Multiple Coexisting Orientations of Flux Lines in Superconductors with Uniaxial Anisotropy

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The self-energy and Gibbs free energy of rigid vortex lines are calculated for a uniaxially anisotropic London superconductor with a magnetic field applied at an angle ϕ from the $\hat{c}$ axis. When the correct elliptical core cutoff is used, the Gibbs free energy may have degenerate double minima as a function of vortex orientation. This indicates the coexistence of flux lines with different orientations at fields just above H_{c1}, applied at an angle $\phi \neq 0$ from the $\hat{c}$ axis. The regime of parameters H, ϕ, M_z/M, and κ where this occurs is mapped out.

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Bitter pattern decoration studies of magnetic flux lines emerging from surfaces of extreme type-II superconductors have recently been carried out intensively. Particularly fascinating are experiments on the high-T_c superconductor $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_8$ [1] where chains of flux lines embedded in an approximately regular hexagonal Abrikosov vortex lattice were seen when the external magnetic field was tilted with respect to the $\hat{c}$ axis. Theoretical scenarios for how such a situation could occur were recently proposed [2,3]: A possible explanation is coexistence of two types of flux lines in the anisotropic superconductor, one type oriented almost parallel to the $\hat{c}$ axis and another type oriented almost parallel to the *ab* planes.

In this paper we demonstrate that such a situation can indeed occur even within the relatively simple framework of anisotropic London theory [4]. We map out the parameter regime of magnitude and direction of applied field (H,ϕ), effective-mass anisotropy M_z/M, and Ginzburg-Landau parameter κ, where such an effect can occur by considering the Gibbs free energy of noninteracting flux lines, i.e., applied fields just above H_{c1} and thus very small inductions. Minima in the Gibbs energy as a function of vortex orientation may occur at *two* different orientations of the flux lines in the superconductor, for large enough anisotropy. Provided that these minima can be brought to be *degenerate*, flux lines may start penetrating the superconductor at $H_{c1}(\phi)$ with two different orientations θ, facilitating the coexistence of multiple "species" of vortices. These observations do not by themselves suffice to explain the experimental results of Bolle *et al.* [1] because the arrangement of these coexisting vortices will depend on their mutual interactions which we do not treat here. Nevertheless, this effect is quite interesting in its own right. For obtaining the desired parameter range it is essential to consider the correct elliptical shape of the vortex core cutoff in London theory [5,6]. We show that an incorrect circular cutoff gives a Gibbs free energy with only one minimum and hence only one permitted orientation of the flux lines for applied fields just above H_{c1}. The core cutoff also has consequences for the self-energy and line tension of a straight vortex line, and we begin by considering the former in some detail, as the self-energy will also be needed for the computation of the Gibbs free energy.

The general expression for the self-energy per unit length $J(\theta)$ of a rigid vortex line running at an arbitrary angle θ from the $\hat{c}$ axis in a uniaxially anisotropic London superconductor is given by [6]

$$J(\theta)=\frac{\Phi_0^2}{2\mu_0}\int\frac{d^2k_\perp}{4\pi^2}V_{zz}(\mathbf{k}_\perp)\,. \tag{1}$$

Here,

$$V_{zz}(\mathbf{k}_\perp)=(1+\lambda_\theta^2k_\perp^2)/[(1+\lambda_{ab}^2k_\perp^2)(1+\lambda_\theta^2k_x^2+\lambda_c^2k_y^2)]\,,$$

λ_{ab} and λ_c are the magnetic penetration lengths for currents along the *ab* plane and $\hat{c}$ axis, respectively, and $\lambda_\theta^2=\lambda_{ab}^2\sin^2\theta+\lambda_c^2\cos^2\theta$. The momenta $k_\perp$ and the x and y axes are oriented orthogonal to the vortex line. Within anisotropic London theory we have $\lambda_c/\lambda_{ab}=\sqrt{M_z/M}\equiv\Gamma=\xi_{ab}/\xi_c$ where ξ_{ab} and ξ_c are the superconducting coherence lengths in the *ab* plane and along the $\hat{c}$ direction, respectively. In general, the line tension of an isolated vortex line is related to the self-energy via the relation [6]

$$P(\theta)=J(\theta)+\frac{\partial^2J(\theta)}{\partial\theta^2}\,, \tag{2}$$

where the last term comes from the fact that in anisotropic superconductors, the self-energy of the vortex line depends explicitly on θ. Thus, contrary to the case of isotropic superconductors, the line tension, and hence the tilt modulus, is *not* simply proportional to the self-energy.

