Scaling of the Critical Current in the Quantum Hall Effect: A Probe of Current Distribution

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The critical current I_c for the breakdown of the quantum Hall effect is found to scale *logarithmically* with the channel width for all Landau levels. We interpret this experimental result as a manifestation of a logarithmic distribution of the Hall potential across the channel. A new perspective on the breakdown mechanism is obtained. Inter-Landau-level transitions, previously thought to predict values of I_c much too high, are shown to be in quantitative agreement with experiments interpreted in this manner.

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The quantum Hall effect (QHE) [1,2] is a state of virtually dissipationless transport in a two-dimensional electron gas (2DEG) in a high magnetic field B. The effect occurs when the filling factor $\nu = nh/eB$ is approximately an integer, or an odd-denominator fraction [3]. Here n is the density of the 2DEG.

Shortly after the discovery of the QHE, it was observed [4–7] that when the current is increased beyond a certain threshold I_c, a sudden onset of dissipation occurs. Over the years, substantial effort has been invested in studying this so-called *breakdown* of the QHE. Experimentally, the phenomenon has been associated with hysteresis [4,5,8,9], instabilities and noise [5,6,8,10–12], and variations among different pairs of contacts on a given sample [13–15]. The reported values of the critical current *density* j_c, obtained simply by dividing I_c by the width of the channel W, vary over more than 2 orders of magnitude [16–18], though typically j_c is about 1 A/m. Furthermore, I_c was found to depend strongly on the precise value of ν within a plateau [4,8,19].

The mechanism causing breakdown remains largely controversial. Among the proposed theories, we briefly mention the models of a runaway heating instability [4,20], Zener transitions between Landau levels (LLs) [21], injection of hot electrons at the current contacts [12,14], and the threshold for emission of acoustical phonons [22–25]. A distinction is necessary between emissions with *intra*-LL and *inter*-LL transitions. For intra-LL transitions [22], the electron drift velocity $v_d = E/B$ must exceed the sound velocity v_s in the host material for such emissions to be kinematically possible. For inter-LL phonon emission processes, similarly to Zener transitions, a much higher value of E, and hence of j_c, is expected, due to the requirement of spatial overlap between initial and final states of different LLs but almost-equal energy [23].

The purpose of our work is to quantify the relation between I_c and W. This is a subject of great interest, since it is closely linked to the nontrivial distribution of currents and fields across the channel in the QHE [26]. However, surprisingly little is actually known about this relation [27,28]. The experiments we report show that I_c scales very sublinearly—apparently *logarithmically*—with W, for a variety of integer filling factors. We can interpret our results using a model for the current distribution across a homogeneous channel in the QHE. While this model suggests that the current tends to concentrate towards the channel boundaries, it should be emphasized that this is a macroscopically distributed current and not a quantum edge channel [26].

The measurements were performed at 40 mK on GaAs/AlGaAs heterojunctions with mobilities of 7 and 9×10^5 cm^2/V s and carrier concentrations of 2.6 and 2.1×10^{11} cm^2, respectively, patterned in Hall bars of different widths W ranging from 5 to 80 μm. W refers to the electrical width of the channels, determined by zero field resistance, which was within 0.5 μm of the lithographic width. In each set samples were closely patterned on the same chip to ensure maximal uniformity.

Figure 1 shows a typical current scan, with the longitudinal voltage drop V_{xx} plotted vs the dc current flowing through the channel. One can easily identify the breakdown point and determine I_c with an accuracy of a few percent. Because of the sensitivity of I_c to the precise filling factor as mentioned above, we first had to determine the value of B giving the *maximal* I_c. An alternative method (not shown) was to sweep B at a sequence of increasing values of the dc current, looking for the current at which the plateau of zero resistivity disappears. Both methods gave the same values for I_c.

It is known that sample geometry and the choice of voltage contacts used can often cause discrepancies in measured breakdown currents, whether upon reversal of the current direction or between voltage probes on opposite edges of the channel [14]. In order to determine values of I_c which could be reliably associated with W, extensive care was taken to separate such effects from our measurements. This was achieved by making long channels and by patterning *four* pairs of voltage contacts, as shown in the inset of Fig. 1. Only the two middle pairs were used to detect breakdown. The additional probes presumably helped establish an equilibrium distribution

