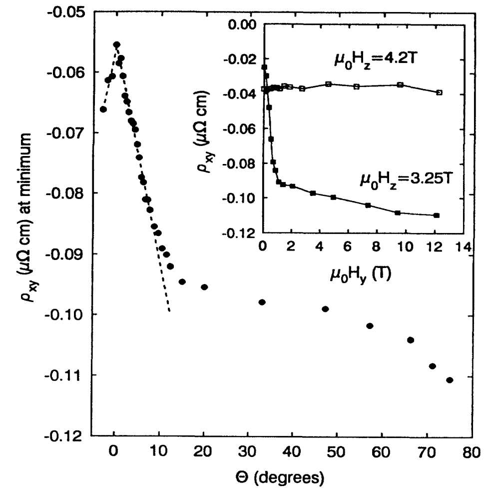

FIG. 4. (Main panel) The angular dependence of the minimum value of ρ_{xy} at 84 K. The steep cusp below 10° (broken lines) reflects depinning from twin boundaries (it is absent in untwinned crystals [16]). The slower decrease of ρ_{xy} at larger angles derives from the increasing upstream force on interlayer segments. The inset shows how ρ_{xy} varies with H_y at fixed H_z. At $B_z = 4.2$ T (open symbols), ρ_{xy} is independent of H_y, but at $B_z = 3.25$ T, it decreases steeply with increasing H_y (solid symbols).

larger than $\sim 10°$. In this regime, twin-wall pinning is insignificant, but ρ_{xy} continues to decrease, at a rate that accelerates as **H** approaches alignment with the layers ($\theta \rightarrow 90°$). Closely similar behavior is observed in untwinned crystals in this regime.

We argue that both the cusp near zero angle and the slower decrease at large angles are evidence for the strong influence of the interlayer segments. Let us start at the large-θ limit. Each flux line is almost aligned with the layers, but has a certain density of pancakes with average spacing fixed by B_z. The preponderance of interlayer segments causes the net Magnus force to point "upstream," so ρ_{xy} is negative (consistent with the curve at 71° in Fig. 2). As θ decreases, the relative population of the pancakes increases. Their positive contribution to the total Magnus forces causes ρ_{xy} to decrease in magnitude, in qualitative agreement with the behavior of $\rho_{xy}^{\min}$ from 75° to 15° in Fig. 4. In the absence of fluctuations, the relative population of the interlayer segments should vary as $\tan\theta$. However, at temperatures close to T_c, entropy causes each vortex line to fluctuate strongly about the average field direction. The fluctuations generate a significant population of interlayer segments that remains even when **H** is normal to the layers (this residual population rapidly decreases if we cool away from T_c). Hence, close to T_c, we expect the relative population of the interlayer segments to change rapidly at large angles, but to "saturate" to a constant value fixed by the fluctuations when θ decreases below $\sim 45°$.

The "cusp" behavior in twinned samples reflects the increased effectiveness of the twin walls as **H** approaches alignment with **c**. In this limit, the twin boundaries effectively suppress the largest transverse fluctuations of the vortex lines. Since this removes the largest contribution to the upstream Magnus force, we expect ρ_{xy} to increase sharply *in the positive direction* as $\theta \rightarrow 0$, in agreement with Fig. 4. Conversely, the cusp is not observed in untwinned crystals because the transverse fluctuations remain unconstrained as $\theta \rightarrow 0$. Hence, the model accounts for the behavior of ρ_{xy} over the full range of θ in both twinned and untwinned crystals. In retrospect, the presence of the cusp in twinned crystals explains why the magnitude of the Hall minimum (with **H**||**c**) is 4–6 times larger in untwinned YBCO crystals ($\rho_{xy} \sim -0.3$ to -0.45 $\mu\Omega$ cm [16,18]) than in twinned ones (-0.06 $\mu\Omega$ cm). As the temperature is decreased, it is known that the Hall anomaly rapidly diminishes (it goes away below ~ 79 K in both twinned and untwinned samples). This is consistent with suppression of the fluctuations (and the residual population of interlayer segments, as discussed above). A prediction of our model is that, at these temperatures, the negative Hall behavior should be resurrected if **H** is tilted by a large angle.

Returning to Fig. 3, we observe that ρ_{xy} collapses to one curve when B_z exceeds 4.1 T. This suggests that the rigidity length perpendicular to the layers is so short ($L_z \sim d$) that the insertion of more interlayer segments leaves ρ_{xy} unchanged. In this limit, the pancakes and interlayer segments are decoupled and the two independent populations drift in opposite directions along the current axis. Since the Hall contacts selectively detect the motion of the former while ignoring the latter, ρ_{xy} is unaffected by further increases in θ (aside from changes caused by reduction in B_z). To make this point, we replot in Fig. 4 (inset) ρ_{xy} versus the in-plane field component $H_y = H\sin\theta$, with H_z held constant. When H_z is fixed at 3.25 T (i.e., below 4.1 T), ρ_{xy} decreases dramatically with increasing H_y (solid symbols). This is consistent with vortex lines being pulled upstream by the increasing population of interlayer segments. However, when H_z exceeds 4.1 T (open symbols), ρ_{xy} becomes independent of H_y. There appears to be an abrupt decoupling of the two populations. An independent demonstration of the decoupling is provided by the six-lead Giaever-transformer experiment. Busch *et al.* [19] showed that vortex lines in $Bi_2Sr_2CaCu_2O_{8+\delta}$ break up when sheared by a strongly inhomogeneous current distribution. Repeating this experiment in YBCO with $\mathbf{H}\parallel\mathbf{c}$, we have confirmed that the breakup occurs at the same decoupling field (4.1 T at 84 K) [16].

The competition implied in Eq. (1) should also affect the angular dependence of out-of-plane Hall resistivity when **H** is nearly aligned with the layers ($\alpha < 20°$, where $\alpha = \pi/2 - \theta$). At alignment ($\alpha = 0$), the flux lines have a large upstream velocity component (Fig. 1). When **H** is tilted out of the plane, each flux line develops "kinks" (attached pancakes) which act like ballasts on the moving line, on account of the downstream force they experience. Thus, the velocity component along **J** (i.e., ρ_{xz}) should decrease *sharply* with increasing α. Our measurements confirm that ρ_{xz} indeed changes rapidly with α, in striking contrast to the $\cos\alpha$ dependence expected if the retarding force from the kinks were absent [20].

In summary, the angular dependence of ρ_{xy} strongly supports the picture that the negative Hall anomaly results from the competition between the Magnus forces acting on interlayer segments and pancakes in a vortex line. Near T_c strong fluctuations generate a significant population of the former even with the field perpendicular to the layers. Slight tilts of **H** away from **c** produce a "cusp" in ρ_{xy} in twinned crystals, but not in untwinned ones. We describe a model that qualitatively accounts for this observation, as well as the variation of ρ_{xy} over the full range of the angle θ. In this view, the negative anomaly derives directly from the striking sign difference between the Hall response of vortices that lie parallel to the layers and perpendicular to it.

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