If an incorrect circular cutoff (circularly symmetric about the vortex axis) is used, one obtains from Eq. (1) the result

$$J(\theta)=J(0)\lambda_\theta/\lambda_c\,, \tag{3}$$

where $J(0)=\Phi_0^2\ln\kappa/4\pi\mu_0\lambda_{ab}^2$. Using Eq. (3) for the self-energy, one finds the following simple relation for the line tension of a rigid vortex line at arbitrary angle θ:

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$$P(\theta)=\frac{J(0)}{\Gamma^2}\left(\frac{\lambda_c}{\lambda_\theta}\right)^3=\frac{J(\theta)}{\Gamma^2}\left(\frac{\lambda_c}{\lambda_\theta}\right)^4 . \quad (4)$$

Thus, the sense of the angular dependence of $J(\theta)$ and $P(\theta)$ is *opposite* in this approximation, a result which also holds if the correct elliptical cutoff is used. In particular, the result Eq. (4) would imply that the tilt modulus of the single vortex vanishes as $1/\Gamma^2$ when $\Gamma\to\infty$ at $\theta=0$, whereas $P(\theta=\pi/2)=\Gamma J(0)$ diverges; i.e., the out-of-plane tilt modulus within the approximation (3) becomes infinite when $\Gamma\to\infty$. Although the assumption of rigid vortex lines in this limit very likely is unphysical, it is tempting to associate this large tilt modulus with a remnant of the "lock in" transition known from the Lawrence-Doniach theory and discussed by Feinberg [7].

In Eq. (1) the contributions from the large **k** region *dominate* the integral. We have seen that, for $\theta=0$, an isotropic cutoff leads to a vanishing tilt modulus of the single flux line along $\hat{c}$ in the limit $\lambda_c/\lambda_{ab}\to\infty$. Therefore, for large Γ any contribution from the correct *elliptical* cutoff is essential and renders the line tension finite even in the case of stacked pancake vortices without Josephson coupling [5,7,8]. As suggested in [4] the integration over k_y,k_x should be cut off on an ellipse with semimajor axis ξ_{ab}^{-1} and semiminor axis ξ_θ^{-1}, where $\xi_\theta^2=\xi_{ab}^2\cos^2\theta+\xi_c^2\sin^2\theta$. This elliptical core shape has now been *derived* from anisotropic Ginzburg-Landau theory [9] by utilizing the Klemm-Clem transformations [10]. When such an elliptical core cutoff is used, the resulting self-energy per unit length is given by

$$J(\theta)=J(0)\frac{\lambda_\theta}{\lambda_c}\nu(\theta;\Gamma,\kappa)\,,$$
$$\nu(\theta)=1+\frac{1}{\ln\kappa}\left[\ln\Gamma+\frac{\lambda_c^2\cos^2\theta}{\lambda_c^2\cos^2\theta+\lambda_\theta^2}\ln\left(\frac{\lambda_c^2+\lambda_\theta^2}{2\Gamma^2\lambda_\theta^2}\right)\right], \quad (5)$$

where the term in square brackets is due to the angular dependence of the anisotropy of the vortex core. If one regards Eq. (5) as the expansion of Eq. (1) in powers of $1/\ln\kappa$, and give the elliptical core cutoff, the expansion truncates exactly at linear order. Since Eq. (1) itself presumes the validity of London theory, our results are limited to large $\kappa\gtrsim 10$, the regime of interest in, e.g., high-T_c superconductors. The result of Eq. (5) was also obtained in a less transparent form in [4] and should be compared with Eq. (3). In the derivation of Eq. (5), we have made the approximations $\ln(1+\kappa^2\Gamma^2)\approx\ln(\kappa^2\Gamma^2)$, and $\ln[1+\kappa^2(\lambda_c^2+\lambda_\theta^2)/2\lambda_\theta^2]\approx\ln[\kappa^2(\lambda_c^2+\lambda_\theta^2)/2\lambda_\theta^2]$. This is generally valid in type-II superconductors with large κ.