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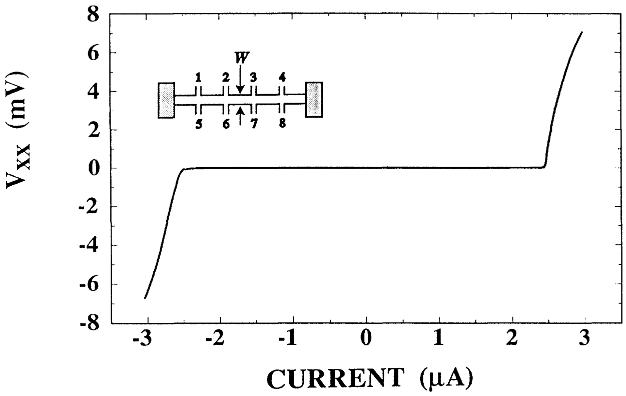

FIG. 1. Determination of the critical current at $\nu=2$ for one of our samples ($W=18\ \mu m$) at $T=40$ mK. The plot shows dc longitudinal voltage V_{xx} vs current at $B=5.05$ T. The voltage is measured between contacts 6 and 7. Breakdown occurs at $I=2.5\pm0.1\ \mu A$. Inset: A schematic view of the sample geometry. The shaded areas are the current contacts. The spacing between voltage probes is 150 μm.

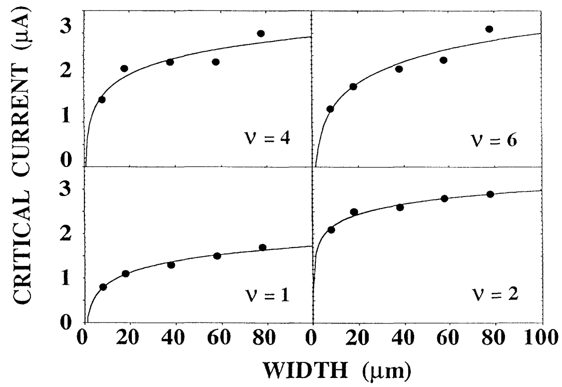

FIG. 2. Critical current vs the channel width at four different filling factors ν. The solid lines are logarithmic fits discussed in the text.

between the current injection contact and the measuring probes. Unlike simpler geometries we have used before, voltage-current characteristics measured in this configuration were indeed identical when measured with opposite voltage probes (V_{23} and V_{67}) and also symmetric in I, as seen in Fig. 1.

Figure 2 shows our results of I_c vs W for several filling factors. A distinct sublinear dependence can be seen for all filling factors. The solid line fit, of the form $I_c(W)=I_0\ln(W/W_0)$, will be motivated below. All sets of samples generally showed very similar behavior.

To interpret these findings, we begin by pointing out that theoretical models of breakdown determine a *critical electric field* E_c, transverse to the direction of current flow, which causes an abrupt rise in dissipation. Unlike the case of the Hall effect in three-dimensional conductors, in the QHE the transverse (Hall) field $E(x)$ in the 2DEG is *not* uniform [26]. This implies a nonuniform current distribution as well, since the current density in the QHE is just $j_y(x,y)=(\nu e^2/h)E_x(x,y)$.

The potential and current distribution in the QHE was studied by MacDonald, Rice, and Brinkman [29] and subsequently by many other workers [26]. For a uniform channel where the current flows in the y direction, and the transverse coordinate x spans from $-W/2$ to $W/2$, the potential $V=V(x)$ satisfies the self-consistency relation

$$V(x)=-\xi\int_{-W/2}^{W/2}dx'V''(x')\ln\left(\frac{2}{W}|x-x'|\right), \quad (1)$$

with $\xi\equiv l_B^2\nu/\pi a$. Here $l_B\equiv\sqrt{\hbar/eB}$ is the magnetic length, $a\equiv\epsilon\hbar^2/me^2$ is the effective Bohr radius, m is the effective mass, and ϵ is the dielectric constant. Thouless [30] solved this equation analytically, finding a logarithmic behavior of $V(x)$ for x far from the edges. Beenakker [26] approximated the near-edge behavior by introducing a cutoff at a distance ξ from the edges, using a *linear* extrapolation of $V(x)$ for $|x|>W/2-\xi$. With our experimental parameters, e.g., when $\nu=2$, we get $\xi\approx 8$ nm while $l_B\approx 11$ nm. Hence we cannot use Beenakker's cutoff, since (1) requires a uniformity at least on the scale of the spread of the wave function, which is $\sim N^{1/2}l_B$. Here N is the LL index, starting from $N=1$, so that $\nu=2N$ (or $2N-1$) for even (odd) filling factors. Applying a different cutoff length δ ($\neq\xi$) is also possible, but it requires a *nonlinear* extrapolation for $|x|>W/2-\delta$, and a different normalization factor, which depends on the precise form of the extrapolation. For illustration, we use simplest nonlinear extrapolation, which is quadratic, namely, $V''(x)=$const for $|x|>W/2-\delta$. We then obtain the solution