In very hard and not too anisotropic superconductors such that $\ln\Gamma/\ln\kappa\ll 1$, the logarithmic terms in square brackets may be disregarded, and the result reduces to the one obtained when the anisotropy of the cutoff was neglected, Eq. (3) [11]. However, note that the θ-dependent part of $\nu(\theta)$ may show *nonmonotonicity* $\sim 1/\ln\kappa$ as a function of θ for large enough Γ, whereas λ_θ/λ_c is monotonically decreasing for $0\in[0,\pi/2]$. If $\ln\kappa$ is not too large and Γ is not too small, one may therefore have nonmonotonic behavior of $J(\theta)$ itself, due to the "squashing" of the vortex core as the flux lines are tilted from $\hat{c}$. The effect naively expected based on Eq. (3), that flux lines will reduce $J(\theta)$ by increasing θ, may be partially compensated by the θ-dependent logarithmic factor in Eq. (5). This fact reflects the cost in energy of deforming the vortex core as the flux line tilts from the $\hat{c}$ axis. It is this extra cost in energy which leads to a delicate cancellation in Eq. (2) and renders the tilt modulus finite even when $\Gamma\to\infty$ [5]. The parameter regime producing nonmonotonicity in $J(\theta)$ will be discussed in detail below.

The angular dependence of the anisotropic cutoff has important consequences for the Gibbs energy, which we now obtain for a gas of noninteracting vortices, i.e., H just above H_{c1}, and compare the results with those obtained using an isotropic cutoff. This will yield information about the angular dependence of the lower critical field, and of preferred vortex orientations at low inductions. In the following we choose a simple geometry which easily admits an exact treatment of demagnetization effects as the external field is tilted, namely, a cylindrical superconductor with the field applied perpendicular to the symmetry axis of the cylinder. Under such circumstances, the Gibbs energy per unit length at $T=0$ is given by

$$G(\theta)=nJ(\theta)-HB\cos(\theta-\phi)$$
$$=J(0)\frac{B}{\Phi_0}\left[\frac{\lambda_\theta}{\lambda_c}\nu(\theta;\Gamma,\kappa)-h\cos(\phi-\theta)\right], \quad (6)$$

where ϕ is the angle of the applied field relative to the $\hat{c}$ axis, $n=B/\Phi_0$, $h=H/H_0$, and $H_0=\Phi_0\ln\kappa/4\pi\mu\lambda_{ab}^2$.

The angular dependence of the Gibbs energy itself may be qualitatively different for the cases of isotropic and anisotropic cutoffs. Only *one* minimum exists for the case of isotropic cutoff, whereas a degenerate double minimum may exist when an anisotropic cutoff is used, as seen in Fig. 1. Observing such double minima requires fine tuning of the parameters (h,ϕ,κ,Γ), in a manner which will be detailed below. In particular, a threshold value of $\Gamma\gtrsim 7$ appears to be necessary to see the double minimum for $\kappa\geq 10$ when the second term in square brackets in Eq. (5) is included. The physical implications of this are quite interesting: It means that at a given magnitude and orientation of an externally applied magnetic field vortices may start penetrating the superconductor at *two* different angles, provided that the two minima in the Gibbs energy are *precisely* degenerate at $G(\theta)=0$. At least in the extreme low-induction regime, one may thus have a "two-component" vortex system characterized by two different orientations of the flux lines from the $\hat{c}$ axis. Such a situation may form a basis for explaining the coexistence of vortex chains and a hexagonal vortex lattice [2] which was recently observed in decoration experiments in tilted fields [1].

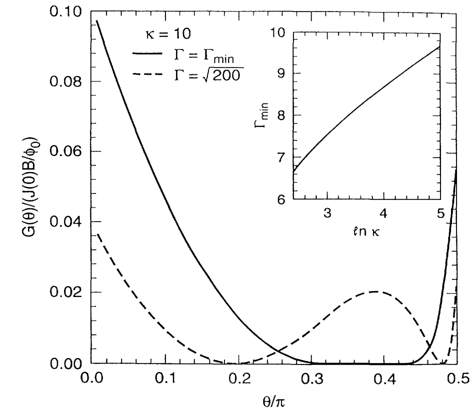

FIG. 1. Gibbs energy G as a function of vortex orientation with anisotropic core cutoff. The applied field H_{c1} and the orientation ϕ of the applied field are chosen to produce degenerate minima in G. Use of isotropic core cutoff gives one minimum in G, whereas anisotropic core cutoff gives the possibility of two (in general nondegenerate) minima in G. The inset shows the minimum penetration length anisotropy Γ necessary to see double minima in G, as a function of $\ln\kappa$.

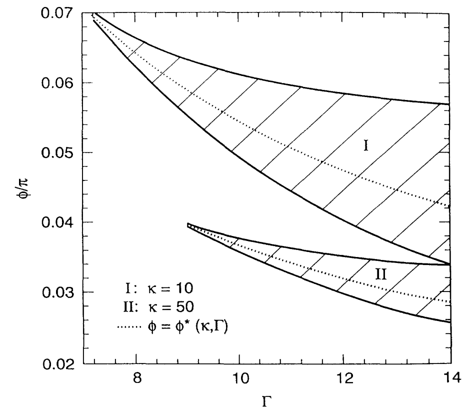

FIG. 2. The hatched regions show the ranges of ϕ where it is possible to see two (not necessarily degenerate) local minima in $G(\theta)$ as a function of Γ, for $\kappa=10$ and 50. The dotted lines in each hatched region indicate the values $\phi=\phi^*(\kappa,\Gamma)$ where the two minima in the Gibbs free energy are *precisely* degenerate at $G(\theta)=0$ in the Gibbs energy.

We shall now investigate the parameter regime (h,ϕ,Γ,κ) for which two distinct vortex orientations are degenerate minima in the Gibbs free energy at a given orientation of the external field ($H_{\rm ext}=H_{c1}$) in more detail. Consider the coupled equations that must be satisfied at the lower critical field H_{c1} for at least one vortex orientation θ,

$$G(\theta)=0;\quad \frac{\partial G(\theta)}{\partial\theta}=0\,. \tag{7}$$

They lead to the following region between ϕ and θ:

$$\tan(\phi-\theta)=-\frac{\lambda'_\theta}{\lambda_\theta}-\frac{v'(\theta;\Gamma,\kappa)}{v(\theta;\Gamma,\kappa)} \tag{8}$$

or equivalently

$$\begin{aligned}\tan\phi&=\Gamma^{-2}\tan\theta-\frac{c}{\cos\theta}\,\frac{(1-a\sin^2\theta)^2}{\cos\theta+c\sin\theta(1-a\sin^2\theta)}\\ &\equiv e^{W(\theta;\Gamma,\kappa)}\,,\end{aligned} \tag{9}$$

$$c\equiv-\frac{v'(\theta;\Gamma,\kappa)}{v(\theta;\Gamma,\kappa)},\quad a\equiv1-\Gamma^{-2},$$