$$V(x)=\frac{IR_H}{2}\left(\ln\frac{W}{\delta}+\frac{\xi+\delta}{2\xi}\right)^{-1}\ln\left|\frac{x+W/2}{x-W/2}\right| \quad (2)$$

for $|x|<W/2-\delta$, with a smooth quadratic continuation to the edge. As an example, we plot this solution in Fig. 3 for $W=20\ \mu m$ and $\delta=11$ nm. Note that the parentheses in (2) contain just a normalization factor, to ensure that $V(W/2)-V(-W/2)=IR_H$. The $\ln(W/\delta)$ term, however, which is independent of the extrapolation scheme, will turn out to be the most important feature for our analysis. The details of the extrapolation scheme are of lesser importance; the critical choice is the value of cutoff length δ. In the following we take the tentative approximation $\delta=N^{1/2}l_B$. We add that evidence for such a nonuniform distribution of the Hall potential has recently been seen in a beautiful experiment by Fontein *et al.* [31].

We can now address the question of breakdown. The assumption is that once a field of E_c is attained at *some* point in the channel dissipation will commence. From (2) we can easily obtain the electric field at any point:

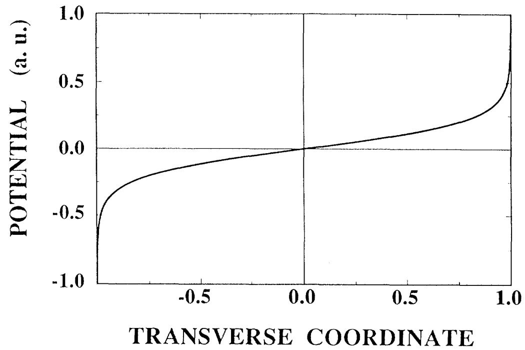

FIG. 3. The calculated potential profile across a 20 μm channel according to Eq. (3). The potential is normalized in units of $V_H/2$. The transverse coordinate is shown in units of half the channel width. The near-edge extrapolation is too small to be visually resolved.

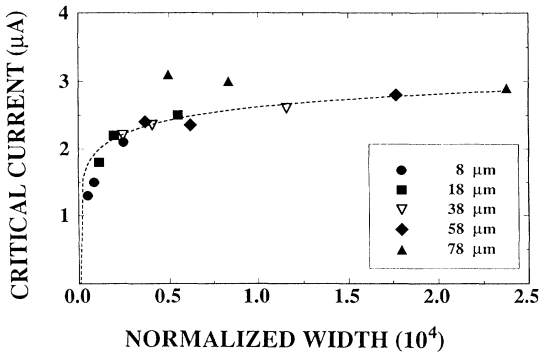

FIG. 4. Measured critical currents for five different widths and three LL indices $N=1$, 2, and 3, vs the normalized width $W/W_0(N)$. The dashed line shows the theoretical dependence $I_c=I_0\ln(W/W_0)$ for $I_0=0.3$ μA (i.e., $\gamma\approx 0.2$).

$$E(x)=V'(x)=\frac{IR_H}{2}\left[\ln\frac{W}{\delta}+\frac{\xi+\delta}{2\xi}\right]^{-1}\frac{W}{(W/2)^2-x^2} \quad \text{for } |x|<W/2-\delta\,. \tag{3}$$

Note that $E(x)$ continues to increase for $|x|>W/2-\delta$, and attains its highest value at the very edge of the channel. Nevertheless, the relevant field should *not* be taken there, but rather at some finite distance d from the edge. This is, once again, because the wave functions are spread in the x direction: E induces transitions from an initial state very near the edge, to a final state slightly further inwards. But even the outermost electrons have a wave function whose center is removed from the edge by about $N^{1/2}l_B$, and therefore the effective field experienced by the electron for such transitions must be evaluated at a distance $d\gtrsim N^{1/2}l_B$ from the edge. By substituting $x=W/2-d$ into (3), and using $d\ll W$, we get the relation between I_c and W,

$$I_c\approx\frac{2d}{R_H}\left[\ln\frac{W}{\delta}+\frac{\xi+\delta}{2\xi}\right]E_c(B)\,, \tag{4}$$

which can be written simply as

$$I_c(W)=I_0\ln(W/W_0)\,. \tag{5}$$

While I_0 and W_0 may have nontrivial dependence on B, N, and possibly on other sample parameters, the *width* dependence is always distinctly logarithmic. The result is quite independent of approximations.