where $\lambda'_\theta=\partial\lambda_\theta/\partial\theta$ and $v'=\partial v/\partial\theta$. In order to satisfy both Eqs. (7) the further constraint $h=\lambda_\theta v(\theta;\Gamma,\kappa)/\lambda_c \times\cos(\theta-\phi)$ between θ and ϕ must be considered. The well-known result for isotropic cutoff [11] is obtained by neglecting the second term on the right-hand side (RHS) of Eq. (9); this is equivalent to setting $v(\theta;\Gamma,\kappa)=1$ and implies that for $(\phi,\theta)\in[0,\pi/2]$ the mapping between ϕ and θ is one-to-one, since the first term on the RHS of Eq. (9) is a *monotonic* function of θ. This explains why only *one* solution $\theta(\phi)$ is found in this case, from which one would conclude that vortices of a unique orientation penetrate the superconductor for $H_{\rm ext}\geq H_{c1}$. For anisotropic cutoff this is not necessarily true since the second term on the RHS $\propto 1/\ln\kappa$ can be nonmonotonic, and multiple solutions θ for a given ϕ may thus be found.

Equation (9) is convenient as a starting point in searching for the region in parameter space (ϕ,Γ,κ) where more than one vortex orientation can occur at a given orientation ϕ of the external magnetic field. A *minimum* requirement to see this *at given* ϕ is that the RHS of Eq. (9) must be nonmonotonic as a function of θ; i.e., $\partial W(\theta;\Gamma,\kappa)/\partial\theta$ must change sign. Hence, $\min_\theta(\partial W/\partial\theta)=0$ determines the critical value of Γ at given κ that is required for multiple solutions of Eq. (9) to occur. This minimum value of $\Gamma\equiv\Gamma_{\min}$ is shown in the inset of Fig. 1 as a function of $\ln\kappa$, and is seen to increases slowly with κ. Because of assumptions in London theory we have restricted our search to $\kappa\geq10$. For $\Gamma>\Gamma_{\min}$, there are two values of θ that satisfy $\partial W/\partial\theta=0$, corresponding to two values of the angle ϕ: $\tan(\phi_i)=\exp[W(\theta_i;\Gamma,\kappa)]$. These two values of ϕ represent the smallest (largest θ) and largest (smallest θ) angles of the applied field where there is *more than one* local minimum of the Gibbs free energy satisfying Eq. (9). The situation is illustrated in Fig. 2, where the hatched regions show the relevant ranges of ϕ as a function of Γ for $\kappa=10$ and 50. It is seen that in general two local minima in the Gibbs energy may occur only in a fairly narrow range of small ϕ values. The *range* of ϕ decreases with increasing κ and increases with increasing Γ. The *values* of ϕ where this occurs decrease with increasing κ and Γ.

At this stage, we have determined a range of parameters (ϕ,Γ,κ) which will produce two local minima in $G(\theta)$ and hence *may* give rise to two coexisting vortex orientations in the superconductor. In general, however, these

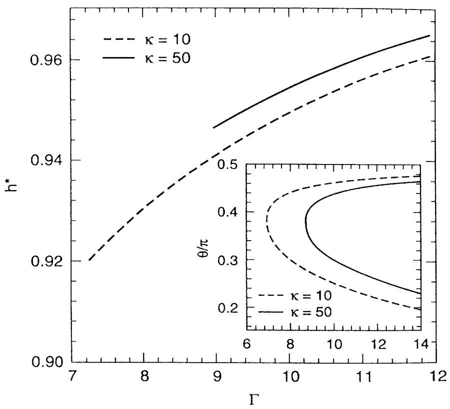

FIG. 3. Normalized field $h=h^*(\kappa,\Gamma)$ which gives two degenerate minima $G(\theta)=0$ in the Gibbs energy, as a function of Γ, for two values $\kappa=10$ and 50. The angle ϕ is chosen at each Γ,κ to be located on the dotted lines of Fig. 2; each $h^*(\kappa,\Gamma)$ terminates at the $\Gamma_{\min}(\kappa)$ shown in Fig. 1. The inset shows the two possible vortex orientations (θ_1,θ_2) vs Γ for $h=h^*$, $\phi=\phi^*$, and $\kappa=10$ and 50, i.e., for parameters such that $G(\theta)$ exhibits two *degenerate* minima $G(\theta)=0$. Note that $\theta_1\neq\theta_2$ is only possible *above* a critical value of Γ. Note also that both θ_1 and θ_2 exceed ϕ^*.

minima are nondegenerate; *multiple* solutions to Eq. (9) with the constraint $h=h^*=\lambda_\theta v(\theta;\Gamma,\kappa)/\lambda_c\cos(\theta-\phi)$ guarantee *one,* but not *two* values of θ such that $G(\theta)=0$. Two nondegenerate minima in $G(\theta)$ mean, as in the case of a single minimum, that only one vortex orientation is possible in the superconductor at applied fields just above H_{c1}. We next determine the parameters $h^*(\kappa,\Gamma),\phi^*(\kappa,\Gamma)$ that will give a minimum $G(\theta)=0$ for *two different vortex orientations* θ. This requires further adjustments of ϕ within the hatched regions shown in Fig. 2 and hence further adjustment of h^* according to the above constraint. Adjusting ϕ to the value ϕ^* satisfying the relation

$$\frac{\lambda_{\theta_1}v(\theta_1(\phi^*);\Gamma,\kappa)}{\lambda_c\cos[\theta_1(\phi^*)-\phi^*]}=\frac{\lambda_{\theta_2}v(\theta_2(\phi^*);\Gamma,\kappa)}{\lambda_c\cos[\theta_2(\phi^*)-\phi^*]} \tag{10}$$

gives two values of $\theta_1\neq\theta_2$ where $G(\theta)=0$; $\partial G/\partial\theta=0$; two examples are shown in Fig. 1. Specific values of $\phi^*(\kappa,\Gamma)$ which give two *degenerate absolute minima* $G(\theta_1)=G(\theta_2)=0$ are given by the dotted lines in Fig. 2 for $\kappa=10$ and 50. Figure 3 shows the corresponding normalized lower critical field h^* that allows two different vortex orientations to occur simultaneously. The inset shows the corresponding two possible vortex orientations (θ_1,θ_2) as a function of Γ, for $h=h^*,\phi=\phi^*$, and two values $\kappa=10$ and 50. Note that θ_1 and θ_2 meet at $\Gamma_{\min}(\kappa)$. By comparing $\theta^*(\Gamma)$, given by the dotted lines in Fig. 2, with the inset of Fig. 3 we conclude that in the case of coexistence neither of the vortex orientations is parallel to the applied field. Both "species" are tilted further away from the $\hat{c}$ axis than the applied field, i.e., $(\theta_1,\theta_2)>\phi^*$.

In summary: (i) Anisotropic London theory *with anisotropic core cutoff* yields the possibility of *coexistence* of flux lines at different θ in uniaxial anisotropic superconductors. This is due to an energy cost associated with the θ-dependent shape of vortex cores affecting the self-energy when it is calculated beyond "logarithmic" accuracy. It partially compensates the effect that flux lines lower their self-energy per unit length by orienting themselves along the *ab* planes at an inclination $\theta\gg\phi$ when $\Gamma\gg1$. (ii) The parameter regime (h,ϕ,Γ,κ) where the effect is seen has been mapped out in a cylindrical superconductor geometry which permits an exact treatment of demagnetization effects, and is found to occur in very anisotropic and not too hard type-II superconductors. (iii) The investigation is carried out at $H_{\rm ext}=H_{c1}$ and shows that no lower threshold value of the *induction* is needed to see coexistence. This implies that the vortex-vortex interaction in anisotropic superconductors is not essential in producing the coexistence. (It will, however, be responsible for destroying coexistence at large enough inductions.) (iv) When the angle $\phi\neq(0,\pi/2)$, no equilibrium vortex orientation is ever parallel to the externally applied field.

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Note added.— In a different context, an interesting cutoff-independent tilt instability of flux lines for vortex orientations within a range of special tilting angles from the $\hat{c}$ axis has recently been reported by Sardella and Moore [12]. The values of these special tilting angles are also determined by Γ and κ.

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