The normalization width $W_0\cong\delta\exp[-(\delta+\xi)/2\xi]$ depends only on ξ and on the cutoff length δ. For our samples (namely, fixed $n=2.6\times10^{11}$ cm^{-2}), $\xi=8N^2$ (nm), and for δ we substitute $N^{1/2}l_B=11N$ (nm). The prefactor $I_0\equiv 2dE_c/R_H$ in Eq. (5) contains, through $E_c(B)$, all the physics of the breakdown mechanism, which is still controversial. Intra-LL transitions are difficult to reconcile with a homogeneous filled LL at such low temperatures. An *inter*-LL phonon-emission mechanism, first studied by Heinonen, Taylor, and Girvin [23] and Smrcka [24], has more intuitive appeal, but was considered in discrepancy with experiments [23], a discrepancy we propose to reexamine. They found that the rate of such transitions (from the lowest LL) depends exponentially on the quantity $m(v_d-v_s)^2/\hbar\omega_c$, so that dissipation becomes significant when it attains a value of order 1 (here $\omega_c\equiv eB/m$ is the cyclotron frequency). This is equivalent to $E\sim\hbar\omega_c/el_B$, namely, a potential drop equal to the cyclotron energy over a distance corresponding to the extent of the wave function. We generalize their result for the Nth LL by replacing l_B with $N^{1/2}l_B$. Thus, for a fixed density 2DEG, $E_c=\gamma\hbar\omega_c/eN^{1/2}l_B$ where γ is a numerical factor of order 1. Taking $d=N^{1/2}l_B$ as proposed before, we obtain $I_0=\gamma(2e\hbar n/m)=1.4\gamma$ (μA), with $n=2.6\times10^{11}$ cm^{-2} and the effective mass in GaAs. Note that I_0 is independent of N.

We can now plot the measured values of I_c vs $W/W_0(N)$, shown in Fig. 4. This plot combines data for all widths and all LL indices, and is an important test of the preceding analysis. The solid line is simply the function $I_0\ln(x)$, with the single fitting parameter $\gamma=I_0/1.4$ μA. The fit yields a value of $\gamma\approx0.2$, which is in good agreement with calculations [23,24]. It is interesting to note that, historically, the main shortcoming of the inter-LL theories was that the predictions were over an order of magnitude above experimental values of j_c deduced by the naive substitution $j_c=I_c/W$.

Despite the appealing agreement with our results, the cutoff length needs careful consideration, taking into account the form of the confining potential at the channel edge. Recent theoretical work [32] suggests that, in the presence of a magnetic field, the *equilibrium* edge potential attains nontrivial structure, though its consequences for large currents, namely the Hall effect, are not yet fully understood. Results from our numerical calculations suggest that a soft edge does not change the Hall potential distribution.

So far we have ignored spin and odd filling factors. As seen in Fig. 2, the logarithmic width dependence is observed for odd ν as well (e.g., $\nu=1$). An analogous quantitative analysis must consider the Zeeman gap instead of the LL gap, and in fact these measurements can be used to *evaluate* the effective gyromagnetic ratio (g factor). This will be the subject of a future report.

Finally, it is not clear yet to what extent our analysis and conclusions can be applied to other published data. The results of Haug, von Klitzing, and Ploog [27] on Hall bars seem to agree with our conclusions, unlike those of Kawaji, Hirakawa, and Nagata [28]. Furthermore, we do not know how breakdown currents of order 100 μA and more (e.g., Refs. [4-7]) can be accounted for in the framework of this model. Most samples reported were significantly wider than ours, and it is possible that inhomogeneities play a more important role for wider samples. Heuristically, if one thinks of inhomogeneity in terms of dividing a wide channel into many narrow ones in parallel, it may increase the total critical current, due to the inherently sublinear width dependence per channel. Furthermore, it may be important that our samples had untypically high mobility compared to most other works on breakdown.

In conclusion, we have observed a systematic logarithmic relation between the channel width and the critical current in the QHE, which is a result, indeed evidence, of the unusual current distribution in a Hall-bar geometry. This distribution has been overlooked in past analyses of the breakdown of the QHE, and our conclusions shed new light on the mechanism causing dissipation. In particular, the model of inter-Landau-level transitions may actually be in good quantitative agreement with experiments.

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